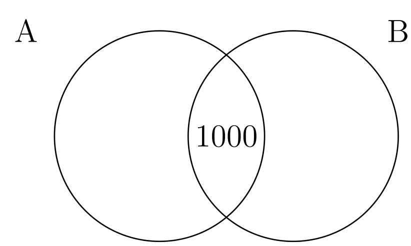
(1) ________ What is the probability of rolling six standard, six-sided dice and getting six distinct numbers? Express your answer as a common fraction.

(2) ________ If a committee of six students is chosen at random from a group of six boys and four girls, what is the probability that the committee contains the same number of boys and girls? Express your answer as a common fraction.

(3) ________ How many three-letter arrangements can be made if the first and third letters each must be one of the 21 consonants, and the middle (second) letter must be one of the five vowels? Two such arrangements are KOM and XAX.

(4) ________ A local restaurant boasts that they have 240 different dinner combinations. A dinner combination consists of an appetizer, entree, and dessert. If the restaurant offers 10 appetizer choices and 6 entree choices, how many different dessert choices does it have?

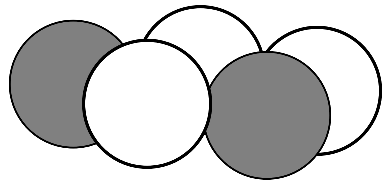
(5) ________ Sets A and B, shown in the Venn diagram, are such that the total number of elements in set A is twice the total number of elements in set B. Altogether, there are 3011 elements in the union of A and B, and their intersection has 1000 elements. What is the total number of elements in set A?



(6) ________ Bob and Tom each roll two standard six-sided dice. What is the probability that the product of their individual sums is greater than 100? Express your answer as a common fraction.

(7) ________ If a fly is buzzing randomly around a room 8 ft long, 12 ft wide and 10 ft high, what is the probability that, at any given time, the fly is within 6 feet of the ceiling? Express your answer as a common fraction.

(8) ________ Bag A contains 3 white and 2 red balls. Bag B contains 6 white and 3 red balls. One of the two bags will be chosen at random, and then two balls will be drawn from that bag at random without replacement. What is the probability that the two balls drawn will be the same color? Express your answer as a common fraction.



(9) ________ How many positive, three-digit integers contain at least one 3 as a digit but do not contain a 5 as a digit?

(10) ________ A pile of marbles has $15^5 + 3$ marbles. It is to be divided into 7 piles with the same number of marbles in each pile. If there are x marbles left over ($0 \leq x < 7$), what is the value of x?

(11) ________ Four standard, six-sided dice are to be rolled. If the product of their values turns out to be an even number, what is the probability their sum is odd? Express your answer as a common fraction.

(12) ________ Elmo is making three-letter "words" using alphabet blocks. Each of the three blocks has a different letter on each of three faces and pictures on the remaining three faces. How many three-letter "words" can Elmo form if none of the blocks have any letters in common?

(13) ________ A bag contains exactly six balls; two are red and four are green. Sam randomly selects one of the six balls and puts it on the table. Then he randomly selects one of the five remaining balls. What is the probability that the two selected balls are of different colors? Express your answer as a common fraction.

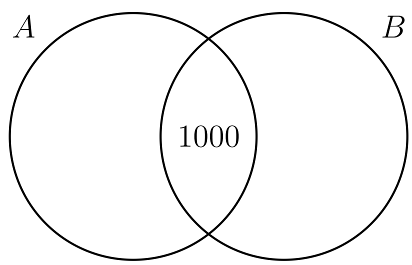
(14) ________ For how many positive values of n are both $\frac{n}{3}$ and $3n$ four-digit integers?

(15) __________ Ms. Marks is selecting students for a committee that must consist of three seniors and five juniors. Six senior volunteers are able to serve on the committee. What is the least number of junior volunteers needed if Ms. Marks wants at least 100 different possible ways to pick the committee?

(16) __________ The enrollment at West Middle School is 450 students. If 300 students participate in sports, 125 students participate in fine arts and 50 students participate in both sports and fine arts, how many students do not participate in either sports or fine arts?

(17) __________ How many subsets of $\{1, 2, 3, 4, 5, 6\}$ have either 4 or 5 as their largest element?

(18) __________ Sets A and B, shown in the Venn diagram, are such that the total number of elements in set A is twice the total number of elements in set B. Altogether, there are 3011 elements in the union of A and B, and their intersection has 1000 elements. What is the total number of elements in set A?



(19) __________ In Pascal's Triangle, each number is the sum of the number just above it and to the left and the number just above it and to the right. So the middle number in Row 2 is 2 because $1 + 1 = 2$. What is the sum of the numbers in Row 8 of Pascal's Triangle?

Row 0: 1

Row 1: 1 1

Row 2: 1 2 1

Row 3: 1 3 3 1

Row 4: 1 4 6 4 1

Row 5: 1 5 10 10 5 1

(20) ________ How many positive integers between 200 and 500 are divisible by each of the integers 4, 6, 10 and 12?

(21) ________ A bag contains exactly three red marbles, five yellow marbles and two blue marbles. If three marbles are to be drawn from the bag at the same time, what is the probability that all three will be the same color? Express your answer as a common fraction.

(22) ________ A bag contains red balls and white balls. If five balls are to be pulled from the bag, with replacement, the probability of getting exactly three red balls is 32 times the probability of getting exactly one red ball. What percent of the balls originally in the bag are red?

(23) ________ Six students (four juniors and two seniors) must be split into three pairs. If the pairs are chosen randomly, what is the probability that the two seniors form one pair? Express your answer as a common fraction.

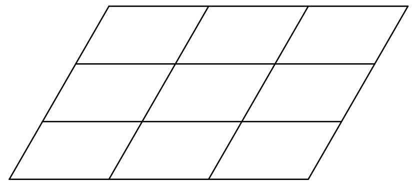
(24) ________ Four dimes and four pennies are randomly placed in a row. What is the probability that the first and eighth coins are both dimes? Express your answer as a common fraction.

(25) ________ What is the probability that a list of five different, single-digit numbers, whose sum is 31, will contain exactly two prime numbers? Express your answer as a common fraction.

(26) ________ All of the possible positive four-digit integers are formed that use only the digits $1, 3, 5$ and 7 with repetition of digits allowed. How many of these four-digit integers are palindromes?

(27) ________ In 2010, the probability of Mark sinking a free throw was twice what it was in 2009, and yet the probability of him sinking exactly two out of three free throws was the same in both years. What was the probability of Mark sinking a free throw in 2010, given that the probability was greater than zero? Express your answer as a common fraction.

(28) _________ In the figure shown, there are parallelograms of many sizes. How many total parallelograms are there in the diagram?



(29) _________ Javier needs to exchange his dollar bill for coins. The cashier has 2 quarters, 10 dimes and 10 nickels. Assuming the cashier gives Javier the correct amount and at least one quarter, how many possible combinations of coins could Javier receive?

(30) _________ A target consists of concentric circles of radii 1 cm, 2 cm and 3 cm. The innermost circle is colored red, the middle ring is colored white, and the outer ring is colored blue. If a point is chosen at random on the target, what is the probability that it lies in the blue region? Express you answer as a common fraction.

Answer Sheet

Number	Answer	Problem ID
1	$\frac{5}{324}$	24CC2
2	8/21	21553
3	2205 Arrangements	AACC2
4	4 choices	5A153
5	2674 elements	42453
6	$\frac{35}{1296}$	4D253
7	3/5	B1453
8	$\frac{9}{20}$	10503
9	200 integers	B4CC2
10	4	34253
11	$\frac{8}{15}$	B5CC2
12	162 "words"	45253
13	$\frac{8}{15}$	DC303
14	112 values	30503
15	6 junior volunteers	54253
16	75 students	DA153
17	24 subsets	CC303
18	2674 items	CB353
19	256	110D3
20	5 positive integers	DACC2
21	$\frac{11}{120}$	C5103
22	80 %	044D3
23	$\frac{1}{5}$	4D103
24	$\frac{3}{14}$	15CC2
25	3/5	3D253
26	16 integers	3C303
27	$\frac{6}{7}$	BD253
28	36 parallelograms	CB3D3
29	11 combinations	1C3D3
30	$\frac{5}{9}$	0D103

Solutions

(1) $\frac{5}{324}$ **ID: [24CC2]**

There are 6^6 different possible rolls exhibited by the six dice. If the six dice yield distinct numbers, then there are 6 possible values that can appear on the first die, 5 that can appear on the second die, and so forth. Thus, there are 6! ways to attain 6 distinct numbers on the die. The desired probability is $\frac{6!}{6^6} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6} = \frac{20}{6 \cdot 6 \cdot 6 \cdot 6} = \frac{5}{2^2 \cdot 3^4} = \boxed{\frac{5}{324}}$.

(2) **8/21** **ID: [21553]**

First let's find the number of ways we can choose 6 students out of this group of 10. There are

$$\binom{10}{6} = \frac{10!}{6! \cdot 4!} = 210$$

ways to select 6 students out of this group of 10. If the committee contains the same number of boys and girls, then it will have 3 boys and 3 girls. There are

$$\binom{4}{3} = \frac{4!}{3! \cdot 1!} = 4$$

ways to choose 3 girls out of the 4 and

$$\binom{6}{3} = \frac{6!}{3! \cdot 3!} = 20$$

ways to select 3 boys out of the 6. Therefore, there are $4 \cdot 20 = 80$ ways to select 3 boys and 3 girls to be on the committee. Thus, the probability that the committee contains the same number of boys and girls is

$$\frac{80}{210} = \boxed{\frac{8}{21}}$$

(3) **2205 Arrangements** **ID: [AACC2]**

Since there are 21 consonants, there are 21 possibilities for the first and third letters. There are 5 possibilities for the second letter because there are 5 vowels. Thus, the total number of three-letter arrangements that satisfy the requirements is $21 \cdot 5 \cdot 21 = \boxed{2205}$.

(4) **4 choices** **ID: [5A153]**

Let there be x different dessert choices. Since the appetizer, entree, and dessert choices are independent, we multiply the number of options for each to get the number of total combinations. Thus we must have $10 \cdot 6 \cdot x = 240$. Solving for x yields $x = 4$; thus the restaurant has $\boxed{4}$ dessert choices.

(5) **2674 elements** **ID: [42453]**

Let's call the total number of elements in set A "a" and the total number of elements in set B "b." We are told that the total number of elements in set A is twice the total number of elements in set B so we can write

$$a = 2b.$$

Since there are 1000 elements in the intersection of set A and set B, there are $a - 1000$ elements that are only in set A and $b - 1000$ elements only in set B. The total number of elements in the union of set A and set B is equal to

$$\text{elements in only set A} + \text{elements in only set B} + \text{elements in intersection}$$

which we can also write as

$$(a - 1000) + (b - 1000) + 1000.$$

Because we know that there is a total of 3011 elements in the union of A and B, we can write

$$(a - 1000) + (b - 1000) + 1000 = 3011$$

which simplifies to

$$a + b = 4011.$$

Because $a = 2b$ or $b = \frac{1}{2}a$, we can write the equation in terms of a and then solve for a. We get

$$\begin{aligned} a + b &= 4011 \\ a + \frac{1}{2}a &= 4011 \\ \frac{3}{2}a &= 4011 \\ a &= 2674 \end{aligned}$$

Therefore, the total number of elements in set A is $\boxed{2674}$.

(6) $\frac{35}{1296}$ **ID: [4D253]**

Bob and Tom can each roll at most a 12. So, we have the following ordered pairs (Bob's sum, Tom's sum):

(12, 12) (11, 11)

(12, 11) (11, 12) (12, 10) (10, 12) (12, 9) (9, 12)

(11, 10) (10, 11)

The probability that one person rolls a 12 is $(1/6)(1/6) = 1/36$; the probability that one person rolls an 11 is $2/36 = 1/18$; the probability that one person rolls a 10 is $3/36 = 1/12$; the probability that one person rolls a 9 is $4/36 = 1/9$.

Thus, the probability of (12, 12) is $(1/36)(1/36) = 1/1296$. We compute the probabilities for the other ordered pairs similarly, and then we sum the probabilities for all 10 ordered pairs above to reach our desired total probability:

$$(1/36)(1/36)+(1/18)(1/18)+2[(1/36)(1/18)+(1/36)(1/12)+(1/36)(1/9)]+2[(1/18)(1/12)] = \boxed{35/1296}.$$

(7) **3/5** **ID: [B1453]**

The fly is within 6 feet of the ceiling when its distance from the ceiling is less than or equal to 6 feet. Since the ceiling is 10 feet high, the probability that the fly is within 6 feet of the ceiling is $\frac{6}{10} = \boxed{\frac{3}{5}}$.

(8) $\frac{9}{20}$ **ID: [10503]**

We need to use casework. Suppose first that bag A is chosen: there is a 1/2 chance of this occurring. There are $\binom{5}{2} = \frac{5 \cdot 4}{2} = 10$ total ways to select two balls from bag A. If the two balls are the same color, then they must be either both white or both red. If both are white, then there are $\binom{3}{2} = 3$ ways to pick the two white balls, and if both are red, then there is 1 way to pick the two red balls. Thus, the probability of selecting two balls of the same color from bag A is $\frac{1+3}{10} = \frac{2}{5}$.

Next, suppose that bag B is chosen, again with 1/2 chance. There are $\binom{9}{2} = \frac{9 \cdot 8}{2} = 36$ ways to pick the two balls. There are $\binom{6}{2} = \frac{6 \cdot 5}{2} = 15$ ways to pick two white balls, and $\binom{3}{2} = 3$ ways to pick two red balls. Thus, the probability that two balls drawn from bag B are the same color is equal to $\frac{15+3}{36} = \frac{1}{2}$.

Thus, the probability that the balls have the same color is $\frac{1}{2} \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{9}{20}}$.

(9) **200 integers** **ID: [B4CC2]**

Let us consider the number of three-digit integers that do not contain 3 and 5 as digits; let this set be S. For any such number, there would be 7 possible choices for the hundreds digit (excluding $0, 3$, and 5), and 8 possible choices for each of the tens and ones digits. Thus, there are $7 \cdot 8 \cdot 8 = 448$ three-digit integers without a 3 or 5.

Now, we count the number of three-digit integers that just do not contain a 5 as a digit; let this set be T. There would be 8 possible choices for the hundreds digit, and 9 for each of the others, giving $8 \cdot 9 \cdot 9 = 648$. By the complementary principle, the set of three-digit integers with at least one 3 and no 5s is the number of integers in T but not S. There are $648 - 448 = \boxed{200}$ such numbers.

(10) **4** **ID: [34253]**

We want to find the remainder when $15^5 + 3$ is divided by 7.

Since 14 is a multiple of 7 that is close to 15, we write 15 as $14 + 1$ and use the binomial theorem to expand:

$(14 + 1)^5 + 3 =$

$\binom{5}{5}14^5 1^0 + \binom{5}{4}14^4 1^1 + \binom{5}{3}14^3 1^2 + \binom{5}{2}14^2 1^3 + \binom{5}{1}14^1 1^4 + \binom{5}{5}14^0 1^5 + 3.$

Notice how the first five terms in the sum are all divisible by 7 because they contain at least one multiple of 14. Thus the only terms that are not divisible by 7 are $\binom{5}{5}14^0 1^5 + 3 = 1 \cdot 1 \cdot 1 + 3 = 4$. Hence the remainder is $\boxed{4}$.

(11) $\frac{8}{15}$ **ID: [B5CC2]**

If the product of their values is even, then at least one of the dice rolls must yield an even number. To find how many ways this is possible, we consider the complementary possibility: suppose that all of the dice rolls yield odd numbers. There are 3^4 ways of this occurring, out of a total of 6^4 possibilities. It follows that there are $6^4 - 3^4$ ways of obtaining at least one even value.

Now, we need to count how many ways we can obtain an odd sum. There must then be an odd number of odd numbers rolled, so there must be either one or three odd numbers rolled. If one odd number is rolled, then there are 4 ways to pick which die yielded the odd number, and 3 possibilities for each dice, yielding a total of $4 \cdot 3^4$ possibilities. If three odd numbers are rolled, then there are again 4 ways to pick which die yielded the even number and 3 possibilities for each dice, yielding $4 \cdot 3^4$. Thus, the desired probability is given by $\frac{4 \cdot 3^4 + 4 \cdot 3^4}{6^4 - 3^4} = \frac{8}{2^4 - 1} = \boxed{\dfrac{8}{15}}$.

(12) **162 "words"** **ID: [45253]**

First, there are three ways to choose a letter on each of the three blocks, for a total of $3^3 = 27$ ways.

Then, there are $3! = 6$ ways to order the three blocks once you've chosen the letters atop them.

Hence there are $27 \cdot 6 = \boxed{162}$ words that Elmo can form.

(13) $\frac{8}{15}$ **ID: [DC303]**

There are 6 balls total and because Sam chooses two balls without replacement, there are $6 \times 5 = 30$ total ways that he could choose two balls (if the order in which Sam chooses the balls matters). Now we consider the number of ways that he could choose a green and a red ball. There are 4 green balls and 2 red balls so there are $4 \times 2 = 8$ ways that Sam could first choose a green ball and then a red ball. There are also $2 \times 4 = 8$ ways that Sam could choose a red ball and then a green ball. Thus, the probability that Sam chooses two different color balls is $\frac{8+8}{30} = \frac{16}{30} = \boxed{\frac{8}{15}}$.

(14) **112 values** **ID: [30503]**

It follows that n must be divisible by 3. Also, as $\frac{n}{3}$ and $3n$ are four-digit integers, then $1000 \le \frac{n}{3}$ and $3n \le 9999$. Simplifying the two inequalities yields the inequality chain $3000 \le n \le 3333$. Since n is divisible by 3, then $n = 3000, 3003, 3006, \ldots, 3333$, which we can check all satisfy the given conditions. There are $\frac{3333-3000}{3} + 1 = \boxed{112}$ such values.

(15) **6 junior volunteers** **ID: [54253]**

Let the number of junior volunteers be x. The number of ways to select five juniors from this group is $\binom{x}{5}$, and similarly, the number of ways to select three seniors from the six seniors is $\binom{6}{3}$. Because selection of juniors and seniors is independent (the events do not affect each other), the total number of ways to select both the grade levels is

$$\binom{x}{5} \cdot \binom{6}{3} = \binom{x}{5} \cdot 20.$$

We want this to be at least 100, so we have $\binom{x}{5} \cdot 20 \ge 100$ or $\binom{x}{5} \ge 5$.

We now try values of x. When $x = 5$, $\binom{x}{5} = \binom{5}{5} = 1$ which is too low. When $x = 6$, $\binom{x}{5} = \binom{6}{5} = 6$ which works. Hence the answer is $\boxed{6}$ junior volunteers.

(16) **75 students ID: [DA153]**

The number of students who participate in at least one of sports or fine arts is $300 + 125 - 50 = 375$. (We subtract 50 because the 50 students who participate in both activities are counted twice in the original 300+125 calculation.) Thus the number of students who do not participate in either activity is $450 - 375 = \boxed{75}$.

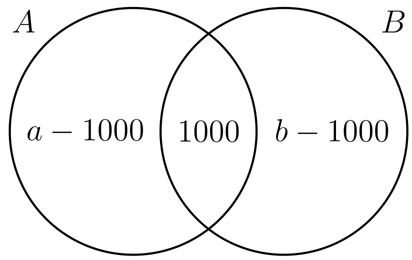
(17) **24 subsets ID: [CC303]**

We first find the number of subsets that have 4 as their largest element. There is one subset that has one element that satisfies this requirement: the subset with 4 as its only element. There are 3 two-element subsets that have 4 as their largest element ($\{1,4\}$, $\{2,4\}$, and $\{3,4\}$). There are 3 three-element subsets that have 4 as their largest element ($\{1,2,4\}$, $\{1,3,4\}$, and $\{2,3,4\}$). Finally there is only one four-element subset with 4 as its largest element ($\{1,2,3,4\}$). Since there aren't four elements less than 4, there cannot be a subset with greater than four elements and 4 as its largest elements. Thus, there are $1+3+3+1 = 8$ subsets with 4 as its largest element.

To find the number of subsets with 5 as their largest element, we notice that these subsets either include 4 or don't include 4 as one of their elements. We already know that there are 8 subsets with 4 as their largest element. In each of these subsets we can replace the 4 with a 5 (for example, $\{1,3,4\}$ becomes $\{1,3,5\}$) so there are 8 subsets with 5 as their largest element that also don't include a 4 as one of their elements. We could also include a 5 in any of the 8 subsets with 4 as their largest element (for example, the subset $\{1,4\}$ becomes $\{1,4,5\}$) to find that there are 8 subsets with 5 as their largest element that also include 4 as an element. Thus, there are $8+8+8 = \boxed{24}$ subsets that have either 4 or 5 as their largest element.

(18) **2674 items ID: [CB353]**

Let a be the number of elements of set A, and let b be the number of elements of set B. There are $a - 1000$ elements in A that are not in B, and there are $b - 1000$ elements in B that are not in A:



Since the total number of elements in all three regions in 3011, we know

$$3011 = (a - 1000) + 1000 + (b - 1000) = a + b - 1000.$$

We also know that $b = \frac{1}{2}a$, so we can solve to get $4011 = \frac{3}{2}a$ and hence $\boxed{a = 2674}$.

(19) **256 ID: [110D3]**

The most obvious way to solve this problem would be to list Pascal's Triangle to Row 8.

Row 0: 1

Row 1: 1 1

Row 2: 1 2 1

Row 3: 1 3 3 1

Row 4: 1 4 6 4 1

Row 5: 1 5 10 10 5 1

Row 6: 1 6 15 20 15 6 1

Row 7: 1 7 21 35 35 21 7 1

Row 8: 1 8 28 56 70 56 28 8 1

We then add $1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = \boxed{256}$.

An interesting note: We can sum the numbers in some of the smaller rows of Pascal's Triangle. Row 0 sums to 1, Row 1 sums to $1 + 1 = 2$, Row 2 sums to $1 + 2 + 1 = 4$, Row 3 sums to $1 + 3 + 3 + 1 = 8$, and Row 4 sums to $1 + 4 + 6 + 4 + 1 = 16$. We start to notice a pattern: the sum of the numbers in Row n of Pascal's Triangle equals 2^n. Sure enough, the sum of the numbers in Row 8 is 256, which is 2^8.

(20) **5 positive integers** **ID: [DACC2]**

For an integer to be divisible by 4, 6, 10, and 12, it must be a multiple of the greatest common divisor of 4, 6, 10, and 12 as well. $4 = 2^2$, $6 = 2 \cdot 3$, $10 = 2 \cdot 5$, and $12 = 2^2 \cdot 3$, so the GCD of those four integers is $2^2 \cdot 3 \cdot 5 = 60$. The only multiples of 60 that between 200 and 500 are 240, 300, 360, 420, and 480, so there are $\boxed{5}$ integers between 200 and 500 that are divisible by 4, 6, 10, and 12.

(21) $\frac{11}{120}$ **ID: [C5103]**

There are 10 marbles total. If we say that the order that we choose the marbles in matters, there are $10 \times 9 \times 8$ ways to choose three marbles. Since there are three red marbles total, there are $3 \times 2 \times 1 = 6$ ways to draw three red marbles. Since there are five yellow marbles total, there are $5 \times 4 \times 3 = 60$ ways to draw three yellow marbles. There are only two blue marbles so it is impossible to draw three blue marbles.

Thus, there is a $\frac{66}{10\times9\times8} = \boxed{\frac{11}{120}}$ chance that the three marbles drawn will be the same color.

(22) **80 %** **ID: [044D3]**

Let p be the probability of drawing a red ball. Then $1-p$ is the probability of drawing a white ball. Because we are drawing with replacement, these probabilities do not change from one draw to the next. The first case is to draw exactly 3 red balls in 5 draws. There are $\binom{5}{3}=10$ ways to do this, and each has probability $p^3(1-p)^2$ of happening. Thus, the total probability of drawing exactly 3 red balls is $10p^3(1-p)^2$. There are $\binom{5}{1}=5$ ways to draw exactly 1 red ball in 5 draws, and each way has probability $p(1-p)^4$ of happening. Thus, the total probability of drawing exactly 1 red ball is $5p(1-p)^4$.

Using the information given in the problem, we now have $10p^3(1-p)^2 = 32\cdot 5p(1-p)^4$. Because the problem states that there are both red and white balls (there aren't 0 of any color), we know that neither p nor $1-p$ is zero. Solving for p, we get

$$\begin{aligned}
10p^3(1-p)^2 &= 32\cdot 5p(1-p)^4\\
10p^3(1-p)^2 &= 160p(1-p)^4\\
p^3(1-p)^2 &= 16p(1-p)^4\\
p^2 &= 16(1-p)^2\\
p^2 &= 16-32p+16p^2\\
0 &= 15p^2-32p+16\\
0 &= (3p-4)(5p-4)\\
p &= \frac{4}{3},\frac{4}{5}
\end{aligned}$$

Because $p<1$, the first solution is impossible. Thus, $p=\frac{4}{5}$. Thus, $\frac{4}{5}$ of the balls are originally red. This is equivalent to $\boxed{80}$%.

(23) $\frac{1}{5}$ **ID: [4D103]**

We first find the probability that the two seniors are in the first pair of students. There are $\binom{6}{2}=\frac{6\times 5}{2\times 1}=15$ ways to choose two students (if order doesn't matter). There is only one way to choose two students such that both are seniors since there are only two seniors to choose from. Therefore, the probability that the two seniors form the first pair of students is 1/15. By similar reasoning, there is a 1/15 chance that the seniors form the second pair of students or the third pair. Thus, the probability that the two seniors form one pair is $3\times\frac{1}{15}=\boxed{\frac{1}{5}}$.

(24) $\frac{3}{14}$ **ID: [15CC2]**

There are $\binom{8}{4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70$ ways to arrange four dimes and four pennies. Of these, if the first and eighth coins are both dimes, it follows that the middle six coins must include two dimes and four pennies. There are $\binom{6}{2} = \frac{6 \cdot 5}{2} = 15$ ways to arrange these coins. Thus, the desired probability is $\frac{15}{70} = \boxed{\dfrac{3}{14}}$.

(25) **3/5 ID: [3D253]**

We start by listing all combinations of five different single-digit numbers. We start from the greatest possible numbers:

$9 + 8 + 7 + 6 + 5 = 5 \cdot 7 = 35$, which is 4 too large. However, $9, 8, 7, 6, 1$ works. This is the only possible list that includes 9, 8, 7, and 6.

The lists that include 9, 8, and 7 are as follows:

$9, 8, 7, 5, 2$

$9, 8, 7, 4, 3$

The lists that include 9 and 8 are as follows:

$9, 8, 6, 5, 3$

The lists that include 9 are as follows:

$9, 7, 6, 5, 4$

We can't have a list with 8 as our largest integer as $8, 7, 6, 5, 4$ sum to 30 which is too small. Hence, there are only 5 lists that work. Out of the five, there are three that contain exactly two prime numbers. Hence the probability is $\boxed{\dfrac{3}{5}}$.

(26) **16 integers ID: [3C303]**

For a four-digit integer to be a palindrome, the thousands digit must be the same as the ones digit. The hundreds digit must also be the same as the tens digit. Thus, our four-digit integer must be in the form $\underline{A}\ \underline{B}\ \underline{B}\ \underline{A}$ where A and B are not necessarily distinct. To find the number of four-digit palindromes we find the number of possible combinations of A and B. There are four possible values of both A and B $(1, 3, 5$ and $7)$, so there are $4 \times 4 = 16$ possible combinations of A and B. There are $\boxed{16}$ four-digit numbers that satisfy the given conditions.

(27) $\frac{6}{7}$ **ID: [BD253]**

Let the probability that Mark sinks a free throw in 2009 be x, so the probability that he does not sink a free throw is $1-x$. The probability he sinks exactly two out of three free throws is $\binom{3}{1}(x^2)(1-x) = 3x^2(1-x)$.

Let the probability that Mark sinks a free throw in 2010 be $2x$, so the probability that he does not sink a free throw is $1-x$. The probability he sinks exactly two out of three free throws is $\binom{3}{1}(2x)^2(1-2x) = 12x^2(1-2x)$.

Since these probabilities are equal, we have $3x^2(1-x) = 12x^2(1-2x)$. We simplify and solve for x:

$$\begin{aligned} x^2(1-x) - 4x^2(1-2x) &= 0 \\ x^2 - x^3 - 4x^2 + 8x^3 &= 0 \\ -3x^2 + 7x^3 &= 0. \end{aligned}$$

We factor: $x^2(-3+7x) = 0$, which yields $x = 0$ or $x = 3/7$. The latter is the valid solution, and the probability that Mark sinks a free throw in 2010 is double this probability, or $\boxed{6/7}$.

(28) **36 parallelograms ID: [CB3D3]**

Let's go case by case for the different sizes of parallelograms. When a parallelogram is referred to as $a \times b$, this means that the parallelogram is a unit parallelograms wide and b tall. 1×1: There are 9 such parallelograms. 1×2: There are 6 such parallelograms. 1×3: There are 3 such parallelograms 2×1: There are 6 such parallelograms 2×2: There are 4 such parallelograms 2×3: There are 2 such parallelograms 3×1: There are 3 such parallelograms 3×2: There are 2 such parallelograms 3×3: There is 1 such parallelogram

Adding all of them up, we get $9+6+3+6+4+2+3+2+1 = \boxed{36}$ parallelograms.

(29) **11 combinations ID: [1C3D3]**

There are two possibilities for the number of quarters Javier could get: 1 or 2. If Javier gets 1 quarter, then he still needs 75 cents. Now we need to find out how many dimes he can have. The most he could have is 7 because 8 would be too much money. The least is 3 because if he had any less, there would not be enough nickels to cover the rest of the change. Thus, he can have 3, 4, 5, 6, or 7 dimes. Once the number of quarters and dimes is chosen, there is only one possibility for the amount of nickels needed to add up to 1 dollar. Thus, with 1 quarter, there are 5 combinations of coins.

If Javier gets 2 quarters, then he still needs 50 cents. Using the same logic as above, the most number of dimes he could have is 5 and the least is 0. Thus, he can have 0, 1, 2, 3, 4, or 5 dimes for a total of 6 possible combinations of coins. Thus, there are a total of $5+6 = \boxed{11}$ combinations.

(30) $\frac{5}{9}$ **ID: [0D103]**

The probability that a point on the target is in the blue ring is equal to the ratio of the area of the blue ring to the area of the whole target. Since the radius of the target is 3 cm, the target's area is $\pi r^2 = 9\pi \text{ cm}^2$. The blue ring is a ring with outer radius 3 cm and inner radius 2 cm, so its area is $\pi(3 \text{ cm})^2 - \pi(2 \text{ cm})^2 = 5\pi \text{ cm}^2$. Thus, the probability that a point chosen is in the blue ring is $\frac{5\pi \text{ cm}^2}{9\pi \text{ cm}^2} = \boxed{\frac{5}{9}}$.