LESSON

5-4 Ratio and Variation

OVERVIEW

*A **ratio** is a comparison by division of two quantities that are measured in the same units. For example, if Mary is 16 years old and her brother Gary is 8 years old, then Mary is 2 times as old as Gary. The ratio of Mary's age to Gary's age is 2 : 1 (read as "2 to 1") since*

$$\frac{\text{Mary's age}}{\text{Gary's age}} = \frac{16 \text{ years}}{8 \text{ years}} = \frac{2}{1} \quad \text{or} \quad 2:1$$

One quantity may be related to another quantity so that either the ratio or product of these quantities always remains the same.

RATIO OF *a* TO *b*

The ratio of a to b ($b \neq 0$) is the fraction $\frac{a}{b}$, which can be written as $a : b$ (read as "a is to b").

EXAMPLE: The ratio of the number of girls to the number of boys in a certain class is 3 : 5. If there is a total of 32 students in the class, how many girls are in the class?

SOLUTION: Since the number of girls is a multiple of 3 and the number of boys is the same multiple of 5, let

$3x$ = the number of girls in the class
and $5x$ = the number of boys in the class

Then

$$3x + 5x = 32$$

$$8x = 32$$

$$x = \frac{32}{8} = 4$$

The number of girls = $3x = 3(4) = 12$.

RATIO OF a TO b TO c

If $a : b$ represents the ratio of A to B and $b : c$ represents the ratio of B to C, then the ratio of A to C is $a : c$, provided that b stands for the same number in both ratios. For example, if the ratio of A to B is 3 : 5 and the ratio of B to C is 5 : 7, then the ratio of A to C is 3 : 7. In this case B represents the number 5 in both ratios.

EXAMPLE: If the ratio of A to B is 3 : 5 and the ratio of B to C is 2 : 7, what is the ratio of A to C?

SOLUTION: Change each ratio into an equivalent ratio in which the term that corresponds to B is the same number.

- The ratio of A to B is 3 : 5, so the term corresponding to B in this ratio is 5. The ratio of B to C is 2 : 7, so the term corresponding to B in this ratio is 2.
- The least common multiple of 5 and 2 is 10. You need to change each ratio into an equivalent ratio in which the term corresponding to B is 10.
- Multiplying each term of the ratio 3 : 5 by 2 gives the equivalent ratio 6 : 10. Multiplying each term of the ratio 2 : 7 by 5 gives 10 : 35.
- Since the ratio of A to B is equivalent to 6 : 10 and the ratio of B to C is equivalent to 10 : 35, the ratio of A to C is 6 : 35.

DIRECT VARIATION

If two quantities change in value so that their ratio always remains the same, then one quantity is said to vary **directly** with the other. When one quantity varies directly with another quantity, a change in one causes a change in the other in the same direction—both increase or both decrease.

EXAMPLE: If 28 pennies weigh 42 grams, what is the weight in grams of 50 pennies?

SOLUTION: The number of pennies and their weight vary directly since multiplying one of the two quantities of pennies by a constant causes the other to be multiplied by the same constant. If x represents the weight in grams of 50 pennies, then

$$\frac{\text{Pennies}}{\text{Grams}} = \frac{28}{42} = \frac{50}{x}$$

Cross-multiply:

$$28x = 42(50)$$

$$x = \frac{2100}{28} = 75$$

The weight of 50 pennies is 75 grams.

INVERSE VARIATION

If two quantities change in opposite directions, so that their product always remains the same, then one quantity is said to vary **inversely** with the other.

EXAMPLE: Four workers can build a house in 9 days. How many days would it take 3 workers to build the same house?

SOLUTION: As the number of people working on the house *decreases*, the number of days needed to build the house *increases*. Since this is an inverse variation, the number of workers times the number of days needed to build the house stays constant.

If d represents the number of days that 3 workers take to build the house, then

$$3 \times d = 4 \times 9$$

$$3d = 36$$

$$d = \frac{36}{3} = 12$$

Three people working together would take 12 days to build the house.

LESSON 5-4 TUNE-UP EXERCISES

Multiple Choice

1 A recipe for 4 servings requires salt and pepper to be added in the ratio of 2 : 3. If the recipe is adjusted from 4 to 8 servings, what is the ratio of the salt and pepper that must now be added?

(A) 4 : 3
(B) 2 : 6
(C) 2 : 3
(D) 3 : 2
(E) 8 : 4

2 On a certain map, $\frac{3}{8}$ of an inch represents 120 miles. How many miles does $1\frac{3}{4}$ inches represent?

(A) 300
(B) 360
(C) 400
(D) 480
(E) 560

3 The population of a bacteria culture doubles in number every 12 minutes. The ratio of the number of bacteria at the end of 1 hour to the number of bacteria at the beginning of that hour is

(A) 64 : 1
(B) 60 : 1
(C) 32 : 1
(D) 16 : 1
(E) 8 : 1

4 At the end of the season, the ratio of the number of games a team has won to the number of games it lost is 4 : 3. If the team won 12 games and each game played ended in either a win or a loss, how many games did the team play during the season?

(A) 9
(B) 15
(C) 18
(D) 21
(E) 24

5 The ratio $3\frac{1}{2} : 8$ is equivalent to which of the following ratios?

(A) $\frac{1}{2} : \frac{8}{3}$
(B) 7 : 16
(C) 2 : 5
(D) $8 : 3\frac{1}{2}$
(E) 7 : 8

6 A school club includes only sophomores, juniors, and seniors, in the ratio of 1 : 3 : 2. If the club has 42 members, how many seniors are in the club?

(A) 6
(B) 7
(C) 12
(D) 14
(E) 21

7 If $\frac{c - 3d}{4} = \frac{d}{2}$, what is the ratio of c to d?

(A) 5 : 1
(B) 3 : 2
(C) 4 : 3
(D) 3 : 4
(E) 2 : 3

8 If 4 pairs of socks costs \$10.00, how many pairs of socks can be purchased for \$22.50?

(A) 5
(B) 7
(C) 8
(D) 9
(E) 10

9 Two boys can paint a fence in 5 hours. How many hours would it take 3 boys to paint the same fence?

(A) $\frac{3}{2}$
(B) 3
(C) $3\frac{1}{3}$
(D) $4\frac{2}{3}$
(E) $7\frac{1}{2}$

10 A car moving at a constant rate travels 96 miles in 2 hours. If the car maintains this rate, how many miles will the car travel in 5 hours?

(A) 480
(B) 240
(C) 210
(D) 192
(E) 144

11 The number of kilograms of corn needed to feed 5000 chickens is 30 less than twice the number of kilograms needed to feed 2800 chickens. How many kilograms of corn are needed to feed 2800 chickens?

(A) 70
(B) 110
(C) 140
(D) 190
(E) 250

12 In an ordered list of five consecutive positive even integers, the ratio of the greatest integer to the least integer is 2 to 1. Which of the following is the middle integer in the list?

(A) 10
(B) 12
(C) 14
(D) 16
(E) 18

13 If the ratio of p to q is 3 : 2, what is the ratio of $2p$ to q?

(A) 1 : 3
(B) 2 : 3
(C) 3 : 3
(D) 3 : 1
(E) 3 : 4

14 $$\frac{x}{z} = \frac{1}{3}$$

If in the equation above x and z are integers, which are possible values of $\frac{x^2}{z}$?

I. $\frac{1}{9}$
II. $\frac{1}{3}$
III. 3

(A) II only
(B) III only
(C) I and III only
(D) II and III only
(E) None

15 If $a - 3b = 9b - 7a$, then the ratio of a to b is

(A) 3 : 2
(B) 2 : 3
(C) 3 : 4
(D) 4 : 3
(E) 1 : 2

16 The ratio of A to B is $a : 8$, and the ratio of B to C is $12 : c$. If the ratio of A to C is $2 : 1$, what is the ratio of a to c?

(A) 2 : 3
(B) 3 : 2
(C) 4 : 3
(D) 3 : 4
(E) 1 : 3

17 If $8^r = 4^t$, the ratio of r to t is

(A) 2 : 3
(B) 3 : 2
(C) 4 : 3
(D) 3 : 4
(E) 1 : 2

18 If $\frac{a+b}{b} = 4$ and $\frac{a+c}{c} = 3$, what is the ratio of c to b?

(A) 2 : 3
(B) 3 : 2
(C) 2 : 1
(D) 3 : 1
(E) 6 : 1

19 In a certain college, the ratio of mathematics majors to English majors is 3 : 8. If in the following school year the number of mathematics majors increases 20% and the number of English majors decreases 15%, what is the new ratio of mathematics majors to English majors?

(A) 4 : 9
(B) 1 : 2
(C) 9 : 17
(D) 17 : 32
(E) 7 : 12

20 At a college basketball game, the ratio of the number of freshmen who attended to the number of juniors who attended is 3 : 4. The ratio of the number of juniors who attended to the number of seniors who attended is 7 : 6. What is the ratio of the number of freshmen to the number of seniors who attended the basketball game?

(A) 7 : 8
(B) 3 : 4
(C) 2 : 3
(D) 1 : 2
(E) 1 : 3

21 Given that y varies inversely as x and x varies directly as z, if z is doubled, then y is:

(A) divided by 4
(B) divided by 2
(C) multiplied by 2
(D) multiplied by 4
(E) unchanged

22 It took 12 men 5 hours to build an airstrip. Working at the same rate, how many additional men could have been hired in order for the job to have taken 1 hour less?

(A) two
(B) three
(C) four
(D) six
(E) eight

23 The price per person to rent a limousine for a prom varies inversely as the number of passengers. If nine people rent the limousine, the cost is \$70 each. How many people are renting the limousine when the cost *per couple* is \$105?

(A) 4
(B) 6
(C) 8
(D) 12
(E) 16

Grid In

1 A string is cut into 2 pieces that have lengths in the ratio of 2 : 9. If the difference between the lengths of the 2 pieces of string is 42 inches, what is the length in inches of the shorter piece?

2 The ratio of a to b is 5 : 9, and the ratio of x to y is 10 : 3. The ratio of ay to bx is equivalent to the ratio of 1 to what number?

3 The ratio of dimes to pennies in a purse is 3 : 4. If 3 pennies are taken out of the purse, the ratio of dimes to pennies becomes 1 : 1. How many dimes are in the purse?

4 For integer values of a and b, $b^a = 8$. The ratio of a to b is equivalent to the ratio of c to d, where c and d are integers. What is the value of c when $d = 10$?

5 If $6a - 8b = 0$ and $c = 12b$, the ratio of a to c is equivalent to the ratio of 1 to what number?

6 Jars A, B, and C each contain 8 marbles. What is the minimum number of marbles that must be transferred among the jars so that the ratio of the number of marbles in jar A to the number in jar B to the number in jar C is 1 : 2 : 3?