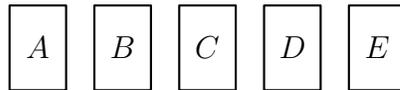


- (1) _____ A dime, 2 nickels, and 3 pennies are in a container. Assume that it is equally likely to shake out any one coin. What is the probability of shaking out a penny each of 4 times if the coin is returned after each shake? Express your answer as a common fraction.
- (2) _____ How many positive integers have cube roots less than $1 + \sqrt{2}$?
- (3) _____ What is the probability that the first three cards drawn from a standard deck of playing cards will all be two's? Express your answer as a common fraction.
- (4) _____ A coin machine returns 5 pennies for every nickel inserted and 5 nickels for every penny inserted. If Ashley starts with one nickel and no pennies, how many coins will she have after using the machine 7 times? Does the answer depend on which coins are used?
- (5) _____ A rectangular field is 50 feet wide and 80 feet long. Fence posts are placed every 10 feet around the field. How many posts are needed?
- (6) _____ Using the digits 1, 2, 3, 4, and 5, how many five-digit positive integers can be formed if no digit can be repeated?
- (7) _____ How many ways can a committee of four members be selected from a club consisting of 6 members?
- (8) _____ In his locker, Andrew has 2 history books and 3 math books. In his rush to get to class, he grabs 1 book, then a second book, without stopping to look. What is the probability that he pulls a math book out first and a history book out second? Express your answer as a common fraction.
- (9) _____ If all multiples of 4 and all multiples of 5 are removed from the set of integers from 1 through 100, how many integers remain?

- (10) _____ What is the maximum number of points of intersection when 5 lines intersect each other?
- (11) _____ Hazel and Basil are playing a game in which either is equally likely to win any given point. Basil has 4 points to Hazel's 3 points. If the first person to get 5 points is the winner, what is the probability that Basil will win?
- (12) _____ In a classroom there are 6 rows of 5 desks each. What percent of the seats does a student have to choose from if he does not want to sit in the first or last seat of any row or in either of the two side rows?
- (13) _____ What is the probability of drawing a card from those pictured where the letter is the first letter of a month of the year? Express your answer as a common fraction.



- (14) _____ Massaru has 3 pencils in his bookbag. Two pencils are blue and one is red. If he randomly selects two pencils, what is the probability that they are the same color? Express your answer as a common fraction.
- (15) _____ Every member of a math club is taking algebra or geometry and 8 are taking both. If there are 17 taking algebra and 13 taking geometry, how many members are in the club.
- (16) _____ Compute: $\frac{7!}{6!} \cdot \frac{3!2!}{(8-1)!}$. Express your answer as a common fraction.
- (17) _____ A game is played with five cards numbered 3, 6, 2, 5, and 1. Face down, the cards are shuffled and then three are turned face up. Your score is the sum of the three numbers showing. What is the probability that your score will be 9?
- (18) _____ How many three-digit numbers do not contain a zero?

- (19) _____ If a marble is chosen from a bag that contains 10 red marbles, 5 blue marbles and 15 white marbles, what is the probability that the marble chosen is blue or red? Express your answer as a common fraction.
- (20) _____ What is the least number of coins you can have and still pay exactly the cost of any purchase less than one dollar?
- (21) _____ The digits 2, 3, 4, 7, and 8 form a five-digit number. The tens digit is odd and the number is divisible by 4. How many numbers are possible?
- (22) _____ What fraction of the multiples of 5 between 1 and 99 are also multiples of 2?
- (23) _____ If two numbers are selected randomly without replacement from the set $\{1, 2, 3, 4, 5, 6\}$, what is the probability that their product will be greater than 15? Express your answer as a common fraction.
- (24) _____ Using one kind of cheese, one kind of meat, and one kind of bread, how many different sandwiches can be made from the following:
Bread: rye, white, wheat, oatmeal
Cheese: Cheddar, swiss
Meat: bologna, turkey, ham
- (25) _____ John's average for making free throws in a basketball game is .80. In a one-and-one free throw situation (he shoots a second basket only if he makes the first), what is the probability that he makes both baskets? Express your answer as a decimal.
- (26) _____ Suppose you toss a coin and get heads five times in a row. What is the probability, expressed as a common fraction, that you will get heads on the sixth toss?
- (27) _____ Simplify: $\frac{8!}{3!4!}$

- (28) _____ On a digital clock showing hours and minutes, how many different readings between noon and midnight contain at least two 3's?
- (29) _____ How many different six-digit numbers can be formed using three 5's, two 4's, and one 6?
- (30) _____ You are buying 15 doughnuts for your MATHCOUNTS team and coach. If the doughnuts are packaged singly or in sets of 3 or 4, in how many different ways can you select your order?

Answer Sheet

| Number | Answer | Problem ID |
|--------|---------------|------------|
| 1 | $1/16$ | 1BAA1 |
| 2 | 14 | 2BB41 |
| 3 | $1/5525$ | B03B |
| 4 | 29 coins, no | D00C |
| 5 | 26 posts | 015B |
| 6 | 120 | 515B |
| 7 | 15 | CBCA |
| 8 | $3/10$ | 323A1 |
| 9 | 60 integers | BCD22 |
| 10 | 10 | ACB02 |
| 11 | $\frac{3}{4}$ | BA3A1 |
| 12 | 40 percent | 4C3B |
| 13 | $2/5$ | BB0B |
| 14 | $\frac{1}{3}$ | 02B41 |
| 15 | 22 members | B4D41 |
| 16 | $1/60$ | 143B |
| 17 | $1/5$ | DACB |
| 18 | 729 | 225A1 |
| 19 | $1/2$ | C2C22 |
| 20 | 9 coins | 3ACB |
| 21 | 12 numbers | 45AB |
| 22 | $9/19$ | 455B |
| 23 | $4/15$ | 2BCB |
| 24 | 24 | 223A1 |
| 25 | .64 | CBC51 |
| 26 | $1/2$ | 5AAB |
| 27 | 280 | 5ABB |
| 28 | 26 | B02B |
| 29 | 60 numbers | 4A2B |
| 30 | 15 ways | D15B |

Solutions

(1) **1/16** ID: [1BAA1]

No solution is available at this time.

(2) **14** ID: [2BB41]

No solution is available at this time.

(3) **1/5525** ID: [B03B]

No solution is available at this time.

(4) **29 coins, no** ID: [D00C]

No solution is available at this time.

(5) **26 posts** ID: [015B]

No solution is available at this time.

(6) **120** ID: [515B]

No solution is available at this time.

(7) **15** ID: [CBCA]

No solution is available at this time.

(8) **3/10** ID: [323A1]

No solution is available at this time.

(9) **60 integers** ID: [BCD22]

No solution is available at this time.

(10) **10** ID: [ACB02]

The maximum occurs when every pair of lines yields a new point of intersection. There are $\binom{5}{2} = \boxed{10}$ ways to choose these pairs.

(11) $\frac{3}{4}$ ID: [BA3A1]
No solution is available at this time.

(12) **40 percent** ID: [4C3B]
No solution is available at this time.

(13) **2/5** ID: [BB0B]
No solution is available at this time.

(14) $\frac{1}{3}$ ID: [02B41]
Massaru has three ways of choosing two pencils. He can either choose two blue pencils, the red pencil and the first blue pencil, or the red pencil and the second blue pencil. Thus, the probability that the two pencils he chooses are the same color is $\frac{1}{3}$.

(15) **22 members** ID: [B4D41]
No solution is available at this time.

(16) **1/60** ID: [143B]
No solution is available at this time.

(17) **1/5** ID: [DACB]
No solution is available at this time.

(18) **729** ID: [225A1]
No solution is available at this time.

(19) **1/2** ID: [C2C22]
No solution is available at this time.

(20) **9 coins** ID: [3ACB]
No solution is available at this time.

(21) **12 numbers** ID: [45AB]

No solution is available at this time.

(22) **9/19** ID: [455B]

No solution is available at this time.

(23) **4/15** ID: [2BCB]

No solution is available at this time.

(24) **24** ID: [223A1]

No solution is available at this time.

(25) **.64** ID: [CBC51]

Since John's average for making free throws is .80, the probability that he makes the first shot is .80. If he makes this shot, he will then get a chance to shoot a second basket. The probability that he makes this second basket (given that he already made the first one) is still .80, so the probability that he makes both baskets is $(.80)(.80) = \boxed{.64}$.

(26) **1/2** ID: [5AAB]

No solution is available at this time.

(27) **280** ID: [5ABB]

No solution is available at this time.

(28) **26** ID: [B02B]

We can divide the number of ways the clock can contain two 3s into three cases.

Case I: The clock reads $x:33$

From noon to midnight, there are twelve hours (that is, x can range from 1 to 12).

Thus, this case gives 12 such readings.

Case II: The clock reads $3:x3$

Because there are 60 minutes in a hour, x can range from 0 to 5, giving 6 such readings. However, note that we have already counted the $3:33$ reading. Thus, this case yields 5 new readings.

Case III: The clock reads $3:3x$

Here, x can range from 0 to 9, giving 10 readings. However, again, the $3:33$ reading has been counted. So, this case gives 9 new readings.

Thus, there are $12 + 5 + 9 = \boxed{26}$ different readings.

(29) **60 numbers** ID: [4A2B]

No solution is available at this time.

(30) **15 ways** ID: [D15B]

No solution is available at this time.