7 Equations and Inequalities

7.1 Writing Equations in One Variable
7.2 Solving Equations Using Addition or Subtraction
7.3 Solving Equations Using Multiplication or Division
7.4 Writing Equations in Two Variables
7.5 Writing and Graphing Inequalities
7.6 Solving Inequalities Using Addition or Subtraction
7.7 Solving Inequalities Using Multiplication or Division

“With the help of your twin brother, I think I have figured it out.”
“Thank you.”

“What is \( 7Q + 3Q \)?”
“You’re welcome.”

“Can someone call the society for cruelty to cats?”
“You weigh 36 dog biscuits.”
Example 1  Evaluate $7x + 3y$ when $x = 2$ and $y = 4$.

$7x + 3y = 7 \cdot 2 + 3 \cdot 4$

Substitute 2 for $x$ and 4 for $y$.

$= 14 + 12$

Using order of operations, multiply from left to right.

$= 26$

Add 14 and 12.

Example 2  Evaluate $5x^2 - 2(y + 1) + 9$ when $x = 2$ and $y = 1$.

$5x^2 - 2(y + 1) + 9 = 5(2)^2 - 2(1 + 1) + 9$

Substitute 2 for $x$ and 1 for $y$.

$= 5(2)^2 - 2 \cdot 2 + 9$

Using order of operations, evaluate within the parentheses.

$= 5 \cdot 4 - 2 \cdot 2 + 9$

Using order of operations, evaluate the exponent.

$= 20 - 4 + 9$

Using order of operations, multiply from left to right.

$= 25$

Subtract 4 from 20. Add the result to 9.

Try It Yourself

Evaluate the expression when $a = \frac{1}{2}$ and $b = 7$.

1. $6ab$
2. $16a - b$
3. $3b - 2a - 9$
4. $b^2 - 16a + 5$

Writing Expressions

Example 3  Write the phrase as an expression.

a. the sum of twice a number $n$ and five

$2n + 5$

b. twelve less than four times a number $y$

$4y - 12$

Try It Yourself

Write the phrase as an expression.

5. six more than three times a number $w$

$6. \text{ the quotient of seven and a number } p$

7. two less than a number $t$

8. the product of a number $x$ and five

9. five more than six divided by a number $r$

10. four less than three times a number $b$
7.1 Writing Equations in One Variable

Essential Question How does rewriting a word problem help you solve the word problem?

ACTIVITY: Rewriting a Word Problem

Work with a partner. Read the problem several times. Think about how you could rewrite the problem. Leave out information that you do not need to solve the problem.

Given Problem (63 words)

Your minivan has a flat, rectangular area in the back. When you fold down the rear seats of the van and move them forward, the width of the rectangular area in the van is increased by 2 feet, as shown in the diagram.

By how many square feet does the rectangular area increase when the rear seats are folded down and moved forward?

Rewritten Problem (28 words)

When you fold down the back seats of a minivan, the added area is a 5-foot by 2-foot rectangle. What is the area of this rectangle?

Can you make the problem even simpler?

Rewritten Problem (20 words)

Added Area = 2 \times 5 = 10 \text{ ft}^2

Explain why your rewritten problem is easier to read.
Work with a partner. Rewrite each problem using fewer words. Leave out information that you do not need to solve the problem. Then solve the problem.

a. (63 words)

A supermarket is having its grand opening on Saturday morning. Every fifth customer will receive a $10 coupon for a free turkey. Every seventh customer will receive a $3 coupon for 2 gallons of ice cream. You are the manager of the store and you expect to have 400 customers. How many of each type of coupon should you plan to give away?

b. (71 words)

You and your friend are at a football game. The stadium is 4 miles from your home. You each brought $5 to spend on refreshments. During the third quarter of the game, you say, “I read that the greatest distance that a baseball has been thrown is 445 feet 10 inches.” Your friend says, “That’s about one and a half times the length of the football field.” Is your friend correct?

c. (90 words)

You are visiting your cousin who lives in the city. To get back home, you take a taxi. The taxi charges $2.10 for the first mile and $0.90 for each additional mile. After riding 13 miles, you decide that the fare is going to be more than the $20 you have with you. So, you tell the driver to stop and let you out. Then you call a friend and ask your friend to come and pick you up. After paying the driver, how much of your $20 is left?

What Is Your Answer?

3. **IN YOUR OWN WORDS** How does rewriting a word problem help you solve the word problem? Make up a word problem that has more than 50 words. Then show how you can rewrite the problem using at most 25 words.

Use what you learned about writing equations to complete Exercises 4 and 5 on page 298.
An **equation** is a mathematical sentence that uses an equal sign, $=$, to show that two expressions are equal.

<table>
<thead>
<tr>
<th>Expressions</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 + 8$</td>
<td>$4 + 8 = 12$</td>
</tr>
<tr>
<td>$x + 8$</td>
<td>$x + 8 = 12$</td>
</tr>
</tbody>
</table>

To write a word sentence as an equation, look for key words or phrases such as *is*, *the same as*, or *equals* to determine where to place the equal sign.

**Example 1** Writing Equations

Write the word sentence as an equation.

a. The sum of a number $n$ and 7 is 15.

   The sum of a number $n$ and 7 is 15.

   $$n + 7 = 15$$  \[ \text{Sum of means addition.} \]

   $\therefore$ An equation is $n + 7 = 15$.

b. A number $y$ decreased by 4 is 3.

   A number $y$ decreased by 4 is 3.

   $$y - 4 = 3$$  \[ \text{Decreased by means subtraction.} \]

   $\therefore$ An equation is $y - 4 = 3$.

c. 12 times a number $p$ equals 48.

   12 times a number $p$ equals 48.

   $$12p = 48$$  \[ \text{Times means multiplication.} \]

   $\therefore$ An equation is $12p = 48$.

**On Your Own**

Write the word sentence as an equation.

1. 9 less than a number $b$ equals 2.

2. The product of a number $g$ and 5 is 30.

3. A number $k$ increased by 10 is the same as 24.

4. The quotient of a number $q$ and 4 is 12.
EXAMPLE 2 Writing an Equation

Ten servers decorate 25 tables for a wedding. Each table is decorated as shown. Let \( c \) be the total number of white and purple candles. Which equation can you use to find \( c \)?

- **A** \( c = 25 + (4 \times 6) \)
- **B** \( c = 25(4 + 6) \)
- **C** \( c = 10(25 + 4 + 6) \)
- **D** \( c = 10(4 + 6) \)

**Words** The total number of candles is the number of tables times the number of candles on each table.

**Variable** Let \( c \) be the total number of candles.

**Equation**

\[
\begin{align*}
\text{Equation:} & \quad c = \quad \underline{25} \quad \times \quad (4 + 6) \\
& \quad c = \quad \underline{25} \quad \times \quad 10
\end{align*}
\]

\( \therefore \) The correct answer is (B).

EXAMPLE 3 Real-Life Application

After two rounds, 24 students are eliminated from a spelling bee. There are 96 students remaining. Write an equation you can use to find the number of students that started the spelling bee.

**Words** The number of students that started minus the number of students eliminated is the number of students remaining.

**Variable** Let \( s \) be the number of students that started.

**Equation**

\[
\begin{align*}
\text{Equation:} & \quad s - \quad 24 \quad = \quad 96
\end{align*}
\]

\( \therefore \) An equation is \( s - 24 = 96 \).

On Your Own

5. You enter an elevator and go down 7 floors. You exit on the 10th floor. Write an equation you can use to find the floor where you entered the elevator.

6. Together you and a friend have $52. Your friend has $28. Write an equation you can use to find how much money you have.

7. A typical person takes about 24,000 breaths each day. Write an equation you can use to find the number of breaths a typical person takes each minute.

Section 7.1 Writing Equations in One Variable 297
7.1 Exercises

**Vocabulary and Concept Check**

1. **VOCABULARY** How are expressions and equations different?

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write “both” equations.

   - 4 less than a number \( n \) is 8.
   - A number \( n \) is 4 less than 8.
   - A number \( n \) minus 4 equals 8.
   - 4 subtracted from a number \( n \) is 8.

3. **OPEN-ENDED** Write a word sentence for the equation \( 28 - n = 5 \).

**Practice and Problem Solving**

Rewrite the problem using fewer words. Leave out information that you do not need to solve the problem. Then solve the problem.

4. In a cross-country race you run at a steady rate of 7 minutes per mile. After 21 minutes, you finish in fourth place. How long is the race?

5. For a science project, you record the high temperature each day. The high temperature on Day 1 was 6° less than on Day 4 and 4° less than on Day 10. The high temperature on Day 10 was 62°F. What was the high temperature on Day 1?

Write the word sentence as an equation.

6. The sum of a number \( x \) and 4 equals 12.

7. A number \( y \) decreased by 9 is 8.

8. 9 times a number \( b \) is 36.

9. A number \( w \) divided by 5 equals 6.

10. 54 equals 9 more than a number \( t \).

11. 5 is one-fourth of a number \( c \).

12. 11 is the quotient of a number \( y \) and 6.

13. 9 less than a number \( n \) equals 27.

14. **ERROR ANALYSIS** Describe the error in writing the sentence as an equation.

   - A number \( n \) is 5 more than 12.
   - \( n + 5 = 12 \)

15. **FUNDRAISING** Students and faculty raised $6042 for band uniforms. The faculty raised $1780. Write an equation you can use to find the amount \( a \) raised by the students.

16. **GOLF** You hit a golf ball 90 yards. It travels three-fourths of the distance to the hole. Write an equation you can use to find the distance \( d \) from the tee to the hole.

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298 Chapter 7 Equations and Inequalities
Evaluate the expression when \( a = 7 \).  

(Section 3.1)

23. \( 6 + a \)  
24. \( a - 4 \)  
25. \( 4a \)  
26. \( \frac{35}{a} \)

27. **MULTIPLE CHOICE** Which expression is equivalent to \( 8(x + 3) \)?  

(Section 3.4)

- **A**: \( 8x + 3 \)  
- **B**: \( 8x + 24 \)  
- **C**: \( 8x + 11 \)  
- **D**: \( x + 24 \)
Essential Question  How can you use addition or subtraction to solve an equation?

When two sides of a scale weigh the same, the scale will balance. When you add or subtract the same amount on each side of the scale, it will still balance.

ACTIVITY: Solving an Equation

Work with a partner.

a. Use a model to solve \( n + 3 = 7 \).
   - Explain how the model represents the equation \( n + 3 = 7 \).

   - How much does one \( \bullet \) weigh? How do you know?
   - The solution is \( n = \square \).

b. Describe how you could check your answer in part (a).

c. Which model below represents the solution of \( n + 1 = 9 \)? How do you know?
2. **ACTIVITY: Solving Equations**

Work with a partner. Solve the equation using the method in Activity 1.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( n + 5 = 10 )</td>
<td>( n = 5 )</td>
</tr>
<tr>
<td>b. ( x + 2 = 11 )</td>
<td>( x = 9 )</td>
</tr>
<tr>
<td>c. ( 6 = y + 3 )</td>
<td>( y = 3 )</td>
</tr>
<tr>
<td>d. ( 8 = m + 8 )</td>
<td>( m = 0 )</td>
</tr>
</tbody>
</table>

3. **ACTIVITY: Solving Equations Using Mental Math**

Work with a partner. Write a question that represents the equation. Use mental math to answer the question. Then check your solution.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Question</th>
<th>Solution</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( x + 1 = 5 )</td>
<td>How many more than 1 is ( x )?</td>
<td>( x = 4 )</td>
<td>True</td>
</tr>
<tr>
<td>b. ( 4 + m = 11 )</td>
<td>What plus 4 is 11?</td>
<td>( m = 7 )</td>
<td>True</td>
</tr>
<tr>
<td>c. ( 8 = a + 3 )</td>
<td>How many plus 3 is 8?</td>
<td>( a = 5 )</td>
<td>True</td>
</tr>
<tr>
<td>d. ( x - 9 = 21 )</td>
<td>How much more than 9 is ( x )?</td>
<td>( x = 30 )</td>
<td>True</td>
</tr>
<tr>
<td>e. ( 13 = p - 4 )</td>
<td>How many more than 4 is 13?</td>
<td>( p = 17 )</td>
<td>True</td>
</tr>
</tbody>
</table>

**What Is Your Answer?**

4. **REPEATED REASONING** In Activity 3, how are parts (d) and (e) different from parts (a)–(c)? Did your process to find the solution change? Explain.

5. Decide whether the statement is *true* or *false*. If false, explain your reasoning.
   - a. In an equation, you can use any letter as a variable. *True*
   - b. The goal in solving an equation is to get the variable by itself. *True*
   - c. In the solution, the variable must always be on the left side of the equal sign. *False*
   - d. If you add a number to one side, you should subtract it from the other side. *True*

6. **IN YOUR OWN WORDS** How can you use addition or subtraction to solve an equation? Give two examples to show how your procedure works.

7. Are the following equations equivalent? Explain your reasoning.

\[
\begin{align*}
x - 5 & = 12 \\
12 & = x - 5
\end{align*}
\]

Use what you learned about solving equations to complete Exercises 12–17 on page 305.
Equations may be true for some values and false for others. A solution of an equation is a value that makes the equation true.

<table>
<thead>
<tr>
<th>Value of x</th>
<th>( x + 3 = 7 )</th>
<th>Are both sides equal?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( 3 + 3 = 7 )</td>
<td>no</td>
</tr>
<tr>
<td>4</td>
<td>( 4 + 3 = 7 )</td>
<td>yes</td>
</tr>
<tr>
<td>5</td>
<td>( 5 + 3 = 7 )</td>
<td>no</td>
</tr>
</tbody>
</table>

So, the value \( x = 4 \) is a solution of the equation \( x + 3 = 7 \).

**EXAMPLE 1 Checking Solutions**

Tell whether the given value is a solution of the equation.

a. \( p + 10 = 38; \ p = 18 \)

\[
18 + 10 \neq 38 \\
28 \neq 38 \quad \times \\
\text{Sides are not equal.}
\]

\( \therefore \) So, \( p = 18 \) is not a solution.

b. \( 4y = 56; \ y = 14 \)

\[
4(14) = 56 \\
56 = 56 \quad \checkmark \\
\text{Sides are equal.}
\]

\( \therefore \) So, \( y = 14 \) is a solution.

**On Your Own**

Tell whether the given value is a solution of the equation.

1. \( a + 6 = 17; \ a = 9 \) 2. \( 9 - g = 5; \ g = 3 \)
3. \( 35 = 7n; \ n = 5 \) 4. \( \frac{q}{2} = 28; \ q = 14 \)
You can use inverse operations to solve equations. Inverse operations “undo” each other. Addition and subtraction are inverse operations.

**Key Ideas**

**Addition Property of Equality**

**Words** When you add the same number to each side of an equation, the two sides remain equal.

**Numbers**

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 = 8</td>
<td>x − 4 = 5</td>
</tr>
<tr>
<td>+ 5 + 5</td>
<td>+ 4 + 4</td>
</tr>
<tr>
<td>13 = 13</td>
<td>x = 9</td>
</tr>
</tbody>
</table>

**Subtraction Property of Equality**

**Words** When you subtract the same number from each side of an equation, the two sides remain equal.

**Numbers**

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 = 8</td>
<td>x + 4 = 5</td>
</tr>
<tr>
<td>− 5 − 5</td>
<td>− 4 − 4</td>
</tr>
<tr>
<td>3 = 3</td>
<td>x = 1</td>
</tr>
</tbody>
</table>

**EXAMPLE 2 Solving Equations Using Addition**

**a. Solve** \( x − 2 = 6 \).

Write the equation. \( x − 2 = 6 \)

Undo the subtraction. + 2 + 2

Addition Property of Equality

\[ x = 8 \]

Simplify.

\[ x = 8 \]

The solution is \( x = 8 \).

**b. Solve** \( 18 = x − 7 \).

Write the equation. \( 18 = x − 7 \)

+ 7 + 7

Addition Property of Equality

\[ 25 = x \]

Simplify.

\[ x = 25 \]

The solution is \( x = 25 \).

**On Your Own**

Solve the equation. Check your solution.

5. \( k − 3 = 1 \)  
6. \( n − 10 = 4 \)  
7. \( 15 = r − 6 \)
EXAMPLE 3 Solving Equations Using Subtraction

a. Solve \( x + 2 = 9 \).

\[
\begin{align*}
\text{Write the equation.} & \quad x + 2 = 9 \\
\text{Undo the addition.} & \quad \rightarrow -2 -2 \\
\text{Subtraction Property of Equality} & \quad x = 7 \\
\text{Simplify.} & \\
\text{The solution is} & \quad x = 7.
\end{align*}
\]

b. Solve \( 26 = 11 + x \).

\[
\begin{align*}
\text{Write the equation.} & \quad 26 = 11 + x \\
\text{Undo the addition.} & \quad \rightarrow -11 -11 \\
\text{Subtraction Property of Equality} & \quad 15 = x \\
\text{Simplify.} & \\
\text{The solution is} & \quad x = 15.
\end{align*}
\]

EXAMPLE 4 Real-Life Application

Your parents give you $20 to help buy the new pair of shoes shown. After you buy the shoes, you have $5.50 left. Write and solve an equation to find how much money you had before your parents gave you $20.

Words
The starting amount plus the amount your parents gave you minus the cost of the shoes is the amount left.

Variable
Let \( s \) be the starting amount.

Equation
\[
\begin{align*}
s + 20 - 59.95 & = 5.50 \\
\text{Write the equation.} & \\
s + 20 - 59.95 + 59.95 & = 5.50 + 59.95 \\
\text{Addition Property of Equality} & \\
s + 20 & = 65.45 \\
\text{Simplify.} & \\
s + 20 - 20 & = 65.45 - 20 \\
\text{Subtraction Property of Equality} & \\
s & = 45.45 \\
\text{Simplify.} & \\
\end{align*}
\]

\( \therefore \) You had $45.45 before your parents gave you money.

On Your Own

Solve the equation. Check your solution.

8. \( s + 8 = 17 \)
9. \( 9 = y + 6 \)
10. \( 13 + m = 20 \)
11. You eat 8 blueberries and your friend eats 11 blueberries from a package. There are 23 blueberries left. Write and solve an equation to find the number of blueberries in a full package.
7.2 Exercises

Vocabulary and Concept Check

1. WRITING How can you check the solution of an equation?

Name the inverse operation you can use to solve the equation.

2. \( x - 8 = 12 \)
3. \( n + 3 = 13 \)
4. \( b + 14 = 33 \)

5. WRITING When solving \( x + 5 = 16 \), why do you subtract 5 from the left side of the equation? Why do you subtract 5 from the right side of the equation?

Practice and Problem Solving

Tell whether the given value is a solution of the equation.

6. \( x + 42 = 85; \ x = 43 \)
7. \( 8b = 48; \ b = 6 \)
8. \( 19 - g = 7; \ g = 15 \)
9. \( \frac{m}{4} = 16; \ m = 4 \)
10. \( w + 23 = 41; \ w = 28 \)
11. \( s - 68 = 11; \ s = 79 \)

Use a scale to model and solve the equation.

12. \( n + 7 = 9 \)
13. \( t + 4 = 5 \)
14. \( c + 2 = 8 \)

Write a question that represents the equation. Use mental math to answer the question. Then check your solution.

15. \( a + 5 = 12 \)
16. \( v + 9 = 18 \)
17. \( 20 = d - 6 \)

Solve the equation. Check your solution.

18. \( y - 7 = 3 \)
19. \( z - 3 = 13 \)
20. \( 8 = r - 14 \)
21. \( p + 5 = 8 \)
22. \( k + 6 = 18 \)
23. \( 64 = h + 30 \)
24. \( f - 27 = 19 \)
25. \( 25 = q + 14 \)
26. \( \frac{3}{4} = x - \frac{1}{2} \)
27. \( x + 2 = \frac{9}{3} = \frac{9}{10} \)
28. \( 1.2 = m - 2.5 \)
29. \( a + 5.5 = 17.3 \)

ERROR ANALYSIS Describe and correct the error in solving the equation.

30. \( x + 7 = \frac{13}{x} + 7 \)
31. \( 34 = y - 12 \)

Section 7.2 Solving Equations Using Addition or Subtraction 305
32. **Penguins** An emperor penguin is 45 inches tall. It is 24 inches taller than a rockhopper penguin. Write and solve an equation to find the height of a rockhopper penguin. Is your answer reasonable? Explain.

33. **Elevator** You get in an elevator and go down 8 floors. You exit on the 16th floor. Write and solve an equation to find what floor you got on the elevator.

34. **Area** The area of Jamaica is 6460 square miles less than the area of Haiti. Write and solve an equation to find the area of Haiti.

35. **Reasoning** The solution of the equation \(x + 3 = 12\) is shown. Explain each step. Use a property, if possible.

Write the word sentence as an equation. Then solve the equation.

36. 13 subtracted from a number \(w\) is 15.

37. A number \(k\) increased by 7 is 34.

38. 9 is the difference of a number \(n\) and 7.

39. 93 is the sum of a number \(g\) and 58.

Solve the equation. Check your solution.

40. \(b + 7 + 12 = 30\)

41. \(y + 4 - 1 = 18\)

42. \(m + 18 + 23 = 71\)

43. \(v - 7 = 9 + 12\)

44. \(5 + 44 = 2 + r\)

45. \(22 + 15 = d - 17\)

**Geometry** Write and solve an addition equation to find \(x\).

46. Perimeter = 48 ft

47. Perimeter = 132 in.

48. Perimeter = 93 ft

49. **Reasoning** Explain why the equations \(x + 4 = 13\) and \(4 + x = 13\) have the same solution.

50. **Reasoning** Explain why the equations \(x - 13 = 4\) and \(13 - x = 4\) do not have the same solution.
51. **Simplifying and Solving** Compare and contrast the two problems.

Simplify the expression $2(x + 3) - 4$.

$$2(x + 3) - 4 = 2x + 6 - 4$$
$$= 2x + 2$$

Solve the equation $x + 3 = 4$.

$$x + 3 = 4$$
$$-3 -3$$
$$x = 1$$

52. **Puzzle** In a magic square, the sum of the numbers in each row, column, and diagonal is the same. Write and solve equations to find the values of $a$, $b$, and $c$.

<table>
<thead>
<tr>
<th></th>
<th>37</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>25</td>
<td>$b$</td>
</tr>
<tr>
<td>34</td>
<td>$c$</td>
<td>28</td>
</tr>
</tbody>
</table>

53. **Fundraiser** You participate in a dance-a-thon fundraiser. After your parents pledge $15.50 and your neighbor pledges $8.75, you have $66.55. Write and solve an equation to find how much money you had before your parents and neighbor pledged.

54. **Money** On Saturday, you spend $33, give $15 to a friend, and receive $20 for mowing your neighbor’s lawn. You have $21 left. Use two methods to find how much money you started with that day.

55. **Amusement Park** You have $15.

- **a.** How much money do you have left if you ride each ride once?
- **b.** Do you have enough money to ride each ride twice? Explain.

56. **Critical Thinking** Consider the equation $x + y = 15$. The value of $x$ increases by 3. What has to happen to the value of $y$ so that $x + y = 15$ remains true?

57. Find the value of the expression. Use estimation to check your answer. (Section 1.1)

<table>
<thead>
<tr>
<th></th>
<th>58.</th>
<th>59.</th>
<th>60.</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 $\times$ 8</td>
<td>$13\times 16$</td>
<td>$75\div 15$</td>
<td>$72\div 3$</td>
</tr>
</tbody>
</table>

61. **Multiple Choice** What is the area of the parallelogram? (Section 1.1)

<table>
<thead>
<tr>
<th></th>
<th>62.</th>
<th>63.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 25 in.$^2$</td>
<td>B 30 in.$^2$</td>
<td>C 50 in.$^2$</td>
</tr>
</tbody>
</table>
How can you use multiplication or division to solve an equation?

### Activity: Finding Missing Dimensions

Work with a partner. Describe how you would find the value of $x$. Then find the value and check your result.

- **a.** rectangle
  - Area = 24 square units
  - $x = 6$
- **b.** parallelogram
  - Area = 20 square units
  - $x = 5$
- **c.** triangle
  - Area = 28 square units
  - $x = 8$

### Activity: Using an Equation to Model a Story

Work with a partner.

- **a.** Use a model to solve the problem.

  Three people go out to lunch. They decide to share the $12 bill evenly. How much does each person pay?

  - What equation does the model represent? Explain how this represents the problem.

  - How much does one weigh? How do you know?

  - Each person pays $4$.

- **b.** Describe how you can check your answer in part (a).
Work with a partner.

- What is the unknown?
- Write an equation that represents each problem.
- What does the variable in your equation represent?
- Explain how you can solve the equation.
- Answer the question.

**Problem**

**Equation**

- **a.** Three robots go out to lunch. They decide to share the $11.91 bill evenly. How much does each robot pay?

- **b.** On Earth, objects weigh 6 times what they weigh on the Moon. A robot weighs 96 pounds on Earth. What does it weigh on the Moon?

- **c.** At maximum speed, a robot runs 6 feet in 1 second. How many feet does the robot run in 1 minute?

- **d.** Four identical robots lie on the ground head-to-toe and measure 14 feet. How tall is each robot?

**What Is Your Answer?**

4. Complete each sentence by matching.

- The inverse operation of addition is multiplication.
- The inverse operation of subtraction is addition.
- The inverse operation of multiplication is division.
- The inverse operation of division is subtraction.

5. **IN YOUR OWN WORDS** How can you use multiplication or division to solve an equation? Give two examples to show how your procedure works.

Use what you learned about solving equations to complete Exercises 15–18 on page 312.
### Key Ideas

**Multiplication Property of Equality**

**Words**  When you multiply each side of an equation by the same nonzero number, the two sides remain equal.

**Numbers**  \( \frac{8}{4} = 2 \)

**Algebra**  \( \frac{x}{4} = 2 \)

\[
\frac{8}{4} \cdot 4 = 2 \cdot 4
\]

\[
x = 8
\]

**Multiplicative Inverse Property**

**Words**  The product of a nonzero number \( n \) and its reciprocal, \( \frac{1}{n} \), is 1.

**Numbers**  \( 5 \cdot \frac{1}{5} = 1 \)

**Algebra**  \( n \cdot \frac{1}{n} = 1 \cdot n = 1, n \neq 0 \)

### Example 1: Solving Equations Using Multiplication

#### a. Solve \( \frac{w}{4} = 12 \).

\[
\frac{w}{4} = 12
\]

Write the equation.

\[
\frac{w}{4} \cdot 4 = 12 \cdot 4
\]

 Undo the division.

\[
w = 48
\]

Simplify.

\( \therefore \) The solution is \( w = 48 \).

#### b. Solve \( \frac{2}{7}x = 6 \).

\[
\frac{2}{7}x = 6
\]

Write the equation.

\[
\frac{7}{2} \cdot \left( \frac{2}{7}x \right) = \frac{7}{2} \cdot 6
\]

Use the Multiplicative Inverse Property.

\[
x = 21
\]

Simplify.

\( \therefore \) The solution is \( x = 21 \).

### On Your Own

Solve the equation. Check your solution.

1. \( \frac{a}{8} = 6 \)
2. \( 14 = \frac{2y}{5} \)
3. \( 3z \div 2 = 9 \)
Key Idea

Division Property of Equality

**Words**  When you divide each side of an equation by the same nonzero number, the two sides remain equal.

**Numbers**  \[ 8 \cdot 4 = 32 \]

**Algebra**  \[ 4x = 32 \]

\[ \frac{8 \cdot 4}{4} = \frac{32}{4} \]

\[ 8 = 8 \]

EXAMPLE 2  Solving an Equation Using Division

Solve \( 5b = 65 \).

\[ 5b = 65 \quad \text{Write the equation.} \]

\[ \frac{5b}{5} = \frac{65}{5} \quad \text{Division Property of Equality} \]

\[ b = 13 \quad \text{Simplify.} \]

\[ \therefore \text{The solution is } b = 13. \]

EXAMPLE 3  Real-Life Application

The area of the parallelogram-shaped courtyard is 2730 square feet. What is the length of the sidewalk?

The height of the parallelogram represents the length of the sidewalk.

\[ A = bh \quad \text{Use the formula for area of a parallelogram.} \]

\[ 2730 = 65h \quad \text{Substitute 2730 for } A \text{ and } 65 \text{ for } b. \]

\[ \frac{2730}{65} = \frac{65h}{65} \quad \text{Division Property of Equality} \]

\[ 42 = h \quad \text{Simplify.} \]

\[ \therefore \text{So, the sidewalk is 42 feet long.} \]

On Your Own

Solve the equation. Check your solution.

4. \( p \cdot 3 = 18 \)
5. \( 12q = 60 \)
6. \( 81 = 9r \)
7. You and four friends buy tickets to a baseball game. The total cost is $70. Write and solve an equation to find the cost of each ticket.
Exercises

Vocabulary and Concept Check

1. **NUMBER SENSE** What number divided by 12 equals 1?

2. **WRITING** What property of equality would you use to solve \( \frac{x}{6} = 7 \)? Explain how you would use the property.

Copy and complete the first step in the solution.

3. \( 4x = 24 \)
   \[ \frac{4x}{4} = \frac{24}{4} \]

4. \( \frac{x}{3} = 11 \)
   \[ \frac{x}{3} \cdot 3 = 11 \cdot 3 \]

5. \( 8 = n \div 3 \)
   \[ 8 \cdot 3 = (n \div 3) \cdot 3 \]

6. **OPEN-ENDED** Write an equation that can be solved using the Division Property of Equality.

Practice and Problem Solving

Solve the equation. Check your solution.

1. \( s \div 10 = 7 \)
2. \( 3a = 12 \)
3. \( 7x = 105 \)
4. \( \frac{x}{3} = 11 \)
5. \( 8 = n \div 3 \)
6. \( \frac{2c}{15} = 8.8 \)
7. \( 6 = \frac{t}{5} \)
8. \( 5 \cdot z = 35 \)
9. \( 5x \div 6 = 20 \)
10. \( 24 = \frac{3r}{4} \)
11. \( 3a = 12 \)
12. \( 5 \cdot z = 35 \)
13. \( 40 = 4y \)
14. \( 42 = 7k \)
15. \( 7x = 105 \)
16. \( 75 = 6 \cdot w \)
17. \( 13 = d \div 6 \)
18. \( 9 = v \div 5 \)
19. \( \frac{2c}{15} = 8.8 \)
20. \( 7b \div 12 = 4.2 \)
21. \( 12.5 \cdot n = 32 \)
22. \( 3.4m = 20.4 \)

23. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

24. **ANOTHER WAY** Show how you can solve the equation \( 3x = 9 \) by multiplying each side by the reciprocal of 3.

25. **BASKETBALL** Forty-five basketball players participate in a tournament. Write and solve an equation to find the number of 3-person teams that they can form.

26. **THEATER** A theater has 1200 seats. Each row has 20 seats. Write and solve an equation to find the number of rows in the theater.
Solve for $x$. Check your answer.

27. rectangle

\[
\text{Area} = 45 \text{ square units}
\]

\[
x = 5
\]

28. rectangle

\[
\text{Area} = 176 \text{ square units}
\]

\[
x = 16
\]

29. parallelogram

\[
\text{Area} = 104 \text{ square units}
\]

\[
x = 13
\]

30. **TEST SCORE** On a test, you correctly answer six 5-point questions and eight 2-point questions. You earn 92% of the possible points on the test. How many points $p$ is the test worth?

31. **CARD GAME** You use index cards to play a homemade game. The object is to be the first to get rid of all your cards. How many cards are in your friend’s stack?

32. **SLUSH DRINKS** A slush drink machine fills 1440 cups in 24 hours.

a. Write and solve an equation to find the number $c$ of cups each symbol represents.

b. To lower costs, you replace the cups with paper cones that hold 20% less. Write and solve an equation to find the number $n$ of paper cones that the machine can fill in 24 hours.

**STRUCTURE** Solve the equation. Explain how you found your answer.

33. $5x + 3x = 5x + 18$

34. $8y + 2y = 2y + 40$

35. The area of the picture is 100 square inches. The length is 4 times the width. Find the length and width of the picture.

36. **NUMBER SENSE** The area of the picture is 100 square inches. The length is 4 times the width. Find the length and width of the picture.

37. **CARD GAME** You use index cards to play a homemade game. The object is to be the first to get rid of all your cards. How many cards are in your friend’s stack?

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b. To lower costs, you replace the cups with paper cones that hold 20% less. Write and solve an equation to find the number $n$ of paper cones that the machine can fill in 24 hours.

**STRUCTURE** Solve the equation. Explain how you found your answer.

33. $5x + 3x = 5x + 18$

34. $8y + 2y = 2y + 40$

35. **NUMBER SENSE** The area of the picture is 100 square inches. The length is 4 times the width. Find the length and width of the picture.

**Fair Game Review** What you learned in previous grades & lessons

Write the word sentence as an equation. *(Section 7.1)*

36. The sum of a number $b$ and 8 is 17.

37. A number $t$ divided by 3 is 7.

38. **MULTIPLE CHOICE** What is the value of $a^3$ when $a = 4$? *(Section 3.1)*

A 12  B 43  C 64  D 81
Writing Equations in Two Variables

How can you write an equation in two variables?

ACTIVITY: Writing an Equation in Two Variables

Work with a partner. You earn $8 per hour working part-time at a store.

a. Complete the table.

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>Money Earned (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

b. Use the values from the table to complete the graph. Then answer each question below.

- What does the horizontal axis represent? What variable did you use to identify it?
- What does the vertical axis represent? What variable did you use to identify it?
- How are the ordered pairs in the graph related to the values in the table?
- How are the horizontal and vertical distances shown on the graph related to the values in the table?

c. How can you write an equation that shows how the two variables are related?

d. What does the green line in the graph represent?
Work with a partner. Use the equation you wrote in Activity 1.

a. How is this equation different from the equations earlier in this chapter?

b. One of the variables in this equation depends on the other variable. Determine which variable is which by answering the following questions:
   - Does the amount of money you earn depend on the number of hours you work?
   - Does the number of hours you work depend on the amount of money you earn?

What do you think is the significance of having two types of variables? How do you think you can use these types of variables in real life?

Work with a partner. Recall that the perimeter of a square is 4 times its side length.

a. Write the formula for the perimeter of a square. Tell what each variable represents.

b. Describe how the perimeter of a square changes as its side length increases by 1 unit. Use a table and a graph to support your answer.

c. In your formula, which variable depends on which?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you write an equation in two variables?

5. The equation \( y = 7.75x \) shows how the number of movie tickets is related to the total amount of money spent. Describe what each part of the equation represents.

6. **CHOOSE TOOLS** In Activity 1, you want to know the amount of money you earn after working 30.5 hours during a week. Would you use the table, the graph, or the equation to find your earnings? What are your earnings? Explain your reasoning.

7. Give an example of another real-life situation that you can model by an equation in two variables.

Use what you learned about equations in two variables to complete Exercises 4 and 5 on page 319.
An **equation in two variables** represents two quantities that change in relationship to one another. A **solution of an equation in two variables** is an ordered pair that makes the equation true.

**EXAMPLE 1** Identifying Solutions of Equations in Two Variables

Tell whether the ordered pair is a solution of the equation.

a. \( y = 2x; (3, 6) \)

\[
6 = 2(3) \quad \text{Substitute.} \\
6 = 6 \quad \text{✓ Compare.}
\]

So, \((3, 6)\) is a solution.

b. \( y = 4x - 3; (4, 12) \)

\[
12 = 4(4) - 3 \\
12 \neq 13 \quad \times
\]

So, \((4, 12)\) is not a solution.

You can use equations in two variables to represent situations involving two related quantities. The variable representing the quantity that can change freely is the **independent variable**. The other variable is called the **dependent variable** because its value *depends* on the independent variable.

**EXAMPLE 2** Using an Equation in Two Variables

The equation \( y = 128 - 8x \) gives the amount \( y \) (in fluid ounces) of milk remaining in a gallon jug after you pour \( x \) cups.

a. Identify the independent and dependent variables.

\[ \because \text{Because the amount } y \text{ remaining depends on the number } x \text{ of cups you pour, } y \text{ is the dependent variable and } x \text{ is the independent variable.} \]

b. How much milk remains in the jug after you pour 10 cups?

Use the equation to find the value of \( y \) when \( x = 10 \).

\[
y = 128 - 8x \quad \text{Write the equation.} \\
= 128 - 8(10) \quad \text{Substitute 10 for } x. \\
= 48 \quad \text{Simplify.}
\]

\[ \because \text{There are 48 fluid ounces remaining.} \]

**On Your Own**

Tell whether the ordered pair is a solution of the equation.

1. \( y = 7x; (2, 21) \)
2. \( y = 5x + 1; (3, 16) \)
3. The equation \( y = 10x + 25 \) gives the amount \( y \) (in dollars) in your savings account after \( x \) weeks.

a. Identify the independent and dependent variables.

b. How much is in your savings account after 8 weeks?

**Key Vocabulary**

- **equation in two variables**, p. 316
- **solution of an equation in two variables**, p. 316
- **independent variable**, p. 316
- **dependent variable**, p. 316
Tables, Graphs, and Equations
You can use tables and graphs to represent equations in two variables.
The table and graph below represent the equation \( y = x + 2 \).

<table>
<thead>
<tr>
<th>Independent Variable, ( x )</th>
<th>Dependent Variable, ( y )</th>
<th>Ordered Pair, ( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>(2, 4)</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>(3, 5)</td>
</tr>
</tbody>
</table>

EXAMPLE 3 Writing and Graphing an Equation in Two Variables
An athlete burns 200 calories weight lifting. The athlete then works out on an elliptical trainer and burns 10 calories for every minute. Write and graph an equation in two variables that represents the total number of calories burned during the workout.

Words
The total number of calories equals calories burned weight lifting plus calories burned per minute times the number of minutes.

Variables
Let \( c \) be the total number of calories burned, and let \( m \) be the number of minutes on the elliptical trainer.

Equation
\[ c = 200 + 10m \]

To graph the equation, first make a table. Then plot the ordered pairs and draw a line through the points.

<table>
<thead>
<tr>
<th>Minutes, ( m )</th>
<th>Equation ( c = 200 + 10m )</th>
<th>Calories, ( c )</th>
<th>Ordered Pair, ( (m, c) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( c = 200 + 10(10) )</td>
<td>300</td>
<td>(10, 300)</td>
</tr>
<tr>
<td>20</td>
<td>( c = 200 + 10(20) )</td>
<td>400</td>
<td>(20, 400)</td>
</tr>
<tr>
<td>30</td>
<td>( c = 200 + 10(30) )</td>
<td>500</td>
<td>(30, 500)</td>
</tr>
</tbody>
</table>

On Your Own
4. It costs $25 to rent a kayak plus $8 for each hour. Write and graph an equation in two variables that represents the total cost of renting the kayak.

Section 7.4 Writing Equations in Two Variables
You can model many rate problems by using the *distance formula* \( d = rt \), where \( d \) is the distance traveled, \( r \) is the speed, and \( t \) is the time. When you are given a speed, you can use the formula to write an equation in two variables that represents the situation.

**Key Idea**

**Distance Formula**

**Words**
To find the distance traveled \( d \), multiply the speed \( r \) by the time \( t \).

**Algebra**

\[ d = rt \]

---

### EXAMPLE 4 Real-Life Application

A train averages 40 miles per hour between two cities. Use a graph to show the relationship between the time and the distance traveled.

**Method 1:** Use a ratio table.

You can use a ratio table and multiplication to find equivalent rates. Then plot the ordered pairs (time, distance) from the table and draw a line through the points.

**Time (hours)**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
</table>

**Distance (miles)**

| 40 | 80 | 160 | 240 |

**Method 2:** Use an equation in two variables.

Use the distance formula to write the equation \( d = 40t \). Use the equation to make a table. Then plot the ordered pairs and draw a line through the points, as shown in the graph above.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>( d = 40t )</th>
<th>Distance (miles), ( d )</th>
<th>Ordered Pair, ((t, d))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( d = 40(1) )</td>
<td>40</td>
<td>((1, 40))</td>
</tr>
<tr>
<td>2</td>
<td>( d = 40(2) )</td>
<td>80</td>
<td>((2, 80))</td>
</tr>
<tr>
<td>4</td>
<td>( d = 40(4) )</td>
<td>160</td>
<td>((4, 160))</td>
</tr>
<tr>
<td>6</td>
<td>( d = 40(6) )</td>
<td>240</td>
<td>((6, 240))</td>
</tr>
</tbody>
</table>

---

5. **WHAT IF?** The train averages 50 miles per hour. Use a graph to show the relationship between the time and the distance traveled.
7.4 Exercises

Vocabulary and Concept Check

1. **VOCABULARY** How are independent variables and dependent variables different?

2. **PRECISION** Explain how to graph an equation in two variables.

3. **WHICH ONE DOESN'T BELONG?** Which one does not belong with the other three? Explain your reasoning.

   - $y = 12x + 25$
   - $c = 10t - 5$
   - $a = 7b + 11$
   - $n = 4n - 6$

Practice and Problem Solving

Write a formula for the given measure. Tell what each variable represents. Identify which variable depends on which in the formula.

4. the perimeter of a rectangle with a length of 5 inches

5. the area of a trapezoid with base lengths of 7 feet and 11 feet

Tell whether the ordered pair is a solution of the equation.

6. $y = 4x$; $(0, 4)$

7. $y = 3x$; $(2, 6)$

8. $y = 5x - 10$; $(3, 5)$

9. $y = x + 7$; $(1, 6)$

10. $y = 7x + 2$; $(2, 0)$

11. $y = 2x - 3$; $(4, 5)$

12. **ERROR ANALYSIS** Describe and correct the error in finding a solution of the equation in two variables.

   - $y = 3x + 2$; $(5, 1)$
   - $5 = 3(1) + 2$
   - $5 = 5$

   So, $(5, 1)$ is not a solution.

Identify the independent and dependent variables.

13. The equation $A = 25w$ gives the area $A$ (in square feet) of a rectangular dance floor with a width of $w$ feet.

14. The equation $c = 0.09s$ gives the amount $c$ (in dollars) of commission a salesperson receives for making a sale of $s$ dollars.

15. The equation $t = 12p + 12$ gives the total cost $t$ (in dollars) of a meal with a tip of $p$ percent (in decimal form).

16. The equation $h = 60 - 4m$ gives the height $h$ (in inches) of the water in a tank $m$ minutes after it starts to drain.

17. **DRUM SET** The equation $b = 540 - 30m$ gives the balance $b$ (in dollars) that you owe on a drum set after $m$ monthly payments. What is the balance after 9 monthly payments?

Section 7.4 Writing Equations in Two Variables 319
OPEN-ENDED  Complete the table by describing possible independent or dependent variables.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. The number of hours you study for a test</td>
<td></td>
</tr>
<tr>
<td>19. The speed you are pedaling a bike</td>
<td></td>
</tr>
<tr>
<td>20.</td>
<td>Your monthly cell phone bill</td>
</tr>
<tr>
<td>21.</td>
<td>The amount of money you earn</td>
</tr>
</tbody>
</table>

22. **PIZZA**  A cheese pizza costs $5. Additional toppings cost $1.50 each. Write and graph an equation in two variables that represents the total cost of a pizza.

23. **GYM MEMBERSHIP**  It costs $35 to join a gym. The monthly fee is $25. Write and graph an equation in two variables that represents the total cost of a gym membership.

24. **TEXTING**  The maximum size of a text message is 160 characters. A space counts as one character.
   - a. Write an equation in two variables that represents the remaining (unused) characters in a text message as you type.
   - b. Identify the independent and dependent variables.
   - c. How many characters remain in the message shown?

25. **CHOOSE TOOLS**  A car averages 60 miles per hour on a road trip. Use a graph to show the relationship between the time and the distance traveled. What method did you use to create your graph?

Write and graph an equation in two variables that shows the relationship between the time and the distance traveled.

26. Moves 2 meters every 3 hours.

27. Rises 5 stories every 6 seconds.

28. Moves 660 feet every 10 seconds.

29. Moves 960 kilometers every 4 minutes.
Fill in the blank so that the ordered pair is a solution of the equation.

30. \( y = 8x + 3; (1, \quad) \)
31. \( y = 12x + 2; (\quad, 14) \)
32. \( y = 22 - 9x; (\quad, 4) \)

33. **CRITICAL THINKING** Can the dependent variable cause a change in the independent variable? Explain.

34. **OPEN-ENDED** Write an equation in two variables that has \((3, 4)\) as a solution.

35. **WALKING** You walk 5 city blocks in 12 minutes. How many city blocks can you walk in 2 hours?

36. **ANT** How fast should the ant walk to go around the rectangle in 4 minutes?

37. **LIGHTNING** To estimate how far you are from lightning (in miles), count the number of seconds between a lightning flash and the thunder that follows. Then divide the number of seconds by 5. Use a graph to show the relationship between the time and the distance. Describe the method you used to create your graph.

38. **PROBLEM SOLVING** You and a friend start biking in opposite directions from the same point. You travel 108 feet every 8 seconds. Your friend travels 63 feet every 6 seconds.
   a. How far apart are you and your friend after 15 minutes?
   b. After 20 minutes, you take a 5-minute rest, but your friend does not. How far apart are you and your friend after 40 minutes? Explain your reasoning.

39. **REASONING** The graph represents the cost \(c\) (in dollars) of buying \(n\) tickets to a baseball game.
   a. Should the points be connected with a line to show all the solutions? Explain your reasoning.
   b. Write an equation in two variables that represents the graph.

---

**Write the fraction as a percent.** *(Section 5.5)*

40. \( \frac{3}{10} \)
41. \( \frac{4}{5} \)
42. \( \frac{9}{20} \)
43. \( \frac{17}{25} \)

44. **MULTIPLE CHOICE** What is the area of the triangle? *(Section 4.2)*
   
   \[ \begin{array}{c}
   \text{A} \quad 36 \text{ cm}^2 \\
   \text{B} \quad 68 \text{ cm}^2 \\
   \text{C} \quad 72 \text{ cm}^2 \\
   \text{D} \quad 76.5 \text{ cm}^2 \\
   \end{array} \]
You can use an example and non-example chart to list examples and non-examples of a vocabulary word or term. Here is an example and non-example chart for equations.

### Equations

<table>
<thead>
<tr>
<th>Examples</th>
<th>Non-Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = 5$</td>
<td>5</td>
</tr>
<tr>
<td>$2a = 16$</td>
<td>$2a$</td>
</tr>
<tr>
<td>$x + 4 = 19$</td>
<td>$x + 4$</td>
</tr>
<tr>
<td>$5 = x + 3$</td>
<td>$x + 3$</td>
</tr>
<tr>
<td>$12 - 7 = 5$</td>
<td>$12 - 7$</td>
</tr>
<tr>
<td>$\frac{3}{4}y = 6$</td>
<td>3</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>4</td>
</tr>
</tbody>
</table>

**On Your Own**

Make example and non-example charts to help you study these topics.

1. inverse operations
2. equations solved using addition or subtraction
3. equations solved using multiplication or division
4. equations in two variables

After you complete this chapter, make example and non-example charts for the following topics.

5. inequalities
6. graphs of inequalities
7. inequalities solved using addition or subtraction
8. inequalities solved using multiplication or division

“I need a good non-example of a cool animal for my example and non-example chart.”
Write the word sentence as an equation. (Section 7.1)
1. A number \(x\) decreased by 3 is 5.
2. A number \(a\) divided by 7 equals 14.

Solve the equation. Check your solution. (Section 7.2 and Section 7.3)
3. \(4 + k = 14\)
4. \(3.5 = m - 2.2\)
5. \(8 = \frac{4w}{3}\)
6. \(31 = 6.2 \cdot y\)

Tell whether the ordered pair is a solution of the equation. (Section 7.4)
7. \(y = 6x; (3, 24)\)
8. \(y = 3x + 4; (4, 16)\)

Write and graph an equation in two variables that shows the relationship between the time and the distance traveled. (Section 7.4)
9. Moves 900 feet every 10 seconds.

10. Rises 4 feet in 9 seconds.

11. RIBBON The length of the blue ribbon is two-thirds the length of the red ribbon. Write an equation you can use to find the length \(r\) of the red ribbon. (Section 7.1)

12. BRIDGES The main span of the Sunshine Skyway Bridge is 360 meters long. The Skyway’s main span is 30 meters shorter than the main span of the Dames Point Bridge. Write and solve an equation to find the length \(l\) of the main span of the Dames Point Bridge. (Section 7.2)

13. SHOPPING At a farmer’s market, you buy 4 pounds of tomatoes and 2 pounds of sweet potatoes. You spend 80% of the money in your wallet. Write and solve an equation to find how much money is in your wallet before you pay. (Section 7.3)

14. SUNDAE A sundae costs $2. Additional toppings cost $0.50 each. Write and graph an equation in two variables that represents the total cost of a sundae. (Section 7.4)
Essential Question  How can you use a number line to represent solutions of an inequality?

1 ACTIVITY: Understanding Inequality Statements

Work with a partner. Read the statement. Circle each number that makes the statement true, and then answer the questions.

a. “Your friend is more than 3 minutes late.”

\[ -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]

- What do you notice about the numbers that you circled?
- Is the number 3 included? Why or why not?
- Write four other numbers that make the statement true.

b. “The temperature is at most 2 degrees.”

\[ -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

- What do you notice about the numbers that you circled?
- Can the temperature be exactly 2 degrees? Explain.
- Write four other numbers that make the statement true.

c. “You need at least 4 pieces of paper for your math homework.”

\[ -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]

- What do you notice about the numbers that you circled?
- Can you have exactly 4 pieces of paper? Explain.
- Write four other numbers that make the statement true.

d. “After playing a video game for 20 minutes, you have fewer than 6 points.”

\[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \]

- What do you notice about the numbers that you circled?
- Is the number 6 included? Why or why not?
- Write four other numbers that make the statement true.
Work with a partner.

a. Consider the statement “x is a number such that \( x < 2 \).”
   - Can the number be exactly 2? Explain.
   - Circle each number that makes the statement true.
     
     \[ -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]
   - Write four other numbers that make the statement true.

b. Consider the statement “x is a number such that \( x \geq 1 \).”
   - Can the number be exactly 1? Explain.
   - Circle each number that makes the statement true.
     
     \[ -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]
   - Write four other numbers that make the statement true.

Work with a partner.

a. Which number line shows \( x > 0 \)? Which number line shows \( x \geq 0 \)? Explain your reasoning.

b. Write the least positive number you can think of that is still a solution of the inequality \( x > 0 \). Explain your reasoning.

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use a number line to represent solutions of an inequality?

5. Write an inequality. Graph all solutions of your inequality on a number line.

6. Graph the inequalities \( x > 9 \) and \( 9 < x \) on different number lines. What do you notice?

Practice

Use what you learned about graphing inequalities to complete Exercises 17–20 on page 329.
An **inequality** is a mathematical sentence that compares expressions. It contains the symbols $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine where to place the inequality symbol.

**Key Vocabulary**
- inequality, p. 326
- solution of an inequality, p. 327
- solution set, p. 327
- graph of an inequality, p. 328

<table>
<thead>
<tr>
<th>Inequality Symbols</th>
<th>Symbol</th>
<th>$&lt;$</th>
<th>$&gt;$</th>
<th>$\leq$</th>
<th>$\geq$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key Phrases</td>
<td>• is less than</td>
<td>• is greater than</td>
<td>• is less than or equal to</td>
<td>• is greater than or equal to</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• is fewer than</td>
<td>• is more than</td>
<td>• is at most</td>
<td>• is at least</td>
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</table>

**EXAMPLE 1 Writing Inequalities**

Write the word sentence as an inequality.

a. A number $c$ is less than $-4$.

A number $c$ is less than $-4$.

\[
\begin{align*}
\text{A number } c & \text{ is less than } -4. \\
\end{align*}
\]

An inequality is $c < -4$.

b. A number $k$ plus 5 is greater than or equal to 8.

A number $k$ plus 5 is greater than or equal to 8.

\[
\begin{align*}
\text{A number } k + 5 & \text{ is greater than or equal to } 8. \\
\end{align*}
\]

An inequality is $k + 5 \geq 8$.

c. Four times a number $q$ is at most 16.

Four times a number $q$ is at most 16.

\[
\begin{align*}
\text{Four times a number } q & \text{ is at most } 16. \\
\end{align*}
\]

An inequality is $4q \leq 16$.

**On Your Own**

Write the word sentence as an inequality.

1. A number $n$ is greater than 1.
2. Twice a number $p$ is fewer than 7.
3. A number $w$ minus 3 is less than or equal to 10.
4. A number $z$ divided by 2 is at least $-6$. 

---

**Check It Out**

Lesson Tutorials

<table>
<thead>
<tr>
<th>Inequality Symbols</th>
<th>Symbol</th>
<th>$&lt;$</th>
<th>$&gt;$</th>
<th>$\leq$</th>
<th>$\geq$</th>
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<td></td>
</tr>
<tr>
<td></td>
<td>• is fewer than</td>
<td>• is more than</td>
<td>• is at most</td>
<td>• is at least</td>
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**Multi-Language Glossary at BigIdeasMath.com**
A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

<table>
<thead>
<tr>
<th>Value of ( x )</th>
<th>( x + 3 \leq 7 )</th>
<th>Is the inequality true?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( 3 + 3 \leq 7 )</td>
<td>( 6 \leq 7 ) ✓</td>
</tr>
<tr>
<td>4</td>
<td>( 4 + 3 \leq 7 )</td>
<td>( 7 \leq 7 ) ✓</td>
</tr>
<tr>
<td>5</td>
<td>( 5 + 3 \leq 7 )</td>
<td>( 8 \not\leq 7 ) ✗</td>
</tr>
</tbody>
</table>

**Example 2** Checking Solutions

Tell whether the given value is a solution of the inequality.

**a.** \( x + 1 > 7; \ x = 8 \)

\[
\begin{align*}
x + 1 & > 7 \\
8 + 1 & > 7 \\
9 & > 7 \ 
\end{align*}
\]

So, 8 is a solution of the inequality.

**b.** \( 7y < 27; \ y = 4 \)

\[
\begin{align*}
7y & < 27 \\
7(4) & < 27 \\
28 & \not< 27 \ 
\end{align*}
\]

So, 4 is *not* a solution of the inequality.

**c.** \( \frac{z}{3} \geq 5; \ z = 15 \)

\[
\begin{align*}
\frac{z}{3} & \geq 5 \\
\frac{15}{3} & \geq 5 \\
5 & \geq 5 \ 
\end{align*}
\]

So, 15 is a solution of the inequality.

**On Your Own**

Tell whether 3 is a solution of the inequality.

5. \( b + 4 < 6 \) 

6. \( 9 - n \geq 6 \) 

7. \( 18 \div x \leq 10 \)
The **graph of an inequality** shows all the solutions of the inequality on a number line. An open circle ○ is used when a number is *not* a solution. A closed circle ● is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

**EXAMPLE 3**  **Graphing an Inequality**

Graph \( g > 2 \).

Use an open circle because 2 is *not* a solution.

Test a number to the left of 2.
\( g = 0 \) is *not* a solution.

Test a number to the right of 2.
\( g = 3 \) is a solution.

Shade the number line on the side where you found the solution. The graph shows there are infinitely many solutions.

**EXAMPLE 4**  **Real-Life Application**

The NASA Solar Probe Plus can withstand temperatures up to and including 2600°F. Write and graph an inequality that represents the temperatures the probe can withstand.

**Words**
- temperatures up to and including 2600°F

**Variable**
- Let \( t \) be the temperatures the probe can withstand.

**Inequality**
\[ t \leq 2600 \]

An inequality is \( t \leq 2600 \).

**On Your Own**

Graph the inequality on a number line.

8. \( a < 4 \)
9. \( f \leq 7 \)
10. \( n > 0 \)
11. \( p \geq -3 \)

Write and graph an inequality for the situation.

12. A cruise ship can carry at most 3500 passengers.
13. A board game is designed for ages 12 and up.
7.5 Exercises

**Vocabulary and Concept Check**

1. **VOCABULARY** How are greater than and greater than or equal to similar? How are they different?

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write “both” inequalities.
   - A number \( n \) is at most 3.
   - A number \( n \) is at least 3.
   - A number \( n \) is less than or equal to 3.
   - A number \( n \) is no more than 3.

3. **WRITING** Explain how the graph of \( x \leq 6 \) is different from the graph of \( x < 6 \).

4. **WRITING** Are the graphs of \( x \leq 5 \) and \( 5 \geq x \) the same or different? Explain.

**Practice and Problem Solving**

Write the word sentence as an inequality.

1. A number \( k \) is less than 10.
2. A number \( a \) is more than 6.
3. A number \( z \) is fewer than \( \frac{3}{4} \).
4. A number \( b \) is at least \( -3 \).
5. One plus a number \( y \) is no more than \( -13 \).
6. A number \( x \) divided by 3 is at most 5.

Tell whether the given value is a solution of the inequality.

1. \( x - 1 \leq 7; \ x = 6 \)
2. \( y + 5 < 13; \ y = 17 \)
3. \( 3z > 6; \ z = 3 \)
4. \( \frac{b}{2} \geq 6; \ b = 10 \)
5. \( c + 2.5 < 4.3; \ c = 1.8 \)
6. \( a \leq 0; \ a = -5 \)

Match the inequality with its graph.

17. \( x \geq 2 \)  
18. \( x < 2 \)  
19. \( x > -2 \)  
20. \( x \leq -2 \)

**A.**

**B.**

**C.**

**D.**

Section 7.5 Writing and Graphing Inequalities 329
Write an inequality and a word sentence that represent the graph.

21. \[ -3 \leq x \leq 2 \]
22. \[ -2 \leq x \leq 3 \]
23. \[ -6 \leq x \leq 4 \]
24. \[ -5 \leq x \leq 20 \]

Graph the inequality on a number line.

25. \( a > 4 \)
26. \( n \geq 8 \)
27. \( 3 \geq x \)
28. \( y < \frac{1}{2} \)
29. \( x < \frac{2}{9} \)
30. \( -3 \geq c \)
31. \( m > -5 \)
32. \( 0 \leq b \)
33. \( 1.5 > f \)
34. \( t \geq -\frac{1}{2} \)
35. \( p > -1.6 \)
36. \( \frac{7}{3} > z \)

ERROR ANALYSIS  Describe and correct the error in graphing the inequality.

37. \( x \geq 1 \)
38. \( x > -1 \)

39. **FISHING** You are fishing and are allowed to catch at most 3 striped bass. Each striped bass must be no less than 18 inches long.
   a. Write and graph an inequality to represent the number of striped bass you are allowed to catch.
   b. Write and graph an inequality to represent the length of each striped bass you are allowed to catch.

40. **MODELING** For a food to be labeled low sodium, there must be no more than 140 milligrams of sodium per serving.
   a. Write and graph an inequality to represent the amount of sodium in a low-sodium serving.
   b. Write and graph an inequality to represent the amount of sodium in a serving that does not qualify as low sodium.
   c. Does the food represented by the nutrition facts label qualify as a low-sodium food? Explain.

**Nutrition Facts**

Serving Size ½ cup (114g)
Servings Per Container 4

<table>
<thead>
<tr>
<th>Amount Per Serving</th>
<th>% Daily Value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calories 90</td>
<td></td>
</tr>
<tr>
<td>Calories from fat 30</td>
<td></td>
</tr>
<tr>
<td>Total Fat 3g</td>
<td>5%</td>
</tr>
<tr>
<td>Saturated Fat 0g</td>
<td>0%</td>
</tr>
<tr>
<td>Cholesterol 0mg</td>
<td>0%</td>
</tr>
<tr>
<td>Sodium 300mg</td>
<td>13%</td>
</tr>
<tr>
<td>Total Carbohydrate 13g</td>
<td>4%</td>
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<tr>
<td>Dietary Fiber 3g</td>
<td>12%</td>
</tr>
<tr>
<td>Sugars 3g</td>
<td></td>
</tr>
<tr>
<td>Protein 3g</td>
<td></td>
</tr>
<tr>
<td>Vitamin A 80%</td>
<td></td>
</tr>
<tr>
<td>Vitamin C 60%</td>
<td></td>
</tr>
<tr>
<td>Calcium 4%</td>
<td></td>
</tr>
<tr>
<td>Iron 4%</td>
<td></td>
</tr>
</tbody>
</table>
41. **SHOPPING** You have $33. You want to buy a necklace and one other item from the list.
   a. Write an inequality to represent the situation.
   b. Can the other item be a T-shirt? Explain.
   c. Can the other item be a book? Explain.

Determine whether the statement is *sometimes, always, or never* true. Explain your reasoning.

42. A number that is a solution of the inequality \( x > 5 \) is also a solution of the inequality \( x \geq 5 \).

43. A number that is a solution of the inequality \( 5 \leq x \) is also a solution of the inequality \( x > 5 \).

44. **BUS RIDE** A bus ride costs $1.50. A 30-day bus pass costs $36. Write an inequality to represent the number of bus rides you would need to take for the bus pass to be a better deal.

45. **MOVIE THEATER** Fifty people are seated in a movie theater. The maximum capacity of the theater is 425 people. Write an inequality to represent the number of additional people who can still be seated.

46. **Critical Thinking** The map shows the elevations above sea level for an area of land.

   a. Graph the possible elevations of A. Write the set of elevations as two inequalities.
   b. Graph the possible elevations of C. How can you write this set of elevations as a single inequality? Explain.
   c. What is the elevation of B? Explain.

---

**Fair Game Review** What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 7.2)*

47. \( x + 3 = 12 \)  
48. \( x - 6 = 8 \)  
49. \( 16 + x = 44 \)  
50. \( 7.6 = x - 6.5 \)

51. **MULTIPLE CHOICE** A stack of boards is 24 inches high. Each board is \( \frac{3}{8} \) inch thick. How many boards are in the stack? *(Section 2.2)*

   A \( \frac{1}{9} \)  
   B \( \frac{1}{6} \)  
   C 9  
   D 64
Essential Question: How can you use addition or subtraction to solve an inequality?

1. **ACTIVITY: Writing an Inequality**

   Work with a partner. In 3 years, your friend will still not be old enough to vote.
   
   a. Which of the following represents your friend’s situation? What does $x$ represent? Explain your reasoning.
      
      $$x + 3 < 18$$  
      $$x + 3 \leq 18$$  
      $$x + 3 > 18$$  
      $$x + 3 \geq 18$$

   b. Graph the possible ages of your friend on a number line. Explain how you decided what to graph.

2. **ACTIVITY: Writing an Inequality**

   Work with a partner. Baby manatees are about 4 feet long at birth. They grow to a maximum length of 13 feet.
   
   a. Which of the following can represent a baby manatee’s growth? What does $x$ represent? Explain your reasoning.
      
      $$x + 4 < 13$$  
      $$x + 4 \leq 13$$  
      $$x - 4 > 13$$  
      $$x - 4 \geq 13$$

   b. Graph the solution on a number line. Explain how you decided what to graph.
5. **IN YOUR OWN WORDS**  How can you use addition or subtraction to solve an inequality?

6. Describe a real-life situation that you can represent with an inequality. Write the inequality. Graph the solution on a number line.

Use what you learned about solving inequalities to complete Exercises 5–7 on page 336.
**Study Tip**

You can solve inequalities the same way you solve equations. Use inverse operations to get the variable by itself.

**Key Ideas**

### Addition Property of Inequality

**Words** When you add the same number to each side of an inequality, the inequality remains true.

**Numbers**  
3 < 5  
+ 2 + 2  
5 < 7

**Algebra**  
x - 4 > 5  
+ 4 + 4  
x > 9

**Graph**

![Graph of x > 9]

### Subtraction Property of Inequality

**Words** When you subtract the same number from each side of an inequality, the inequality remains true.

**Numbers**  
3 < 5  
- 2 - 2  
1 < 3

**Algebra**  
x + 4 > 5  
- 4 - 4  
x > 1

**Graph**

![Graph of x > 1]

These properties are also true for ≤ and ≥.

**EXAMPLE 1**

**Solving an Inequality Using Addition**

Solve \( x - 3 > 1 \). Graph the solution.

\[
\begin{align*}
x - 3 &> 1 \\
x &> 4
\end{align*}
\]

**Check:**

- \( x = 3: \quad 3 - 3 \neq 1 \)  
- \( x = 5: \quad 5 - 3 > 1 \)  

The solution is \( x > 4 \).

**On Your Own**

Solve the inequality. Graph the solution.

1. \( x - 2 < 3 \)
2. \( x - 6 \geq 4 \)
3. \( 10 \geq x - 1 \)
EXAMPLE 2  Solving an Inequality Using Subtraction

Solve \(15 \geq 6 + x\). Graph the solution.

\[
egin{align*}
15 & \geq 6 + x \\
-6 & \quad -6 \\
9 & \geq x
\end{align*}
\]

Undo the addition.

\[
\begin{align*}
x \leq 9
\end{align*}
\]

The solution is \(x \leq 9\).

Reading

The inequality \(x \leq 9\) is the same as \(9 \geq x\).

On Your Own

Solve the inequality. Graph the solution.

4. \(x + 3 > 7\)  
5. \(y + 2 < 17\)  
6. \(16 \leq m + 9\)

EXAMPLE 3  Real-Life Application

A flea market advertises that it has more than 250 vending booths. Of these, 184 are currently filled. Write and solve an inequality to represent the number of vending booths still available.

Words

The number of booths filled plus the number of remaining booths is greater than the total number of booths.

Variable

Let \(b\) be the number of remaining booths.

Inequality

\[
184 + b > 250
\]

\[
184 + b > 250 \quad \text{Write the inequality.}
\]

\[
-184 \quad -184
\]

\[
b > 66 \quad \text{Subtraction Property of Inequality}
\]

More than 66 vending booths are still available.

On Your Own

7. You have already spent $24 shopping online for clothes. Write and solve an inequality to represent the additional amount you must spend to get free shipping.
7.6 Exercises

Vocabulary and Concept Check

1. OPEN-ENDED Write an inequality that can be solved by subtracting 7 from each side.

2. WRITING Explain how to solve the inequality \( x - 6 > 3 \).

3. WRITING Describe the graph of the solution of \( x + 3 \leq 4 \).

4. OPEN-ENDED Write an inequality that the graph represents. Then use the Subtraction Property of Inequality to write another inequality that the graph represents.

Practice and Problem Solving

Solve the inequality. Graph the solution.

5. \( x - 4 < 5 \)
6. \( 5 + h > 7 \)
7. \( 3 \geq y - 2 \)
8. \( 9 \leq c + 1 \)
9. \( 18 > 12 + x \)
10. \( 37 + z \leq 54 \)
11. \( y - 21 < 85 \)
12. \( g - 17 \geq 17 \)
13. \( 7.2 < x + 4.2 \)
14. \( 12.7 \geq s - 5.3 \)
15. \( \frac{3}{4} \leq \frac{1}{2} + n \)
16. \( \frac{1}{3} + b > \frac{3}{4} \)

17. ERROR ANALYSIS Describe and correct the error in solving the inequality.

18. AIR TRAVEL Your carry-on bag can weigh at most 40 pounds. Write and solve an inequality to represent how much more weight you can add to the bag and still meet the requirement.

19. SHOPPING It costs \( x \) for a round-trip bus ticket to the mall. You have $24. Write and solve an inequality to represent how much money you can spend for the bus fare and still have enough to buy the baseball cap.

Write the word sentence as an inequality. Then solve the inequality.

20. Five more than a number is less than 17.
21. Three less than a number is more than 15.

336 Chapter 7 Equations and Inequalities
Solve the inequality. Graph the solution.

22. \( x + 9 - 3 \leq 14 \)  
23. \( 44 > 7 + s + 26 \)  
24. \( 6.1 - 0.3 \geq c + 1 \)

25. **VIDEO GAME** The high score for a video game is 36,480. Your current score is 34,280. Each dragonfly you catch is worth 1 point. You also get a 1000-point bonus for reaching 35,000 points. Write and solve an inequality to represent the number of dragonflies you must catch to earn a new high score.

26. **PICKUP TRUCKS** You can register a pickup truck as a passenger vehicle if the truck is not used for commercial purposes and the weight of the truck with its contents does not exceed 8500 pounds.
   a. Your pickup truck weighs 4200 pounds. Write an inequality to represent the number of pounds your truck can carry and still qualify as a passenger vehicle. Then solve the inequality.
   b. A cubic yard of sand weighs about 1600 pounds. How many cubic yards of sand can you haul in your truck and still qualify as a passenger vehicle? Explain your reasoning.

27. **TRIATHLON** You complete two events of a triathlon. Your goal is to finish with an overall time of less than 100 minutes.
   a. Write and solve an inequality to represent how many minutes you can take to finish the running event and still meet your goal.
   b. The running event is 3.1 miles long. Estimate how many minutes it would take you to run 3.1 miles. Would this time allow you to reach your goal? Explain your reasoning.

28. **Number Sense** The possible values of \( x \) are given by \( x - 3 \geq 2 \). What is the least possible value of \( 5x \)?

29. \( \frac{t}{12} = 4 \)  
30. \( 6 = \frac{2s}{9} \)  
31. \( 8x = 72 \)  
32. \( 9 = 1.5z \)

33. **MULTIPLE CHOICE** Which brand of turkey is the best buy? (Section 5.4)
   - **A** Brand A
   - **B** Brand B
   - **C** Brand C
   - **D** Brand D

<table>
<thead>
<tr>
<th>Brand</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost (dollars)</strong></td>
<td>10.38</td>
<td>13.47</td>
<td>21.45</td>
<td>34.93</td>
</tr>
<tr>
<td><strong>Pounds</strong></td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

**Fair Game Review** What you learned in previous grades & lessons

Solve the equation. Check your solution. (Section 7.3)

Section 7.6 Solving Inequalities Using Addition or Subtraction
Essential Question: How can you use multiplication or division to solve an inequality?

1. **ACTIVITY: Writing an Inequality**

Work with a partner. A store has a clearance rack of shirts that each cost the same amount. You buy 2 shirts and have money left after paying with a $20 bill.

   a. Which of the following represents your purchase? What does $x$ represent? Explain your reasoning.

   \[
   \begin{align*}
   2x &< 20 \\
   2x &> 20 \\
   2x &\leq 20 \\
   2x &\geq 20
   \end{align*}
   \]

   b. Graph the possible values of $x$ on a number line. Explain how you decided what to graph.

   c. Can you buy a third shirt? Explain your reasoning.

2. **ACTIVITY: Writing an Inequality**

Work with a partner. One of your favorite stores is having a 75% off sale. You have $20. You want to buy a pair of jeans.

   a. Which of the following represents your ability to buy the jeans with $20? What does $x$ represent? Explain your reasoning.

   \[
   \begin{align*}
   \frac{1}{4}x &< 20 \\
   \frac{1}{4}x &> 20 \\
   \frac{1}{4}x &\leq 20 \\
   \frac{1}{4}x &\geq 20
   \end{align*}
   \]

   b. Graph the possible values of $x$ on a number line. Explain how you decided what to graph.

   c. Can you afford a pair of jeans that originally costs $100? Explain your reasoning.
3 Activity: Solving Inequalities

Work with a partner. Complete the following steps for Activity 1. Then repeat the steps for Activity 2.

- Use your inequality from part (a). Replace the inequality symbol with an equal sign.
- Solve the equation.
- Replace the equal sign with the original inequality symbol.
- Graph this new inequality.
- Compare the graph with your graph in part (b). What do you notice?

4 Activity: Matching Inequalities

Work with a partner. Match the inequality with its graph. Explain your method.

- a. $3x < 9$
- b. $3x \leq 9$
- c. $\frac{x}{2} \geq 1$
- d. $6 < 2x$
- e. $12 \leq 4x$
- f. $\frac{x}{2} < 2$

Use what you learned about solving inequalities to complete Exercises 8–11 on page 342.

Section 7.7 Solving Inequalities Using Multiplication or Division

5. In your own words How can you use multiplication or division to solve an inequality?
**Key Ideas**

### Multiplication Property of Inequality

**Words**  When you multiply each side of an inequality by the same positive number, the inequality remains true.

**Numbers**

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8 &gt; 6$</td>
<td>$\frac{x}{4} &lt; 2$</td>
</tr>
<tr>
<td>$8 \times 2 &gt; 6 \times 2$</td>
<td>$\frac{x \cdot 4}{4} &lt; 2 \cdot 4$</td>
</tr>
<tr>
<td>$16 &gt; 12$</td>
<td>$x &lt; 8$</td>
</tr>
</tbody>
</table>

### Division Property of Inequality

**Words**  When you divide each side of an inequality by the same positive number, the inequality remains true.

**Numbers**

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8 &gt; 6$</td>
<td>$4x &lt; 8$</td>
</tr>
<tr>
<td>$8 \div 2 &gt; 6 \div 2$</td>
<td>$\frac{4x}{4} &lt; \frac{8}{4}$</td>
</tr>
<tr>
<td>$4 &gt; 3$</td>
<td>$x &lt; 2$</td>
</tr>
</tbody>
</table>

These properties are also true for $\leq$ and $\geq$.

### Example 1

**Solving an Inequality Using Multiplication**

Solve $\frac{x}{5} \leq 2$. Graph the solution.

```
\frac{x}{5} \leq 2  \quad \text{Write the inequality.}
```

```
\frac{x}{5} \cdot 5 \leq 2 \cdot 5  \quad \text{Undo the division.}
```

```
\frac{x}{5} \cdot 5 \leq 2 \cdot 5  \quad \text{Multiplication Property of Inequality}
```

```
x \leq 10  \quad \text{Simplify.}
```

:The solution is $x \leq 10$.

```
\text{The solution is } x \leq 10.
```

### On Your Own

**Solve the inequality. Graph the solution.**

1. $p + 3 > 2$
2. $\frac{3}{5}q \leq 6$
3. $1 < \frac{s}{7}$
EXAMPLE 2 Solving an Inequality Using Division

Solve $4n > 32$. Graph the solution.

$$4n > 32 \quad \text{Write the inequality.}$$

$$\frac{4n}{4} > \frac{32}{4} \quad \text{Division Property of Inequality}$$

$$n > 8 \quad \text{Simplify.}$$

The solution is $n > 8$.

EXAMPLE 3 Real-Life Application

A one-way bus ride costs $1.75. A 30-day bus pass costs $42.

a. Write and solve an inequality to find the least number of one-way rides you must take for the 30-day pass to be a better deal.

b. You ride the bus an average of 20 times each month. Is the pass a better deal? Explain.

a. Words The price of a times the number of one-way rides is more than $42.

Variable Let $r$ be the number of one-way rides.

Inequality $1.75 \cdot r > 42 \quad \text{Write the inequality.}$

$$\frac{1.75r}{1.75} > \frac{42}{1.75} \quad \text{Division Property of Inequality}$$

$$r > 24 \quad \text{Simplify.}$$

So, you need to take more than 24 one-way rides for the pass to be a better deal.

b. No. The cost of 20 one-way rides is less than $42. So, the pass is not a better deal.

On Your Own

Solve the inequality. Graph the solution.

4. $11k \leq 33$  
5. $5 \cdot j > 20$  
6. $50 \leq 2m$

7. The sign shows the toll for driving on Alligator Alley. Write and solve an inequality to represent the number of times someone can drive on Alligator Alley with $15.
7.7 Exercises

Vocabulary and Concept Check

1. REASONING How is the graph of the solution of \(2x \geq 10\) different from the graph of the solution of \(2x = 10\)?

Name the property you should use to solve the inequality.

2. \(3x \leq 27\)
3. \(7x > 49\)
4. \(\frac{x}{2} < 36\)

5. OPEN-ENDED Write two inequalities that have the same solution set: one that you can solve using division and one that you can solve using multiplication.

Practice and Problem Solving

Solve the inequality. Graph the solution.

6. \(\frac{m}{8} < 4\)
7. \(n \div 6 > 2\)
8. \(\frac{f}{3} \geq 15\)
9. \(\frac{1}{11}c \geq 9\)

10. \(12x < 96\)
11. \(5x \geq 25\)
12. \(8 \cdot w \leq 72\)
13. \(7p \leq 42\)

14. \(\frac{3}{4}b > 15\)
15. \(6x < 90\)
16. \(3s \geq 36\)
17. \(\frac{5}{9}v \leq 45\)

18. \(4t > 72\)
19. \(\frac{3}{4}w \leq 24\)
20. \(12m < 132\)
21. \(\frac{5x}{8} \geq 30\)

22. ERROR ANALYSIS Describe and correct the error in solving the inequality.

23. GEOMETRY The length of a rectangle is 8 feet, and its area is less than 168 square feet. Write and solve an inequality to represent the width of the rectangle.

24. PLAYGROUND Students at a playground are divided into 5 equal groups with at least 6 students in each group. Write and solve an inequality to represent the number of students at the playground.

Write the word sentence as an inequality. Then solve the inequality.

25. Eight times a number \(n\) is less than 72.
26. A number \(t\) divided by 32 is at most 4.25.
27. 225 is no less than 12 times a number \(w\).
Graph the numbers that are solutions to both inequalities.

28. \( x + 7 > 9 \) and \( 8x \leq 64 \)

29. \( x - 3 \leq 8 \) and \( 6x < 72 \)

30. **THRILL RIDE** A thrill ride at an amusement park holds a maximum of 12 people per ride.
   a. Write and solve an inequality to find the least number of rides needed for 15,000 people.
   b. Do you think it is possible for 15,000 people to ride the thrill ride in 1 day? Explain.

31. **FOOTBALL** A winning football team more than doubled the offensive yards gained by its opponent. The opponent gained 272 offensive yards. The winning team had 80 offensive plays. Write and solve an inequality to find the possible number of yards per play for the winning team.

32. **LOGIC** Explain how you know that \( 7x < 7x \) has no solution.

33. **OPEN-ENDED** Give an example of a real-life situation in which you can list all the solutions of an inequality. Give an example of a real-life situation in which you cannot list all the solutions of an inequality.

34. **FUNDRAISER** You are selling items from a catalog for a school fundraiser. Write and solve two inequalities to find the range of sales that will earn you between $40 and $50.

Let \( a > b \) and \( x > y \). Tell whether the statement is always true.
Explain your reasoning.

35. \( a + x > b + y \)

36. \( a - x > b - y \)

37. \( ax > by \)

38. \( \frac{a}{x} > \frac{y}{b} \)

### Fair Game Review

What you learned in previous grades & lessons

**Classify the quadrilateral.** *(Skills Review Handbook)*

39.

40.

41.

42. **MULTIPLE CHOICE** On a normal day, 12 airplanes arrive at an airport every 15 minutes. Which rate does not represent this situation? *(Section 5.3)*
   - [A] 24 airplanes every 30 minutes
   - [B] 4 airplanes every 5 minutes
   - [C] 6 airplanes every 5 minutes
   - [D] 48 airplanes each hour
Write the word sentence as an inequality.  
1. A number $x$ is greater than 0.  
2. Twice a number $c$ is at least $-8$.

Tell whether the given value is a solution of the inequality. 
3. $2n > 16; \ n = 9$  
4. $x - 1 \leq 9; \ x = 10$

Graph the inequality on a number line.  
5. $y > -4$  
6. $m \leq \frac{3}{5}$

Solve the inequality. Graph the solution.  
7. $x + 4 \leq 8$  
8. $18 > 16 + g$

Write the word sentence as an inequality. Then solve the inequality.  
9. Two less than a number is more than 15.  
10. Seven more than a number is less than or equal to 27.

Solve the inequality. Graph the solution.  
11. $\frac{3a}{2} < 24$  
12. $121 \geq 11s$

Write the word sentence as an inequality. Then solve the inequality.  
13. Three times a number $x$ is more than 18.  
14. 84 is no less than 7 times a number $k$.

15. WATER PARK Each visit to a water park costs $19.95. An annual pass to the park costs $89.95. Write an inequality to represent the number of times you would need to visit the park for the pass to be a better deal.  

16. GARDEN You want to use a square section of your yard for a garden. You have at most 52 feet of fencing to surround the garden. Write and solve an inequality to represent the possible lengths of each side of the garden.

17. DELIVERY You were planning to spend $12 on a pizza. Write and solve an inequality to represent the additional amount you must spend to get free delivery.
Review Key Vocabulary

- equation, p. 296
- solution, p. 302
- inverse operations, p. 303
- equation in two variables, p. 316
- solution of an equation in two variables, p. 316
- independent variable, p. 316
- dependent variable, p. 316
- inequality, p. 326
- solution of an inequality, p. 327
- solution set, p. 327
- graph of an inequality, p. 328

Review Examples and Exercises

**7.1 Writing Equations in One Variable (pp. 294–299)**

Write the word sentence “The quotient of a number $b$ and 6 is 9” as an equation.

The quotient of a number $b$ and 6 is 9.

\[ b \div 6 = 9 \]

Quotient of means division.

\[ \therefore \text{ An equation is } b \div 6 = 9. \]

**Exercises:**

Write the word sentence as an equation.

1. The product of a number $m$ and 2 is 8.
2. 6 less than a number $t$ is 7.
3. A number $m$ increased by 5 is 7.
4. 8 is the quotient of a number $g$ and 3.

**7.2 Solving Equations Using Addition or Subtraction (pp. 300–307)**

Solve $z + 5 = 13$.

\[ z + 5 = 13 \]

Write the equation.

\[ \begin{align*}
5 & \quad -5 \\
\hline
\end{align*} \]

Subtraction Property of Equality

\[ z = 8 \]

Simplify.

\[ \therefore \text{ The solution is } z = 8. \]

**Exercises:**

Solve the equation. Check your solution.

5. $x - 1 = 8$
6. $m + 7 = 11$
7. $21 = p - 12$
7.3 Solving Equations Using Multiplication or Division (pp. 308–313)

Solve $4c = 32$.

$4c = 32$ Write the equation.

$\frac{4c}{4} = \frac{32}{4}$ Division Property of Equality

$c = 8$ Simplify.

Check

$4c = 32$

$4(8) \overset{?}{=} 32$

$32 = 32$ ✓

Exercises:

Solve the equation. Check your solution.

8. $7 \cdot q = 42$  
9. $7k \div 3 = 21$  
10. $\frac{5a}{7} = 25$

7.4 Writing Equations in Two Variables (pp. 314–321)

Tell whether $(6, 16)$ is a solution of the equation $y = 3x - 4$.

$16 \overset{?}{=} 3(6) - 4$ Substitute.

$16 \neq 14$ Compare.

✓: So, $(6, 16)$ is not a solution.

Exercises:

Tell whether the ordered pair is a solution of the equation.

11. $y = 3x + 1; (2, 7)$  
12. $y = 7x - 4; (4, 22)$  
13. TAXI A taxi ride costs $3 plus $2.50 per mile. Write and graph an equation in two variables that represents the total cost of a taxi ride.

7.5 Writing and Graphing Inequalities (pp. 324–331)

Write the word sentence as an inequality.

a. A number $x$ is more than $-9$.  

b. A number $r$ divided by 2 is at most 4.

$A \text{ number } x \text{ is more than } -9.$  

$A \text{ number } r \text{ divided by 2 is at most } 4.$

$x > -9$  

$\frac{r}{2} \leq 4$

✓: An inequality is $x > -9$.  

✓: An inequality is $\frac{r}{2} \leq 4$.

Exercises:

Write the word sentence as an inequality.

14. A number $m$ is less than 5.  
15. A number $h$ is at least $-12$.

Graph the inequality on a number line.

16. $x < 0$  
17. $a \geq 3$  
18. $n \leq -1$
7.6 Solving Inequalities Using Addition or Subtraction (pp. 332–337)

Solve \( 1 \leq x - 4 \). Graph the solution.

\[
\begin{align*}
1 & \leq x - 4 & \text{Write the inequality.} \\
1 + 4 & \leq x - 4 + 4 & \text{Addition Property of Inequality} \\
5 & \leq x & \text{Simplify.}
\end{align*}
\]

The inequality \( 5 \leq x \) is the same as \( x \geq 5 \).

\( \therefore \) The solution is \( x \geq 5 \).

7.7 Solving Inequalities Using Multiplication or Division (pp. 338–343)

Solve \( 7n < 42 \). Graph the solution.

\[
\begin{align*}
7n & < 42 & \text{Write the inequality.} \\
\frac{7n}{7} & < \frac{42}{7} & \text{Division Property of Inequality} \\
n & < 6 & \text{Simplify.}
\end{align*}
\]

\( \therefore \) The solution is \( n < 6 \).

Exercises:

Solve the inequality. Graph the solution.

19. \( x + 1 \geq 3 \) \hspace{1cm} 20. \( k - 7 \leq 0 \) \hspace{1cm} 21. \( y + 8 \geq 9 \)

22. \( 24 < 11 + x \) \hspace{1cm} 23. \( 4 \leq n - 4 \) \hspace{1cm} 24. \( x - 20 > 24 \)

25. \( b + 12 \leq 26 \) \hspace{1cm} 26. \( s - 1.5 < 2.5 \) \hspace{1cm} 27. \( \frac{1}{4} + m \leq \frac{1}{2} \)

28. \( x \div 2 < 4 \) \hspace{1cm} 29. \( 9n \geq 63 \) \hspace{1cm} 30. \( \frac{5}{3}x \leq 10 \)

31. \( 9 \geq 3b \) \hspace{1cm} 32. \( 10p > 40 \) \hspace{1cm} 33. \( \frac{3}{11}k < 15 \)

34. **TICKETS** The cost of three tickets to a movie is at least $20. Write and solve an inequality that represents the situation.
Write the word sentence as an equation.

1. 7 times a number \( s \) is 84.
2. 13 is one-third of a number \( m \).

Solve the equation. Check your solution.

3. \( 15 = 7 + b \)
4. \( v - 6 = 16 \)
5. \( 5x = 70 \)
6. \( 3b = 45 \)
7. \( \frac{6m}{7} = 30 \)
8. \( \frac{8k}{3} = 32 \)

Tell whether the ordered pair is a solution of the equation.

9. \( y = 9x; (3, 27) \)
10. \( y = 4x + 2; (8, 36) \)

Write an inequality for the situation.

11. An MP3 player holds up to 300 songs.
12. Riders must be at least 48 inches tall.

Graph the inequality on a number line.

13. \( x \geq 5 \)
14. \( m \leq -2 \)

Solve the inequality. Graph the solution.

15. \( x - 3 < 7 \)
16. \( 12 \geq n + 6 \)
17. \( \frac{4}{3}b \leq 12 \)
18. \( 72 > 12p \)

19. **SCHOOL DANCE** Each ticket to a school dance is $4. The total amount collected in ticket sales is $332. Write and solve an equation to find the number of students attending the dance.

20. **T-SHIRTS** A soccer team will sell T-shirts for a fundraiser. The company that makes the T-shirts charges $10 per shirt plus a $20 shipping fee per order.
   a. Write and graph an equation in two variables that represents the total cost of ordering the shirts.
   b. Choose an ordered pair that lies on your graph in part (a). Interpret it in the context of the problem.

21. **HURRICANE** A hurricane has wind speeds that are greater than or equal to 74 miles per hour. Write an inequality to represent the possible wind speeds during a hurricane.
1. What is the area of the balcony shown below?

![Balcony Diagram]

A. 9 ft²  
B. 18 ft²  
C. 26 ft²  
D. 52 ft²

2. You are making identical fruit baskets using 16 apples, 24 pears, and 32 bananas. What is the greatest number of baskets you can make using all the fruit?

F. 2  
G. 4  
H. 8  
I. 16

3. Which equation represents the word sentence below?

The sum of 18 and 5 is equal to 9 less than a number y.

A. 18 − 5 = 9 − y  
B. 18 + 5 = 9 − y  
C. 18 + 5 = y − 9  
D. 18 − 5 = y − 9

4. Which number line is a graph of the solution of the inequality below?

\[ x \geq 5 \]

F.  
G.  
H.  
I.  

Test-Taking Strategy

Work Backwards

You like taking x catnaps each day, where 3x = 24. How many is that?

A. 6  
B. 7  
C. 8  
D. 9

“Work backwards by trying 6, 7, 8, and 9. You will see that 3(8) = 24. So, C is correct.”
5. The steps your friend took to divide two mixed numbers are shown below.

\[
\begin{align*}
3 \frac{3}{5} \div 1 \frac{1}{2} &= \frac{18}{5} \times \frac{3}{2} \\
&= \frac{27}{5} \\
&= 5 \frac{2}{5}
\end{align*}
\]

What should your friend change in order to divide the two mixed numbers correctly?

A. Find a common denominator of 5 and 2.
B. Multiply by the reciprocal of \( \frac{18}{5} \).
C. Multiply by the reciprocal of \( \frac{3}{2} \).
D. Rename \( 3 \frac{3}{5} \) as \( 2 \frac{8}{5} \).

6. An inequality is graphed on the number line below.

![Number line with a blue shaded area from 7 to 11.]

What is the least whole number value that is a solution of the inequality?

7. A company ordering parts receives a charge of $25 for shipping and handling plus $20 per part. Which equation represents the cost \( c \) of ordering \( p \) parts?

F. \( c = 25 + 20p \)
G. \( c = 20 + 25p \)
H. \( p = 25 + 20c \)
I. \( p = 20 + 25c \)

8. Which property is illustrated by the statement below?

\[ 5(3 + 6) = 5(3) + 5(6) \]

A. Associative Property of Multiplication
B. Commutative Property of Multiplication
C. Commutative Property of Addition
D. Distributive Property
9. What is the value of the expression below?

\[ 46.8 \div 0.156 \]

10. In a fish tank, 75% of the fish are goldfish. How many fish are in the tank if there are 24 goldfish?

F. 6  
H. 32  
G. 18  
I. 96

11. What are the coordinates of point \( P \) in the coordinate plane below?

A. \((-3, -2)\)  
B. \((3, -2)\)  
C. \((-2, -3)\)  
D. \((-2, 3)\)

12. What is the first step in evaluating the expression below?

\[ 3 \cdot (5 + 2)^2 \div 7 \]

F. Multiply 3 and 5.  
H. Evaluate 5\(^2\).  
G. Add 5 and 2.  
I. Evaluate 2\(^2\).

13. Jeff wants to save $4000 to buy a used car. He has already saved $850. He plans to save an additional $150 each week.

Part A Write and solve an equation to represent the number of weeks remaining until he can afford the car.

Jeff saves $150 per week by saving \( \frac{3}{4} \) of what he earns at his job each week. He works 20 hours per week.

Part B Write an equation to represent the amount per hour that Jeff must earn to save $150 per week. Explain your reasoning.

Part C What is the amount per hour that Jeff must earn? Show your work and explain your reasoning.