### Essential Question
How can you derive a general formula for solving a quadratic equation?

### Deriving the Quadratic Formula

**Work with a partner.** Analyze and describe what is done in each step in the development of the Quadratic Formula.

<table>
<thead>
<tr>
<th>Step</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ax^2 + bx + c = 0 )</td>
<td>( ax^2 + bx = -c )</td>
</tr>
<tr>
<td>( x^2 + \frac{b}{a}x = \frac{-c}{a} )</td>
<td>( x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2 )</td>
</tr>
<tr>
<td>( x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2} )</td>
<td>( \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} )</td>
</tr>
<tr>
<td>( x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} )</td>
<td>( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} )</td>
</tr>
</tbody>
</table>

The result is the Quadratic Formula.

### Using the Quadratic Formula

**Work with a partner.** Use the Quadratic Formula to solve each equation.

- a. \( x^2 - 4x + 3 = 0 \)  
- b. \( x^2 - 2x + 2 = 0 \)  
- c. \( x^2 + 2x - 3 = 0 \)  
- d. \( x^2 + 4x + 4 = 0 \)  
- e. \( x^2 - 6x + 10 = 0 \)  
- f. \( x^2 + 4x + 6 = 0 \)

### Communicate Your Answer

3. How can you derive a general formula for solving a quadratic equation?  
4. Summarize the following methods you have learned for solving quadratic equations: graphing, using square roots, factoring, completing the square, and using the Quadratic Formula.
**What You Will Learn**

- Solve quadratic equations using the Quadratic Formula.
- Analyze the discriminant to determine the number and type of solutions.
- Solve real-life problems.

**Solving Equations Using the Quadratic Formula**

Previously, you solved quadratic equations by completing the square. In the Exploration, you developed a formula that gives the solutions of any quadratic equation by completing the square once for the general equation $ax^2 + bx + c = 0$. The formula for the solutions is called the **Quadratic Formula**.

**Core Concept**

**The Quadratic Formula**

Let $a$, $b$, and $c$ be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Example 1** Solving an Equation with Two Real Solutions

Solve $x^2 + 3x = 5$ using the Quadratic Formula.

**Solution**

\[
\begin{align*}
\text{Write original equation.} & \quad x^2 + 3x = 5 \\
\text{Write in standard form.} & \quad x^2 + 3x - 5 = 0 \\
\text{Quadratic Formula} & \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
& \quad x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)} \\
& \quad x = \frac{-3 \pm \sqrt{29}}{2} \\
\text{Simplify.} & \quad x = \frac{-3 + \sqrt{29}}{2} \approx 1.19 \quad \text{and} \quad x = \frac{-3 - \sqrt{29}}{2} \approx -4.19.
\end{align*}
\]

So, the solutions are $x = \frac{-3 + \sqrt{29}}{2} \approx 1.19$ and $x = \frac{-3 - \sqrt{29}}{2} \approx -4.19$.

**Check** Graph $y = x^2 + 3x - 5$. The $x$-intercepts are about $-4.19$ and about $1.19$.

**Monitoring Progress**

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Solve the equation using the Quadratic Formula.

1. $x^2 - 6x + 4 = 0$  
2. $2x^2 + 4 = -7x$  
3. $5x^2 = x + 8$
Solving an Equation with One Real Solution

Solve \(25x^2 - 8x = 12x - 4\) using the Quadratic Formula.

**SOLUTION**

\[
25x^2 - 8x = 12x - 4
\]

Write original equation.

\[
25x^2 - 20x + 4 = 0
\]

Write in standard form.

\[
x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(4)}}{2(25)}
\]

\[
a = 25, \ b = -20, \ c = 4
\]

\[
x = \frac{20 \pm \sqrt{0}}{50}
\]

Simplify.

\[
x = \frac{2}{5}
\]

Simplify.

So, the solution is \(x = \frac{2}{5}\). You can check this by graphing \(y = 25x^2 - 20x + 4\). The only 
\(x\)-intercept is \(\frac{2}{5}\).

Solving an Equation with Imaginary Solutions

Solve \(-x^2 + 4x = 13\) using the Quadratic Formula.

**SOLUTION**

\[
-x^2 + 4x = 13
\]

Write original equation.

\[
-x^2 + 4x - 13 = 0
\]

Write in standard form.

\[
x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-13)}}{2(-1)}
\]

\[
a = -1, \ b = 4, \ c = -13
\]

\[
x = \frac{-4 \pm \sqrt{-36}}{-2}
\]

Simplify.

\[
x = \frac{-4 \pm 6i}{-2}
\]

Write in terms of \(i\).

\[
x = 2 \pm 3i
\]

Simplify.

The solutions are \(x = 2 + 3i\) and \(x = 2 - 3i\).

**Monitoring Progress**

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Solve the equation using the Quadratic Formula.

4. \(x^2 + 41 = -8x\)  
5. \(-9x^2 = 30x + 25\)  
6. \(5x - 7x^2 = 3x + 4\)

**ANOTHER WAY**

You can also use factoring to solve \(25x^2 - 20x + 4 = 0\) because the left side factors as \((5x - 2)^2\).

**COMMON ERROR**

Remember to divide the real part and the imaginary part by \(-2\) when simplifying.
Analyzing the Discriminant

In the Quadratic Formula, the expression $b^2 - 4ac$ is called the **discriminant** of the associated equation $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You can analyze the discriminant of a quadratic equation to determine the number and type of solutions of the equation.

**Core Concept**

**Analyzing the Discriminant of $ax^2 + bx + c = 0$**

<table>
<thead>
<tr>
<th>Value of discriminant</th>
<th>$b^2 - 4ac &gt; 0$</th>
<th>$b^2 - 4ac = 0$</th>
<th>$b^2 - 4ac &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and type of solutions</td>
<td>Two real solutions</td>
<td>One real solution</td>
<td>Two imaginary solutions</td>
</tr>
<tr>
<td>Graph of $y = ax^2 + bx + c$</td>
<td>Two x-intercepts</td>
<td>One x-intercept</td>
<td>No x-intercept</td>
</tr>
</tbody>
</table>

**EXAMPLE 4** Analyzing the Discriminant

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

a. $x^2 - 6x + 10 = 0$

b. $x^2 - 6x + 9 = 0$

c. $x^2 - 6x + 8 = 0$

**SOLUTION**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Discriminant</th>
<th>Solution(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax^2 + bx + c = 0$</td>
<td>$b^2 - 4ac$</td>
<td>$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</td>
</tr>
<tr>
<td>a. $x^2 - 6x + 10 = 0$</td>
<td>$(-6)^2 - 4(1)(10) = -4$</td>
<td>Two imaginary: $3 \pm i$</td>
</tr>
<tr>
<td>b. $x^2 - 6x + 9 = 0$</td>
<td>$(-6)^2 - 4(1)(9) = 0$</td>
<td>One real: $3$</td>
</tr>
<tr>
<td>c. $x^2 - 6x + 8 = 0$</td>
<td>$(-6)^2 - 4(1)(8) = 4$</td>
<td>Two real: $2, 4$</td>
</tr>
</tbody>
</table>

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Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

7. $4x^2 + 8x + 4 = 0$

8. $\frac{1}{2}x^2 + x - 1 = 0$

9. $5x^2 = 8x - 13$

10. $7x^2 - 3x = 6$

11. $4x^2 + 6x = -9$

12. $-5x^2 + 1 = 6 - 10x$
EXAMPLE 5  Writing an Equation

Find a possible pair of integer values for $a$ and $c$ so that the equation $ax^2 - 4x + c = 0$ has one real solution. Then write the equation.

**SOLUTION**

In order for the equation to have one real solution, the discriminant must equal 0.

\[ b^2 - 4ac = 0 \]

Write the discriminant.

\[ (-4)^2 - 4ac = 0 \]

Substitute $-4$ for $b$.

\[ 16 - 4ac = 0 \]

Evaluate the power.

\[ -4ac = -16 \]

Subtract 16 from each side.

\[ ac = 4 \]

Divide each side by $-4$.

Because $ac = 4$, choose two integers whose product is 4, such as $a = 1$ and $c = 4$.

So, one possible equation is $x^2 - 4x + 4 = 0$.

**ANOTHER WAY**

Another possible equation in Example 5 is $4x^2 - 4x + 1 = 0$. You can obtain this equation by letting $a = 4$ and $c = 1$.

Check  Graph $y = x^2 - 4x + 4$. The only $x$-intercept is 2. You can also check by factoring.

\[ x^2 - 4x + 4 = 0 \]

\[ (x - 2)^2 = 0 \]

\[ x = 2 \]

Join the Check

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13. Find a possible pair of integer values for $a$ and $c$ so that the equation $ax^2 + 3x + c = 0$ has two real solutions. Then write the equation.

The table shows five methods for solving quadratic equations. For a given equation, it may be more efficient to use one method instead of another. Suggestions about when to use each method are shown below.

**Concept Summary**

**Methods for Solving Quadratic Equations**

<table>
<thead>
<tr>
<th>Method</th>
<th>When to Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphing</td>
<td>Use when approximate solutions are adequate.</td>
</tr>
<tr>
<td>Using square roots</td>
<td>Use when solving an equation that can be written in the form $u^2 = d$, where $u$ is an algebraic expression.</td>
</tr>
<tr>
<td>Factoring</td>
<td>Use when a quadratic equation can be factored easily.</td>
</tr>
<tr>
<td>Completing the square</td>
<td>Can be used for any quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and $b$ is an even number.</td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td>Can be used for any quadratic equation.</td>
</tr>
</tbody>
</table>

Section 3.4  Using the Quadratic Formula  125
Solving Real-Life Problems

The function \( h = -16t^2 + h_0 \) is used to model the height of a dropped object. For an object that is launched or thrown, an extra term \( v_0t \) must be added to the model to account for the object’s initial vertical velocity \( v_0 \) (in feet per second). Recall that \( h \) is the height (in feet), \( t \) is the time in motion (in seconds), and \( h_0 \) is the initial height (in feet).

\[
\begin{align*}
    h &= -16t^2 + h_0 \quad \text{Object is dropped.} \\
    h &= -16t^2 + v_0t + h_0 \quad \text{Object is launched or thrown.}
\end{align*}
\]

As shown below, the value of \( v_0 \) can be positive, negative, or zero depending on whether the object is launched upward, downward, or parallel to the ground.

As shown below, the value of \( v_0 \) can be positive, negative, or zero depending on whether the object is launched upward, downward, or parallel to the ground.

\[V_0 > 0 \quad V_0 < 0 \quad V_0 = 0\]

**EXAMPLE 6  Modeling a Launched Object**

A juggler tosses a ball into the air. The ball leaves the juggler’s hand 4 feet above the ground and has an initial vertical velocity of 30 feet per second. The juggler catches the ball when it falls back to a height of 3 feet. How long is the ball in the air?

**SOLUTION**

Because the ball is thrown, use the model \( h = -16t^2 + v_0t + h_0 \). To find how long the ball is in the air, solve for \( t \) when \( h = 3 \).

\[
\begin{align*}
    h &= -16t^2 + v_0t + h_0 \\
    3 &= -16t^2 + 30t + 4 \\
    0 &= -16t^2 + 30t + 1
\end{align*}
\]

Write the height model.

Substitute 3 for \( h \), 30 for \( v_0 \), and 4 for \( h_0 \).

Write in standard form.

This equation is not factorable, and completing the square would result in fractions. So, use the Quadratic Formula to solve the equation.

\[
t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(1)}}{2(-16)} = \frac{a = -16, b = 30, c = 1}{2(-16)}
\]

\[
t = \frac{-30 \pm \sqrt{964}}{-32}
\]

Simplify.

\[
t \approx -0.033 \quad \text{or} \quad t \approx 1.9
\]

Use a calculator.

Reject the negative solution, \(-0.033\), because the ball’s time in the air cannot be negative. So, the ball is in the air for about 1.9 seconds.

**Monitoring Progress**

**WHAT IF?** The ball leaves the juggler’s hand with an initial vertical velocity of 40 feet per second. How long is the ball in the air?
3.4 Exercises

Vocabulary and Core Concept Check

1. COMPARE THE SENTENCE When a, b, and c are real numbers such that \( a \neq 0 \), the solutions of the quadratic equation \( ax^2 + bx + c = 0 \) are \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

2. COMPLETE THE SENTENCE You can use the ____________ of a quadratic equation to determine the number and type of solutions of the equation.

3. WRITING Describe the number and type of solutions when the value of the discriminant is negative.

4. WRITING Which two methods can you use to solve any quadratic equation? Explain when you might prefer to use one method over the other.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–18, solve the equation using the Quadratic Formula. Use a graphing calculator to check your solution(s). (See Examples 1, 2, and 3.)

5. \( x^2 - 4x + 3 = 0 \)  
6. \( 3x^2 + 6x + 3 = 0 \)
7. \( x^2 + 6x + 15 = 0 \)  
8. \( 6x^2 - 2x + 1 = 0 \)
9. \( x^2 - 14x = -49 \)  
10. \( 2x^2 + 4x = 30 \)
11. \( 3x^2 + 5 = -2x \)  
12. \( -3x = 2x^2 - 4 \)
13. \( -10x = -25 - x^2 \)  
14. \( -5x^2 - 6 = -4x \)
15. \( -4x^2 + 3x = -5 \)  
16. \( x^2 + 121 = -22x \)
17. \( -z^2 = -12z + 6 \)  
18. \( -7w + 6 = -4w^2 \)

In Exercises 19–26, find the discriminant of the quadratic equation and describe the number and type of solutions of the equation. (See Example 4.)

19. \( x^2 + 12x + 36 = 0 \)  
20. \( x^2 - x + 6 = 0 \)
21. \( 4n^2 - 4n - 24 = 0 \)  
22. \( -x^2 + 2x + 12 = 0 \)
23. \( 4x^2 = 5x - 10 \)  
24. \( -18p = p^2 + 81 \)
25. \( 24x = -48 - 3x^2 \)  
26. \( -2x^2 - 6 = x \)

27. USING EQUATIONS What are the complex solutions of the equation \( 2x^2 - 16x + 50 = 0 \)?
   - A. \( 4 + 3i, 4 - 3i \)
   - B. \( 4 + 12i, 4 - 12i \)
   - C. \( 16 + 3i, 16 - 3i \)
   - D. \( 16 + 12i, 16 - 12i \)

28. USING EQUATIONS Determine the number and type of solutions to the equation \( x^2 + 7x = -11 \).
   - A. two real solutions
   - B. one real solution
   - C. two imaginary solutions
   - D. one imaginary solution

ANALYZING EQUATIONS In Exercises 29–32, use the discriminant to match each quadratic equation with the correct graph of the related function. Explain your reasoning.

29. \( x^2 - 6x + 25 = 0 \)  
30. \( 2x^2 - 20x + 50 = 0 \)
31. \( 3x^2 + 6x - 9 = 0 \)  
32. \( 5x^2 - 10x - 35 = 0 \)

A. \( \frac{5}{2}, 5 \)  
B. \( 10, 5 \)  
C. \( -1, -6 \)  
D. \( 6, 4 \)

Section 3.4 Using the Quadratic Formula
ERROR ANALYSIS  In Exercises 33 and 34, describe and correct the error in solving the equation.

33.  
\[ x^2 + 10x + 74 = 0 \]
\[ x = \frac{-10 \pm \sqrt{10^2 - 4(1)(74)}}{2(1)} \]
\[ x = \frac{-10 \pm \sqrt{-196}}{2} \]
\[ x = \frac{-10 \pm 14}{2} \]
\[ x = -12 \text{ or } 2 \]

34.  
\[ x^2 + 6x + 8 = 2 \]
\[ x = \frac{-6 \pm \sqrt{6^2 - 4(1)(8)}}{2(1)} \]
\[ x = \frac{-6 \pm \sqrt{4}}{2} \]
\[ x = -6 \text{ or } 2 \]
\[ x = -2 \text{ or } -4 \]

OPEN-ENDED  In Exercises 35–40, find a possible pair of integer values for \( a \) and \( c \) so that the quadratic equation has the given solution(s). Then write the equation. (See Example 5.)

35.  \( ax^2 + 4x + c = 0 \); two imaginary solutions
36.  \( ax^2 + 6x + c = 0 \); two real solutions
37.  \( ax^2 - 8x + c = 0 \); two real solutions
38.  \( ax^2 - 6x + c = 0 \); one real solution
39.  \( ax^2 + 10x = c \); one real solution
40.  \(-4x + c = -ax^2\); two imaginary solutions

USING STRUCTURE  In Exercises 41–46, use the Quadratic Formula to write a quadratic equation that has the given solutions.

41.  \[ x = \frac{-8 \pm \sqrt{-176}}{-10} \]
42.  \[ x = \frac{15 \pm \sqrt{-215}}{22} \]
43.  \[ x = \frac{-4 \pm \sqrt{-124}}{-14} \]
44.  \[ x = \frac{-9 \pm \sqrt{137}}{4} \]
45.  \[ x = \frac{-4 \pm 2}{6} \]
46.  \[ x = \frac{2 \pm 4}{-2} \]

COMPARING METHODS  In Exercises 47–58, solve the quadratic equation using the Quadratic Formula. Then solve the equation using another method. Which method do you prefer? Explain.

47.  \[ 3x^2 - 21 = 3 \]
48.  \[ 5x^2 + 38 = 3 \]
49.  \[ 2x^2 - 54 = 12x \]
50.  \[ x^2 = 3x + 15 \]
51.  \[ x^2 - 7x + 12 = 0 \]
52.  \[ x^2 + 8x - 13 = 0 \]
53.  \[ 5x^2 - 50x = -135 \]
54.  \[ 8x^2 + 4x + 5 = 0 \]
55.  \[-3 = 4x^2 + 9x \]
56.  \[-31x + 56 = -x^2 \]
57.  \[ x^2 = 1 - x \]
58.  \[ 9x^2 + 36x + 72 = 0 \]

MATHEMATICAL CONNECTIONS  In Exercises 59 and 60, find the value for \( x \).

59.  Area of the rectangle = 24 m²

60.  Area of the triangle = 8 ft²

61. MODELING WITH MATHEMATICS  A lacrosse player throws a ball in the air from an initial height of 7 feet. The ball has an initial vertical velocity of 90 feet per second. Another player catches the ball when it is 3 feet above the ground. How long is the ball in the air? (See Example 6.)

62. NUMBER SENSE  Suppose the quadratic equation \( ax^2 + 5x + c = 0 \) has one real solution. Is it possible for \( a \) and \( c \) to be integers? rational numbers? Explain your reasoning. Then describe the possible values of \( a \) and \( c \).
63. **MODELING WITH MATHEMATICS** In a volleyball game, a player on one team spikes the ball over the net when the ball is 10 feet above the court. The spike drives the ball downward with an initial vertical velocity of 55 feet per second. How much time does the opposing team have to return the ball before it touches the court?

64. **MODELING WITH MATHEMATICS** An archer is shooting at targets. The height of the arrow is 5 feet above the ground. Due to safety rules, the archer must aim the arrow parallel to the ground.

   ![Diagram of archery target](image)

   a. How long does it take for the arrow to hit a target that is 3 feet above the ground?

   b. What method did you use to solve the quadratic equation? Explain.

65. **PROBLEM SOLVING** A rocketry club is launching model rockets. The launching pad is 30 feet above the ground. Your model rocket has an initial vertical velocity of 105 feet per second. Your friend’s model rocket has an initial vertical velocity of 100 feet per second.

   a. Use a graphing calculator to graph the equations of both model rockets. Compare the paths.

   b. After how many seconds is your rocket 119 feet above the ground? Explain the reasonableness of your answer(s).

66. **PROBLEM SOLVING** The number $A$ of tablet computers sold (in millions) can be modeled by the function $A = 4.5t^2 + 43.5t + 17$, where $t$ represents the year after 2010.

   ![Image of tablet computers](image)

   a. In what year did the tablet computer sales reach 65 million?

   b. Find the average rate of change from 2010 to 2012 and interpret the meaning in the context of the situation.

   c. Do you think this model will be accurate after a new, innovative computer is developed? Explain.

67. **MODELING WITH MATHEMATICS** A gannet is a bird that feeds on fish by diving into the water. A gannet spots a fish on the surface of the water and dives 100 feet to catch it. The bird plunges toward the water with an initial vertical velocity of $-88$ feet per second.

   ![Image of gannet](image)

   a. How much time does the fish have to swim away?

   b. Another gannet spots the same fish, and it is only 84 feet above the water and has an initial vertical velocity of $-70$ feet per second. Which bird will reach the fish first? Justify your answer.

68. **USING TOOLS** You are asked to find a possible pair of integer values for $a$ and $c$ so that the equation $ax^2 - 3x + c = 0$ has two real solutions. When you solve the inequality for the discriminant, you obtain $ac < 2.25$. So, you choose the values $a = 2$ and $c = 1$. Your graphing calculator displays the graph of your equation in a standard viewing window. Is your solution correct? Explain.

69. **PROBLEM SOLVING** Your family has a rectangular pool that measures 18 feet by 9 feet. Your family wants to put a deck around the pool but is not sure how wide to make the deck. Determine how wide the deck should be when the total area of the pool and deck is 400 square feet. What is the width of the deck?

   ![Image of pool with deck](image)
70. **HOW DO YOU SEE IT?** The graph of a quadratic function \( y = ax^2 + bx + c \) is shown. Determine whether each discriminant of \( ax^2 + bx + c = 0 \) is positive, negative, or zero. Then state the number and type of solutions for each graph. Explain your reasoning.

a. \( x \)

b. \( y \)

c. \( x \)

71. **CRITICAL THINKING** Solve each absolute value equation.

a. \( |x^2 - 3x - 14| = 4 \)

b. \( x^2 = |x| + 6 \)

72. **MAKING AN ARGUMENT** The class is asked to solve the equation \( 4x^2 + 14x + 11 = 0 \) by completing the square. Your friend decides to use the Quadratic Formula. Whose method is more efficient? Explain your reasoning.

73. **ABSTRACT REASONING** For a quadratic equation \( ax^2 + bx + c = 0 \) with two real solutions, show that the mean of the solutions is \( \frac{-b}{2a} \). How is this fact related to the symmetry of the graph of \( y = ax^2 + bx + c \)?

74. **THOUGHT PROVOKING** Describe a real-life story that could be modeled by \( h = -16t^2 + v_0t + h_0 \). Write the height model for your story and determine how long your object is in the air.

75. **REASONING** Show there is no quadratic equation \( ax^2 + bx + c = 0 \) such that \( a \), \( b \), and \( c \) are real numbers and \( 3i \) and \( -2i \) are solutions.

76. **MODELING WITH MATHEMATICS** The Stratosphere Tower in Las Vegas is 921 feet tall and has a “needle” at its top that extends even higher into the air. A thrill ride called Big Shot catapults riders 160 feet up the needle and then lets them fall back to the launching pad.

a. The height \( h \) (in feet) of a rider on the Big Shot can be modeled by \( h = -16t^2 + v_0t + 921 \), where \( t \) is the elapsed time (in seconds) after launch and \( v_0 \) is the initial vertical velocity (in feet per second). Find \( v_0 \) using the fact that the maximum value of \( h \) is 921 + 160 = 1081 feet.

b. A brochure for the Big Shot states that the ride up the needle takes 2 seconds. Compare this time to the time given by the model \( h = -16t^2 + v_0t + 921 \), where \( v_0 \) is the value you found in part (a). Discuss the accuracy of the model.

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**Maintaining Mathematical Proficiency**

Reviewing what you learned in previous grades and lessons

Solve the system of linear equations by graphing. **(Skills Review Handbook)**

77. \(-x + 2y = 6\)
\(x + 4y = 24\)

78. \(y = 2x - 1\)
\(y = x + 1\)

79. \(3x + y = 4\)
\(6x + 2y = -4\)

80. \(y = -x + 2\)
\(-5x + 5y = 10\)

Graph the quadratic equation. Label the vertex and axis of symmetry. **(Section 2.2)**

81. \(y = -x^2 + 2x + 1\)

82. \(y = 2x^2 - x + 3\)

83. \(y = 0.5x^2 + 2x + 5\)

84. \(y = -3x^2 - 2\)