

**PreCalculus Honors  
Curriculum  
Sayreville War Memorial High School  
Full Year**

## **Statement of Purpose**

PreCalculus Honors is the next course in the mathematics sequence for students who have completed Algebra 2 and intend to take Advanced Calculus and/or Statistics the following year. In each topic studied, multiple perspectives (verbal, algebraic, graphical, and numeric) are explored to allow students to develop creative problem-solving skills and recognize the equivalence of different representations.

The students will engage in relevant activities that will utilize their reasoning and critical thinking strategies as they apply them in problem solving both independently and in-group. Students will encounter several types of assessment within this course: unit quizzes, tests, exams and projects which will require more details and work outside of the classroom. The honors course will be a rigorous course, preparing all of its students to take Advanced Math courses the following year. It will be taught at a faster pace to further challenge a student who has mastered all of the algebra 2 concepts.

## **Summary of Course**

Honors PreCalculus is the fourth course in the honors college preparatory sequence. This course is designed for students who have the appropriate background to understand the concepts and techniques in advanced college preparatory mathematics. The topics covered include logarithms, trigonometric functions, and identities, solving trigonometric equations, applications involving triangles, inverse trigonometric functions, trigonometric addition formulas, advanced graphing techniques, polar coordinates, vectors, sequences and series, and limits. This course is intended to prepare students for a post-secondary education. It emphasizes higher-level mathematical thinking necessary to pursue the study of Advanced Placement Calculus and/or Statistics.

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Unit 09 Logarithmic Functions

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# Unit 01 Trigonometric Functions

Content Area: **Mathematics**  
Course(s): **Adv. Concepts in Com. Sci. (s), English 9, English 9 CP**  
Time Period: **1st Marking Period**  
Length: **30-35 Days**  
Status: **Not Published**

## Summary of the Unit

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Analyze, describe different types of angles. Analyze and describe angles and angular movement and relate this information to real life phenomena.

|              |   |
|--------------|---|
| MA.F-TF.A.3  | Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for $\pi - \theta$ , $\pi + \theta$ , and $2\pi - \theta$ in terms of their values for $\theta$ , where $\theta$ is any real number. |
| MA.F-TF.A.4  | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.  |
| MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles.   |
| MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.  |
| MA.F-TF.A.1  | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.   |
| MA.F-TF.A.2  | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.   |

## Enduring Understandings

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Students will understand that...

- An angle is determined by rotating a ray about its endpoint and can be measured in degrees, radians, and revolutions.
- The co-terminal relationship can be used to model infinitely many angles.
- The formula for the length of a circular arc can be used to analyze the motion of a particle moving at a constant speed along a circular path.
- Trigonometry describes the relationship between the sides and angles in right triangles.
- Many relationships exist between angles in trigonometry that can be used to make predictions about trigonometric values at a variety of angles.
- Reference angles are used to model and make predictions about trigonometric functions of any angle.

## Essential Questions

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How do you describe angles and angular movement?

What are real world applications of linear and angular speed?

How do you use trigonometry to find unknown side lengths and angles in right triangles?

How can the different trigonometric functions be used to solve real world applications?

How do you evaluate trigonometric functions at any angle?

How do you evaluate trigonometric functions by using the unit circle?

## **Summative Assessment and/or Summative Criteria**

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Describe Learning Vertically

Identify Key Building Blocks

Make Connections (between and among key building blocks)

Short/Extended Constructed Response Items

Multiple-Choice Items (where multiple answer choices may be correct)

Drag and Drop Items

Use of Equation Editor Quizzes

Journal Entries/Reflections/Quick-Writes

Accountable talk

Projects Portfolio

Observation

Graphic Organizers/Concept Mapping

Presentations

Role Playing

Teacher-Student and Student-Student Conferencing Homework

## **Resources**

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- Cengage Learning: Precalculus with Limits - A Graphing Approach, Sixth Edition
- <https://www.ixl.com/standards/new-jersey/math/high-school>
- [http://www.mathwords.com/index\\_adv\\_alg\\_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- <http://www.hershey.k12.pa.us/Page/3608>
- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/prec calculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>
- [https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured\\_content](https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content)
- Statistics Education Web (STEW). <http://www.amstat.org/education/STEW/>
- The Data and Story Library (DASL). <http://lib.stat.cmu.edu/DASL/>

## **Unit Plan**

| Topic/Selection | General Objectives | Instructional Activities | B |
|-----------------|--------------------|--------------------------|---|
|-----------------|--------------------|--------------------------|---|

| Timeframe                              |  |   |  |
|--|--|---|--|
| Angles and angular movement.<br>6 days | <p>SWBAT describe angles and use radian measure.</p> <p>SWBAT use degree measure and convert between degree and radian measure.</p> <p>SWBAT will use angles to model and solve real-life problems.</p>  | <p>Remind students of different types of angles, complementary and supplementary.</p> <p>Using a regular analog clock Have students create a set of directions using terms such as clockwise, counterclockwise, positive, negative, etc, to describe a time on the clock. Then another student will take the directions and try to guess the time.</p> <p>Show the definition of arc length as suggested in technological initiatives. Illustrating the radius of a circle traveling the length of the intercepted arc.</p> | <p>Group assignment source</p> <p>Home textbook source</p> <p>Introduction Activity day to know understand</p> <p>Quiz</p> |
| The Unit Circle<br>7days               | <p>SWBAT will identify the unit circle and describe its relationships to real numbers.</p> <p>SWBAT will evaluate trigonometric functions using the unit circle.</p> <p>SWBAT will use domain and period to evaluate sine and cosine functions and use a calculator to evaluate trigonometric functions.</p> | <p>Using a paper plate, ruler, and protractor recreate the unit circle and the four quadrants.</p> <p>Using the class room floor as a coordinate plane, have the students lay out all the particular values of the unit circle until the entire circle is complete. Each student will be responsible for certain values. Do not begin with the first quadrant as it will be too easy for the students to complete and will require them not to think too deeply.</p>  | <p>Group assignment source</p> <p>Home textbook source</p> <p>Introduction Activity day to know understand</p> <p>Quiz</p> |
| Functions of Any Angle<br>9 days       | <p>SWBAT will evaluate trigonometric functions of any angle.</p> <p>SWBAT will find reference angles.</p> <p>SWBAT will evaluate trigonometric functions of real numbers.</p>  | <p>Sketching several angles with their reference angles helps reinforce the fact that the reference angle is the acute angle formed with the horizontal.</p>  | <p>Group assignment source</p> <p>Home textbook source</p> <p>Introduction Activity day to know understand</p>             |

|                                       |  |   |  |
|---------------------------------------|--|---|--|
|                                       |  |   | Quiz   |
| Right Triangle Trigonometry<br>5 days | <p>SWBAT will evaluate trigonometric functions of acute angles and use a calculator to evaluate trigonometric functions.</p> <p>SWBAT will use the fundamental trig identities.</p> <p>SWBAT will use trigonometric functions to model and solve real-life problems.</p> | <p>Multiple activities that have both degree and radian mode type questions so students begin to differentiate between the two.</p> <p>Create visuals to better represent angles of elevation and depression.</p> | <p>Group assignments</p> <p>Home textbooks</p> <p>Introduction</p> <p>Activities</p> <p>day to know</p> <p>under</p> <p>Quiz</p> |

### **Suggested Modifications for Special Education, ELL and Gifted Students**

- Have students work in pairs for exploration activities and classwork assignments. This will allow students to bounce ideas off of each other and gain confidence with the material.
- Provide the opportunity for students to practice problems both with and without graphically calculator technology.
- Assign problems at varying levels of difficulty to challenge all learners.
- Create differentiated groups for application based activities, assigning harder problems to groups who have demonstrated understanding of the material.
- Provide a list of resources, like the ones listed in the technology section, for students to turn to for additional help, if needed.
- Assign challenge problems on the related topic for those students who finish early.

### **Suggested Technological Innovations/Use**

The following links can provide visual and interactive models for understanding what a radian is and why it is important. Students can follow along using their chromebooks:

<http://zonalandeducation.com/mmts/trigonometryRealms/radianDemo1/RadianDemo1.html>



<http://www.mathsisfun.com/geometry/radians.html>

The following video demonstrates what linear and angular velocity are and how they are affected by different size circles. Students who are struggling with the difference can view this video for extra help. The link can be placed on the teacher's google classroom or OnCourse Connect Classroom:

<https://www.youtube.com/watch?v=yDHM6rd8P94>

## **Cross Curricular/21st Century Connections**

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Social Studies and ELA Literacy Connection:

- Name of Task: Americans' spending: NJCCCS: 9.1.12.A.9; W.11-12.1

From July 1998 to July 1999, Americans' spending rose from 5.82 trillion dollars to 6.20 trillion dollars

- Let  $x = 0$  represent July 1998,  $x = 1$  represent August 1998, ..., and  $x = 12$  represent July 1999. Write a linear equation for Americans' spending in terms of the month  $x$
- Use the equation in (a) to predict Americans' spending in July 2002.
- Based on the model created in (a) when would the aggregate expenditure exceed 10 trillion dollars?
- What part of the US GDP is spent by the Americans in 2013?

CRP.K-12.CRP2

Apply appropriate academic and technical skills.

CRP.K-12.CRP4

Communicate clearly and effectively and with reason.

CRP.K-12.CRP8

Utilize critical thinking to make sense of problems and persevere in solving them.

CRP.K-12.CRP6

Demonstrate creativity and innovation.

# Unit 02 Graphs of Trigonometric Functions

Content Area: **Mathematics**  
Course(s): **Adv. Concepts in Com. Sci. (s), English 9, English 9 CP, Pre Calculus Honors**  
Time Period: **1st Marking Period**  
Length: **18 - 20 Days**  
Status: **Not Published**

## Summary of the Unit

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Analyze, describe different types of angles. Model and make predictions about periodic behavior using the graphs of trigonometric functions.

|             |  |
|-------------|--|
| MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.   |
| MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.   |
| MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. |
| MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.   |
| MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.   |

## Enduring Understandings

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- There exists a relationship between the graphs of sine and cosine.
- The amplitude of a sine or cosine graph is useful when making predictions about the vertical shrink or stretch of the graph.
- Sine and cosine functions are used to model waves and periodic behavior.
- There exists more than one way to write the equation of a sinusoidal function given its graph.
- The domain and range of a function are useful in predicting the graph of a function.
- The domains of sine, cosine, and tangent must be restricted in order for the function to be one-to-one and have an inverse.
- Inverse functions are necessary to determine the angle for a given trigonometric value which has uses in many real life applications.
- Inverse properties are useful when making predictions.
- The relationship between the sides and angles of right triangles can be used when converting a trigonometric expression into an algebraic expression

## Essential Questions

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How are the graphs of sine and cosine related?

How can the amplitude and period of a sinusoidal function be used to sketch its graph?

How is the graph of a sinusoidal function related to its equation?

What real life scenarios are best modeled by trigonometric functions?

How is the graph of a trigonometric function a useful tool for modeling and interpreting data?

How are domain and range useful when analyzing trigonometric graphs?

What is the relationship between a function and its inverse?

What does finding the inverse of a function mean graphically?

Why is finding the inverse of a function useful?

## **Summative Assessment and/or Summative Criteria**

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Describe Learning Vertically

Identify Key Building Blocks

Make Connections (between and among key building blocks)

Short/Extended Constructed Response Items

Multiple-Choice Items (where multiple answer choices may be correct)

Drag and Drop Items

Use of Equation Editor Quizzes

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## **Resources**

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- [http://www.mathwords.com/index\\_adv\\_alg\\_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- <http://www.hershey.k12.pa.us/Page/3608>
- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/prec calculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>
- [https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured\\_content](https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content)
- Statistics Education Web (STEW). <http://www.amstat.org/education/STEW/>

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## Unit Plan

| Topic/Selection<br>Timeframe                                | General Objectives   | Instructional Activities   | E   |
|---|--|--|---|
| Graphing Basic Sine and Cosine Curves<br>4 days             | SWBAT define Amplitude, period, zeros, maximum and minimum values.<br>SWBAT determine when a graph is a typical or image (reflection) curve. | To emphasize determining and locating key points have student mark each point on their graphs and then check their graphs with their graphing utilities.<br>Relate graphing as being very closely related to an algebraic 5 point process. Stress points as beginning, middle and end, one-quarter, and three-quarter cycle. | Group<br>assign<br>source<br><br>Home<br>textb<br>source<br><br>Intro<br>Activ<br>day t<br>know<br>unde<br><br>Quiz |
| Translations of graphs of sine and cosine curves.<br>6 days | SWBAT describe both horizontal and vertical translations to a graph.<br>SWBAT graph functions over multiple cycles and intervals.            | While the "C" term may introduce the shift of a curve, a student can set up two specific equations to algebraically determine both the "shifted" starting and end point of a particular curve for one cycle.<br>Have students graph additional cycles by adding the appropriate values of "pi".                              | Group<br>assign<br>source<br><br>Home<br>textb<br>source<br><br>Intro<br>Activ<br>day t<br>know<br>unde<br><br>Quiz |
| Graphing reciprocal   | SWBAT sketch graphs of tangent functions.  | Relate the equations as reciprocals. Talk about undefined terms and where they are on the graph.   |   |

|   |  |   |  |
|---|--|---|--|
| <p>trigonometric functions.<br/>6 days</p>        | <p>SWBAT sketch graphs of cotangent functions.<br/>SWBAT sketch graphs of secant and cosecant.<br/>SWBAT sketch the graph of damped trigonometric functions.</p> | <p>Discuss asymptotes and boundaries.</p>   | <p>Group<br/>assignment<br/>source<br/><br/>Home<br/>textbook<br/>source<br/><br/>Intro<br/>Activity<br/>day to<br/>know<br/>under<br/><br/>Quiz</p> |
| <p>Inverse Trigonometric Functions<br/>5 days</p> | <p>SWBAT evaluate and graph inverse sine functions.<br/>SWBAT evaluate and graph other inverse functions.<br/>SWBAT evaluate composition of functions.</p>       | <p>Activity in writing in the 2 alternate forms. Students must be comfortable and familiar with both.<br/>Activity relating to and increasing the understanding of the inverse functions by relating them to triangle definitions of the trigonometric functions.<br/>Discuss range of trig functions and how they vary.<br/>Students should know these ranges well to ensure that their answers are within the correct range. Referencing the graphs of the inverse functions is also helpful. .</p> | <p>Group<br/>assignment<br/>source<br/><br/>Home<br/>textbook<br/>source<br/><br/>Intro<br/>Activity<br/>day to<br/>know<br/>under<br/><br/>Quiz</p> |

### **Suggested Modifications for Special Education, ELL and Gifted Students**

- Have students work in pairs for exploration activities and classwork assignments. This will allow students to bounce ideas

off of each other and gain confidence with the material.

- Relate the trigonometric graphs to numerical and analytical approaches to solving applications.
- Provide the opportunity for students to practice problems both with and without graphically calculator technology.
- Assign problems at varying levels of difficulty to challenge all learners.
- Create differentiated groups for application based activities, assigning harder problems to groups who have demonstrated understanding of the material.
- Provide a list of resources, like the ones listed in the technology section, for students to turn to for additional help, if needed.
- Assign challenge problems on the related topic for those students who finish early.

## Suggested Technological Innovations/Use

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- Have students complete a Graphing Calculator Discovery Lab on their own to test hypotheses on how the equation of a sine or cosine function relates to its graph. This can be done to illustrate amplitude, period, and vertical and phase shifts. For example, to illustrate amplitude, students should compare the graphs of  $y = \sin x$ ,  $y = 3\sin x$ ,  $y = -3\sin x$ , and  $y = -1/2 \sin x$ . Teachers should make sure students set their calculators in radian mode and have a uniform window.
- In pairs, have students complete an Applications of Sine and Cosine Functions Project. Students will be assigned applications of sine and cosine functions and will solve and fully explain their solutions through a ShowMe video on their chromebook.
- Have students graph  $y = \tan x$ ,  $y = \csc x$ ,  $y = \sec x$ , and  $y = \cot x$  on their graphing calculator. Have them state the domain, range, period, and asymptotes of each of these functions. Then, discuss results and conclusions as a class.

## Cross Curricular/21st Century Connections

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Science Connection:

- Name of Task: Myoglobin and Hemoglobin: NJCCCS: HS-LS1-2; HS-LS1-4

Myoglobin and hemoglobin are oxygen-carrying molecules in the human body. Hemoglobin is found inside red blood cells, which flow from the lungs to the muscles through the bloodstream. Myoglobin is found in muscle cells. The function  $M(p) = \frac{p}{1+p}$  calculates the fraction of myoglobin saturated with oxygen at a given pressure  $p$  Torr. For example, at a pressure of 1 Torr,  $M(1) = 0.5$ , which means half of the myoglobin (i.e. 50%) is oxygen saturated. (Note: More precisely, you need to use something called the “partial pressure”, but the distinction is not important for this problem.) Likewise, the function  $H(p) = \frac{p}{1+p}$  calculates the fraction of hemoglobin saturated with oxygen at a given pressure  $p$ . [UW]

- The graphs of  $M(P)$  and  $H(P)$  are given here on the domain  $0 \leq p \leq 100$ . Which is which?
- If the pressure in the lungs is 100 Torr, what is the level of oxygen saturation of the hemoglobin in the lungs?
- The pressure in an active muscle is 20 Torr. What is the level of oxygen saturation of myoglobin in an active muscle? What is the level of hemoglobin in an active muscle? d. Define the efficiency of oxygen transport at a given pressure  $p$  to be  $M(p) - H(p)$ . What is the oxygen transport efficiency at 20 Torr? At 40 Torr? At 60 Torr? Sketch the graph of  $M(p) - H(p)$ ; are there conditions under which transport efficiency is maximized (explain)?

CRP.K-12.CRP2

Apply appropriate academic and technical skills.

CRP.K-12.CRP8

Utilize critical thinking to make sense of problems and persevere in solving them.

CRP.K-12.CRP6

Demonstrate creativity and innovation.

CRP.K-12.CRP3

Attend to personal health and financial well-being.

# Unit 03 Analytic Trigonometry

Content Area: **Mathematics**  
Course(s): **Adv. Concepts in Com. Sci. (s), English 9, English 9 CP, Pre Calculus Honors**  
Time Period: **1st Marking Period**  
Length: **8 - 10 Days**  
Status: **Not Published**

## Summary of the Unit

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Model real-life scenarios using trigonometric equations and solve the equation using trigonometric relationships and algebraic techniques.

|              |   |
|--------------|---|
| MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  |
| MA.A-REI.B.4 | Solve quadratic equations in one variable.  |
| MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - x^4$ as $(x^2)^2 - (x^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - x^2)(x^2 + x^2)$ .                                |
| MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  |
| MA.F-TF.C.8  | Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ given $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ and the quadrant of the angle.                      |
| MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context.  |
| MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.   |
| MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |
| MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  |

## Enduring Understandings

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There exists many relationships between trigonometric functions that can be rewritten to evaluate or simplify trigonometric expressions and verify trigonometric identities.

Many previously learned algebraic methods can be used to solve trigonometric equations.

Due to the periodic nature of trigonometric functions, trigonometric equations often have infinitely many solutions that can be expressed using general solution format.

The solutions to trigonometric equations can be approximated using graphing technology.

There is a direct relationship between  $k$  in a multiple angle equation the period the graph of the equation.

Trigonometric equations can be used to solve real-life scenarios that are periodic in nature.

The sum and difference formulas can be used to find exact values of trigonometric expressions at additional angles on the unit circle.



## Essential Questions

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- How do you use relationships to rewrite trigonometric expressions in order to simplify and evaluate trigonometric functions?
- How do you use trigonometric relationships and algebraic methods to verify trigonometric identities?
- How can algebraic techniques be applied to solving trigonometric equations?
- How are the graphs of trigonometric functions related to the solutions of the corresponding trigonometric equation?
- What real life problems can be modeled by and solved using trigonometric equations?
- Describe a situation where extraneous solutions may exist. How do you verify if a solution is extraneous to an equation?
- How can you use trigonometry to apply the sum and difference formulas to find exact values of angles?
- How are multiple angle formulas used to transform a trigonometric equation and create a new method to solve the equation?

## Summative Assessment and/or Summative Criteria

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Describe Learning Vertically

Identify Key Building Blocks

Make Connections (between and among key building blocks)

Short/Extended Constructed Response Items

Multiple-Choice Items (where multiple answer choices may be correct)

Drag and Drop Items

Use of Equation Editor Quizzes

Journal Entries/Reflections/Quick-Writes

Accountable talk

Projects Portfolio

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## Resources

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- [http://www.mathwords.com/index\\_adv\\_alg\\_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- <http://www.hershey.k12.pa.us/Page/3608>
- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/prec calculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>

- [https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured\\_content](https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content)
- Statistics Education Web (STEW). <http://www.amstat.org/education/STEW/>
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## Unit Plan

| Topic/Selection<br>Timeframe                                | General Objectives  | Instructional Activities   |
|---|---|--|
| Applying the fundamental Trigonometric Identities<br>5 days | SWBAT recognize and write the fundamental trigonometric identities.<br>SWBAT use the fundamental trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and rewrite trigonometric expressions. | For these fundamental identities stress that "u" can be a real number, or a variable.<br>When simplifying expressions, remind students that any identity introduced must be a known trigonometric identity or a known identity.<br>Use graphing utility to check answers. Y1 would be the original expression Y2 would be the simplified expression. Select path style for Y1 and path style for Y2. Graph both in same viewing window. If the two graphs appear to coincide the expressions are equivalent.<br>For auditory learners, instruct them to read each identity aloud. Hearing that they can hear how it is used in the expression. Narrating the identity as it is implemented will help students better recall the case in which each identity is used. |
| Verifying trigonometric identities.<br>5 days               | SWBAT verify trigonometric identities by proving one side is equivalent to an other side.   | Remind students to work with one side of equation only. It is often better to use the more complicated side.<br>Look for opportunities to factor expressions, add fractions, rationalize a binomial, or create a monomial denominator.<br>Look for opportunities to use the fundamental identities.<br>"a little algebra a little identity" The Ping Pong Method.<br>If no algebra or direct identities can be applied Convert to sines and cosines.<br>Stress to students always to try and do something.<br>Have class discussion about conditional equations over conditional expressions.  |
| Solving trigonometric equations                             | SWBAT use common algebraic techniques to solve trigonometric equations.<br>SWBAT solve trigonometric equations of the   | Conduct discussion about exact "pi" answers opposed to approximations. Including discussion on using values from different quadrants. Stress checking against the range of the function.   |

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| <p>7 days</p> | <p>quadratic type.<br/>         SWBAT solve trigonometric equations involving multiple angles.<br/>         SWBAT use inverse trigonometric functions to solve trigonometric equations.</p> | <p>interval specified. If no interval is stated, solutions must be given in the interval <math>[-\pi, \pi]</math> or <math>[0, 2\pi]</math> unless otherwise specified. If no interval is stated, solutions must be given in the interval <math>[-\pi, \pi]</math> or <math>[0, 2\pi]</math> unless otherwise specified. For students having difficulty working with the equation, suggest turning the equation into an algebraic equation by using the identity <math>\sin^2 x + \cos^2 x = 1</math> then solve for <math>x</math>. Students can convert everything back to degrees if a solution is gained.<br/>         Include discussions of dividing by a trig function is the same as dividing by a variable and how solutions can be lost.<br/>         Use graphing utilities to confirm solutions. Graph both <math>\sin x</math> and <math>\cos x</math> and find the <math>x</math> coordinates of the points at which they intersect. Then graph a combination of the sides and find the <math>x</math> intercepts of the graph.</p> |
|---------------|---|---|

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|--|---|--|
| <p>Applying the sum and difference formulas.<br/>         5 days</p> | <p>SWBAT use the sum and difference formula to evaluate trig functions, verify trigonometric identities, and solve trigonometric equations.</p> | <p>Activity on radical operations and exact values rather than decimal form.<br/>         Graphing utility activity graphing trig functions added and subtracted together to see if the forms are equivalent i.e. <math>\sin(x + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\sin x + \cos x)</math></p> |
|--|---|--|

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|   |   |  |
| <p>Multiple angle formulas<br/>5 days</p> | <p>SWBAT apply the double angle formulas to rewrite and evaluate trigonometric functions.<br/>SWBAT apply the sum to product and product to sum formulas to rewrite and evaluate trigonometric functions.</p> | <p>Activity showing the equivalence of all three forms of the double angle formula of <math>\cos 2u</math>.<br/>Graphing utility activity where in the same viewing window the graphs that coincided are equivalent.</p> |

### **Suggested Modifications for Special Education, ELL and Gifted Students**

- Have students work in pairs for exploration activities and classwork assignments. This will allow students to bounce ideas off of each other and gain confidence with the material.
- Relate the solutions of a trigonometric equation to its corresponding graph.
- Provide the opportunity for students to practice problems both with and without graphically calculator technology.
- Assign problems at varying levels of difficulty to challenge all learners.
- Create differentiated groups for application based activities, assigning harder problems to groups who have demonstrated understanding of the material.
- Provide a list of resources, like the ones listed in the technology section, for students to turn to for additional help, if needed.
- Assign challenge problems on the related topic for those students who finish early.

### **Suggested Technological Innovations/Use**

Derivation of Pythagorean Identities at the link below can be used as a guide when deriving the Pythagorean identities as a class:  
<http://www.mathalino.com/reviewer/derivation-of-formulas/derivation-of-pythagorean-identities>

## **Cross Curricular/21st Century Connections**

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Science Connection:

Name of Task: Moving Boxes NJSLS: 5.2.12.E.1 In a warehouse a box is being pushed up a  $15^\circ$  inclined plane with a force of 2.5lb parallel to the inclined plane.

1- Find the component of the force.

2- Give an interpretation of the horizontal and vertical components of the force.

3- If the box is towed up the inclined plane with a force making a  $33^\circ$  with a horizontal line, Find the force w needed in order for the component of the force parallel to the incline plane to be 2.5 lb. Give the answer in component form

CRP.K-12.CRP2

Apply appropriate academic and technical skills.

CRP.K-12.CRP5

Consider the environmental, social and economic impacts of decisions.

CRP.K-12.CRP4

Communicate clearly and effectively and with reason.

CRP.K-12.CRP8

Utilize critical thinking to make sense of problems and persevere in solving them.

CRP.K-12.CRP6

Demonstrate creativity and innovation.

# Unit 04 Oblique Triangles

Content Area: **Mathematics**  
Course(s): **Adv. Concepts in Com. Sci. (s), English 9, English 9 CP, Pre Calculus Honors**  
Time Period: **1st Marking Period**  
Length: **8 - 10 Days**  
Status: **Not Published**

## Summary of the Unit

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Analyze, describe different types of angles. Apply trigonometric formulas to model and solve real life problems involving oblique triangles.

|               |  |
|---------------|--|
| MA.G-SRT.D.9  | Derive the formula $A = (1/2)ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.   |
| MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems.  |
| MA.G-MG.A.3   | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). |
| MA.G-SRT.C.8  | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.   |
| MA.G-SRT.D    | Apply trigonometry to general triangles  |

## Enduring Understandings

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- The Law of Sines and the Law of Cosines can be used to solve for missing sides and angles of oblique triangles.
- In order to solve an oblique triangle, you need to know the measure of at least one side and the measures of any two other parts of the triangle – two sides, two angles, or one angle and one side.
- When two sides of a triangle and an angle opposite one of them are given and a second angle is to be derived, the resulting triangle may not be unique, or even exist at all and thus this situation results in ambiguity.
- In surveying and navigation, directions are generally given in terms of bearings.
- Trigonometry can be used to model many real life situations involving navigation and surveying.

## Essential Questions

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How do you use trigonometry to solve and find the areas of oblique triangles?

What situations create the ambiguous case for the Law of Sines?

What is a directional bearing and how is it applied to real life situations?

How do you use trigonometric functions to solve real life problems?

## Summative Assessment and/or Summative Criteria

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Describe Learning Vertically

Identify Key Building Blocks  
 Make Connections (between and among key building blocks)  
 Short/Extended Constructed Response Items  
 Multiple-Choice Items (where multiple answer choices may be correct)  
 Drag and Drop Items  
 Use of Equation Editor Quizzes  
 Journal Entries/Reflections/Quick-Writes  
 Accountable talk  
 Projects Portfolio  
 Observation  
 Graphic Organizers/Concept Mapping  
 Presentations  
 Role Playing  
 Teacher-Student and Student-Student Conferencing Homework

## Resources

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- <https://www.ixl.com/standards/new-jersey/math/high-school>
- [http://www.mathwords.com/index\\_adv\\_alg\\_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- <http://www.hershey.k12.pa.us/Page/3608>
- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/prec calculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>
- [https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured\\_content](https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content)
- Statistics Education Web (STEW). <http://www.amstat.org/education/STEW/>
- The Data and Story Library (DASL). <http://lib.stat.cmu.edu/DASL/>

## Unit Plan

| Topic/Selection<br>Timeframe | General Objectives  | Instructional Activities  |
|------------------------------|---|---|
| Law of Sines<br>4 days       | <p>SWBAT use the law of sines to solve oblique triangles.</p> <p>SWBAT recognize and understand the ambiguous SSA case and the types of solutions gained.</p> <p>SWBAT use the law of sines to model and solve real-life problems including bearing problems.</p> | <p>Activity on sketching triangles and dimension. Remember that the longest side is opposite the largest angle and shortest opposite the smallest angle.</p> <p>Encourage students to also find the height of the triangle as a reference.</p> <p>Activity on one solution, two solutions, and no solution.</p> <p>Compare to range of functions.</p> |

|                                  |   |  |
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|                                  |   |  |
| <p>Law of Cosines<br/>7 days</p> | <p>SWBAT will use the law of cosines to solve oblique triangles.<br/>SWBAT use the law of cosines to solve real-life problems including bearing problems<br/>SWBAT apply Heron's Formula to find area of triangles.</p> | <p>Lesson and activity deriving the third form of the law. Have students relate this to the Pythagorean Theorem. students to set up a system where the largest angle/side is always solved for first.<br/>Discussion on the alternative forms and how they are algebraically manipulated to better solve for angles.<br/>Lesson on the three types of formulas for area of a triangle. All three forms have advantages and disadvantages, depending on the angle or sides given.</p> |

### **Suggested Modifications for Special Education, ELL and Gifted Students**

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- Have students work in pairs for exploration activities and classwork assignments. This will allow students to bounce ideas off of each other and gain confidence with the material.
- Show analytic and visual proofs of the Law of Sines and Law of Cosines so students understand where each of these formulas comes from and how they relate to solving right triangles.
- Assign problems at varying levels of difficulty to challenge all learners.
- Create differentiated groups for application based activities, assigning harder problems to groups who have demonstrated understanding of the material.
- Provide a list of resources, like the ones listed in the technology section, for students to turn to for additional help, if needed.
- Assign challenge problems on the related topic to those students who finish early.



## **Suggested Technological Innovations/Use**

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8.1.12.A.4, 8.1.12.D.5, 8.1.12.E.1, 8.1.12.F.1

### Research and Information Literacy

- Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

Example: Students will be able to produce a position statement about the real world problem by developing systematic plan of investigation with peers and experts synthesizing information from multiple source.

## **Cross Curricular/21st Century Connections**

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Social Studies Connection:

Name of Task: INCA Dwelling NJSLS: 9.1.12.A.9

Inca dwelling found in Machu Picchu, about 50 miles northwest of Cuzco, Peru.

All that remains of the ancient city today are stone ruins. 1- Express the area of the triangular portion of the side of the dwelling as a function of  $\sin(\theta)$  and  $\cos(\theta)$

2- Express the area as a function of  $\sin \theta$ . Then solve for  $\theta$  assuming that the area is 132 square feet. 73

CRP.K-12.CRP2

Apply appropriate academic and technical skills.

CRP.K-12.CRP5

Consider the environmental, social and economic impacts of decisions.

CRP.K-12.CRP4

Communicate clearly and effectively and with reason.

CRP.K-12.CRP6

Demonstrate creativity and innovation.

# Unit 05 Vectors

Content Area: **Mathematics**  
Course(s): **Pre Calculus Honors**  
Time Period: **3rd Marking Period**  
Length: **13 - 15 Days**  
Status: **Not Published**

## Summary of the Unit

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The study of vectors is crucial in applied mathematics. Vectors relate to geometry, trigonometry, and physics.

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|--------------|--|
| MA.N-VM.B.5a | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $a(\langle x, y \rangle)$ = $\langle ax, ay \rangle$ .   |
| MA.N-VM.B.4c | Understand vector subtraction $\mathbf{a} - \mathbf{b}$ as $\mathbf{a} + (-\mathbf{b})$ , where $-\mathbf{b}$ is the additive inverse of $\mathbf{b}$ , with the same magnitude as $\mathbf{b}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. |
| MA.N-VM.A    | Represent and model with vector quantities.  |
| MA.N-VM.A.1  | Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{a}$ , $ \mathbf{a} $ , $\ \mathbf{a}\ $ , $a$ ).   |
| MA.N-VM.A.2  | Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.   |
| MA.N-VM.A.3  | Solve problems involving velocity and other quantities that can be represented by vectors.   |
| MA.N-VM.B.4  | Add and subtract vectors.  |
| MA.N-VM.B.5  | Multiply a vector by a scalar.   |
| MA.N-VM.B.4a | Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.   |
| MA.N-VM.B.4b | Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.   |

## Enduring Understandings

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Students will be able to

- Represent vectors as directed line segments.
- Write the component forms of vectors.
- Perform basic vector operations and represent vectors graphically.
- Write vectors as linear combination of unit vectors.
- Find the direction angles of vectors.
- Use vectors to model and solve real-life problems.
- Find the dot product of two vectors and use the properties of the dot product.
- Find the angle between two vectors and determine whether two vectors are orthogonal.
- Write vectors as the sums of two vector components.
- Use vectors to find the work done by a force.
- Find the distance between two points in space.

## Essential Questions

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- What is a vector in the plane?
- How do you represent and perform operations with vector quantities?
- How do you write a vector as a sum of two vector components?
- What is the dot product? How is it used to analyze vectors?
- What makes a vector orthogonal?
- What do vector quantities signify in real life situations?
- How do you locate points, and find distances in three dimensions?

## Summative Assessment and/or Summative Criteria

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Describe Learning Vertically

Identify Key Building Blocks

Make Connections (between and among key building blocks)

Short/Extended Constructed Response Items

Multiple-Choice Items (where multiple answer choices may be correct)

Drag and Drop Items

Use of Equation Editor Quizzes

Journal Entries/Reflections/Quick-Writes

Accountable talk

Projects Portfolio

Observation

Graphic Organizers/Concept Mapping

Presentations

Role Playing

Teacher-Student and Student-Student Conferencing Homework

## Resources

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- Cengage Learning: Precalculus with Limits - A Graphing Approach, Sixth Edition
- <https://www.ixl.com/standards/new-jersey/math/high-school>
- [http://www.mathwords.com/index\\_adv\\_alg\\_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- <http://www.hershey.k12.pa.us/Page/3608>
- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/prec calculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>
- [https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured\\_content](https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content)
- Statistics Education Web (STEW). <http://www.amstat.org/education/STEW/>

- The Data and Story Library (DASL). <http://lib.stat.cmu.edu/DASL/>

## Unit Plan

| Topic/Selection<br>Timeframe                         | General Objectives  | Instructional Activities   | Be   |
|--|---|--|--|
| Vectors in the Plane<br><br>6 days                   | <p>SWBAT represent vectors as directed line segments.</p> <p>SWBAT write the component forms of vectors.</p> <p>SWBAT perform basic vector operations and represent vectors graphically.</p> <p>SWBAT write vectors as linear combination of unit vectors.</p> <p>SWBAT find the direction angles of vectors.</p> <p>SWBAT use vectors to model and solve real-life problems.</p> | <p>Discuss the properties of vectors algebraically and geometrically.</p> <p>Explain and model the definition of two-dimensional vectors.</p> <p>Model vector addition numerically and geometrically.</p> <p>Multiplying a Vector by a Scalar.</p>                       | <p>Group daily, f</p> <p>Homev textboo</p> <p>Introdu will be student unders</p> <p>Quiz o</p> |
| Vectors and Dot Products<br><br>5 days               | <p>SWBAT find the dot product of two vectors and use the properties of the dot product.</p> <p>SWBAT find the angle between two vectors and determine whether two vectors are orthogonal.</p> <p>SWBAT write vectors as the sums of two vector components.</p> <p>SWBAT use vectors to find the work done by a force.</p>   | <p>Explain and model the definition of two-dimensional vectors.</p> <p>Geometric Interpretation of Dot Product: Applies the dot product to scenarios such as determining the angle between two vectors and using the dot product to prove if a vector is orthogonal.</p> | <p>Group daily, f</p> <p>Homev textboo</p> <p>Introdu will be student unders</p> <p>Quiz o</p> |
| The three dimensional Coordinate System<br><br>1 day | <p>SWBAT find the distance between two points in space.</p>   | <p>Explain and model the definition of two-dimensional vectors.</p>  | <p>Group daily, f</p> <p>Homev textboo</p> <p>Introdu will be student unders</p>               |

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|  |  |  | <p>Quiz o</p> <p>Studen<br/>assesse<br/>concep<br/>on two<br/>contain<br/>which<br/>respon<br/>questio</p> |
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## **Suggested Modifications for Special Education, ELL and Gifted Students**

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- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
- Introduce vocabulary before lessons, so that ELLs have access to and can build understand of the math language.
- Support ELL students by modeling, creating visuals, describing with words, symbols, or numbers. Chart key vocabulary and examples with students. Leave these anchor-charts up so you and the students can refer back to them.
- Supports such as sentence stems, think-pair-shares, intentional groupings for cooperative learning activities, and guided math

groups so ELLs can be successful.

- The ability to synthesize concepts and make real world and cross- curricular connections through examples for gifted students
- Elevated contextual complexity for gifted students
- Sometimes independent activities, sometimes direct instruction for gifted students
- Inquiry based or open ended assignments and projects for gifted students
- Using supplementary materials in addition to the normal range of resources for gifted students
- Choices for students for gifted students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly) for gifted students

## **Suggested Technological Innovations/Use**

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TECH.8.1.12

Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

## **Cross Curricular/21st Century Connections**

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CRP.K-12.CRP2

Apply appropriate academic and technical skills.

CRP.K-12.CRP4

Communicate clearly and effectively and with reason.

CRP.K-12.CRP7

Employ valid and reliable research strategies.

# Unit 06 Parametric Equations

Content Area: **Mathematics**  
Course(s): **Pre Calculus Honors**  
Time Period: **3rd Marking Period**  
Length: **8 - 10 Days**  
Status: **Not Published**

## Summary of the Unit

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Parametric equations can be used to model the path of an object. These objects can be represented by conics (ellipses, parabolas, and hyperbolas). Conics are applied in real-world situations, such as orbits of planets, flashlights, and satellites. Since conics are not always functions, they can be defined using parametric equations.

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|----------------|--|
| MA.9-12.I.E    | Parametric, polar, and vector functions  |
| MA.9-12.II.E.2 | Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration |

## Enduring Understandings

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Students will be able to

- Evaluate sets of parametric equations for given values of the parameter.
- Graph curves that are represented by sets of parametric equations with and without graphing calculators.
- Rewrite sets of parametric equations as single rectangular equations by eliminating the parameters.
- Use time as a parameter in parametric equations.
- Find parametric equations for curves defined by rectangular equations.

## Essential Questions

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- How do you write equations to describe the motion of a point in a plane?
- Why the function and relations represented by parametric equations?
- When is the parametric form of an equation more useful than rectangular equation, and what additional information it can provide?

## Summative Assessment and/or Summative Criteria

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Describe Learning Vertically  
Identify Key Building Blocks  
Make Connections (between and among key building blocks)  
Short/Extended Constructed Response Items  
Multiple-Choice Items (where multiple answer choices may be correct)  
Drag and Drop Items  
Use of Equation Editor Quizzes  
Journal Entries/Reflections/Quick-Writes

Accountable talk  
 Projects Portfolio  
 Observation  
 Graphic Organizers/Concept Mapping  
 Presentations  
 Role Playing  
 Teacher-Student and Student-Student Conferencing Homework

## Resources

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- Cengage Learning: Precalculus with Limits - A Graphing Approach, Sixth Edition
- <https://www.ixl.com/standards/new-jersey/math/high-school>
- [http://www.mathwords.com/index\\_adv\\_alg\\_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- <http://www.hershey.k12.pa.us/Page/3608>
- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/precalculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>
- [https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured\\_content](https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content)
- Statistics Education Web (STEW). <http://www.amstat.org/education/STEW/>
- The Data and Story Library (DASL). <http://lib.stat.cmu.edu/DASL/>

## Unit Plan

| Topic/Selection<br>Timeframe          | General Objectives  | Instructional Activities  | Be   |
|---------------------------------------|---|---|--|
| Parametric<br>Equations<br><br>3 days | <p>SWBAT evaluate sets of parametric equations for given values of the parameter.</p> <p>SWBAT graph curves that are represented by sets of parametric equations with and without graphing calculators.</p> <p>SWBAT rewrite sets of parametric equations as single rectangular equations</p> | <p>When is it advantageous to define curves parametrically:<br/>         Students will work in groups to determine the orientation of a curve, and thus the usefulness of parametric equations.</p> | <p>Group<br/>daily, t</p> <p>Homev<br/>textbo</p> <p>Introdu<br/>will be<br/>studen<br/>unders</p> |



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|  | <p>by eliminating the parameters.</p> <p>SWBAT use time as a parameter in parametric equations.</p> <p>SWBAT find parametric equations for curves defined by rectangular equations.</p> |  | <p>Quiz o</p> <p>Studen<br/>assess<br/>concep<br/>on para</p> <p>All pro<br/>includ</p> |
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## **Suggested Modifications for Special Education, ELL and Gifted Students**

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
- Introduce vocabulary before lessons, so that ELLs have access to and can build understand of the math language.
- Support ELL students by modeling, creating visuals, describing with words, symbols, or numbers. Chart key vocabulary and examples with students. Leave these anchor-charts up so you and the students can refer back to them.
- Supports such as sentence stems, think-pair-shares, intentional groupings for cooperative learning activities, and guided math groups so ELLs can be successful.

- The ability to synthesize concepts and make real world and cross- curricular connections through examples for gifted students
- Elevated contextual complexity for gifted students
- Sometimes independent activities, sometimes direct instruction for gifted students
- Inquiry based or open ended assignments and projects for gifted students
- Using supplementary materials in addition to the normal range of resources for gifted students
- Choices for students for gifted students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly) for gifted students

## **Suggested Technological Innovations/Use**

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### Technology Operations and Concepts

- Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.

Example: Using technology, students are able to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in statistics and probability in a short amount of time.

TECH.8.1.12

Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

## **Cross Curricular/21st Century Connections**

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Name of Task: Rock Toss NJSL: HS-LS1-2;

HS-LS1-4 A rock is thrown straight up from level ground. The velocity of the rock at any time  $t$  (sec) is  $v(t) = 48 - 32t$  ft/sec

- 1- Graph the velocity function.
- 2- At what time does the rock reach its maximum height?
- 3- Find how far the rock has traveled at its maximum height.
- 4- Graph the pathway followed by the rock 5- How far away from the launching point would the rock land?

CRP.K-12.CRP2

Apply appropriate academic and technical skills.

CRP.K-12.CRP4

Communicate clearly and effectively and with reason.

CRP.K-12.CRP7

Employ valid and reliable research strategies.



# Unit 07 Polar Coordinates and Graphs of Polar Equations

Content Area: **Mathematics**  
Course(s): **Pre Calculus Honors**  
Time Period: **3rd Marking Period**  
Length: **13 -15 Days**  
Status: **Not Published**

## Summary of the Unit

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The polar coordinate system builds on the unit circle. Traditionally, we establish a point using horizontal and vertical components. On polar graphs, we instead establish a point using the distance from the origin (pole) and an angle in standard position. The polar coordinate system connects directly with both the rectangular and complex coordinate systems, allowing for conversion between each system.

|                |  |
|----------------|--|
| MA.9-12.I.E    | Parametric, polar, and vector functions  |
| MA.9-12.II.E.2 | Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration |

## Enduring Understandings

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Students will be able to

- Plot points and find multiple representations of points in the polar coordinate system.
- Convert points from rectangular to polar form and vice versa.
- Convert equations from rectangular to polar form and vice versa.
- Graph polar equations by point plotting.
- Use symmetry and zeros as sketching aids.
- Recognize special polar graphs.

## Essential Questions

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- How do you describe the position of a point in a plane using distance and angle rather than x- and y-coordinates?
- How are the rectangular, and polar coordinate system related, algebraically and graphically?
- How do you sketch graphs of polar equations?
- How do you represent conic sections in polar coordinates?
- How does the study of trigonometry and polar graphs relate to real-world?

## Summative Assessment and/or Summative Criteria

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Describe Learning Vertically  
Identify Key Building Blocks

Make Connections (between and among key building blocks)  
 Short/Extended Constructed Response Items  
 Multiple-Choice Items (where multiple answer choices may be correct)  
 Drag and Drop Items  
 Use of Equation Editor Quizzes  
 Journal Entries/Reflections/Quick-Writes  
 Accountable talk  
 Projects Portfolio  
 Observation  
 Graphic Organizers/Concept Mapping  
 Presentations  
 Role Playing  
 Teacher-Student and Student-Student Conferencing Homework

## Resources

- Cengage Learning: Precalculus with Limits - A Graphing Approach, Sixth Edition
- <https://www.ixl.com/standards/new-jersey/math/high-school>
- [http://www.mathwords.com/index\\_adv\\_alg\\_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- <http://www.hershey.k12.pa.us/Page/3608>
- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/prec calculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>
- [https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured\\_content](https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content)
- Statistics Education Web (STEW). <http://www.amstat.org/education/STEW/>
- The Data and Story Library (DASL). <http://lib.stat.cmu.edu/DASL/>

## Unit Plan

| Topic/Selection<br>Timeframe    | General Objectives  | Instructional Activities  | Be   |
|---------------------------------|---|---|--|
| Polar Coordinates<br><br>6 days | <p>SWBAT plot points and find multiple representations of points in the polar coordinate system.</p> <p>SWBAT convert points from rectangular to polar form and vice versa.</p> <p>SWBAT convert equations from rectangular to polar form and vice versa.</p> | <p>Explain and model Polar coordinate system.</p> <p>Have students investigate the general representation of a point in polar coordinate system.</p> <p>Students will compare polar coordinates to that of rectangular coordinates. They will locate points on a polar coordinate plane and convert the point to one on a rectangular coordinate plane, and vice versa.</p> | <p>Group daily, t</p> <p>Homev textbo</p> <p>Introdu will be studen unders</p> |

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|---|---|--|---|
|   |   |  | Quiz o  |
| Graphs of Polar Equations<br><br>6 days | SWBAT graph polar equations by point plotting.<br><br>SWBAT use smmetry and zeros as sketching aids.<br><br>SWBAT recognize special polar graphs. | Introduce Sketching Graphs of Polar Equations by hand and using Desmos.<br><br>Students will graph and identify characteristics of simple polar equations including lines, circles, cardioids, limacons, and roses | Group daily, f<br><br>Homev textbo<br><br>Introd will be studen unders<br><br>Unit P Polar E Desmo Studen comple will be or her |

### **Suggested Modifications for Special Education, ELL and Gifted Students**

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary

- Sentence stems to provide additional language support for ELL students.
- Introduce vocabulary before lessons, so that ELLs have access to and can build understand of the math language.
- Support ELL students by modeling, creating visuals, describing with words, symbols, or numbers. Chart key vocabulary and examples with students. Leave these anchor-charts up so you and the students can refer back to them.
- Supports such as sentence stems, think-pair-shares, intentional groupings for cooperative learning activities, and guided math groups so ELLs can be successful.
- The ability to synthesize concepts and make real world and cross- curricular connections through examples for gifted students
- Elevated contextual complexity for gifted students
- Sometimes independent activities, sometimes direct instruction for gifted students
- Inquiry based or open ended assignments and projects for gifted students
- Using supplementary materials in addition to the normal range of resources for gifted students
- Choices for students for gifted students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly) for gifted students

## **Suggested Technological Innovations/Use**

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TECH.8.1.12

Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

## **Cross Curricular/21st Century Connections**

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CRP.K-12.CRP2

Apply appropriate academic and technical skills.

CRP.K-12.CRP4

Communicate clearly and effectively and with reason.

CRP.K-12.CRP7

Employ valid and reliable research strategies.

# Unit 08 Limits and Continuity

Content Area: **Mathematics**  
Course(s): **Pre Calculus Honors**  
Time Period: **3rd Marking Period**  
Length: **15 - 18 Days**  
Status: **Not Published**

## Summary of the Unit

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Limits are an essential concept underlying theorems used throughout calculus. The ability to evaluate both one-sided and two-sided limits analytically, graphically, and numerically sets a foundation for differential and integral calculus. Understanding continuity and special limits provides a deeper understanding of how functions work, in addition to special limits.

|                |  |
|----------------|--|
| MA.9-12.I.C.1  | Understanding asymptotes in terms of graphical behavior  |
| MA.9-12.I      | Functions, Graphs, and Limits  |
| MA.9-12.I.A    | Analysis of graphs   |
| MA.9-12.I.B    | Limits of functions (including one-sided limits)   |
| MA.9-12.I.B.1  | An intuitive understanding of the limiting process   |
| MA.9-12.I.B.2  | Calculating limits using algebra   |
| MA.9-12.I.B.3  | Estimating limits from graphs or tables of data  |
| MA.9-12.I.C    | Asymptotic and unbounded behavior  |
| MA.9-12.I.C.2  | Describing asymptotic behavior in terms of limits involving infinity   |
| MA.9-12.I.D    | Continuity as a property of functions  |
| MA.9-12.I.D.1  | An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)     |
| MA.9-12.I.D.2  | Understanding continuity in terms of limits  |
| MA.9-12.II.B.1 | Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents. |

## Enduring Understandings

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Students will be able to

- Identify asymptotes from domain restrictions.
- Understand the limit concept.
- Use the definition of a limit to estimate limits.
- Determine a limit by looking at a graph.
- Determine whether limits of a functions exist.
- Use properties of limits and direct substitution to evaluate limits.
- Evaluate limits at infinity of rational functions.
- Use different techniques to evaluate limits of functions.
- Find one-sided limits.
- Distinguish between continuous and discontinuous functions
- Graph discontinuous functions both removable and non-removable by hand and using technology.
- Evaluate limits of difference quotients from calculus.
- Find the formula of the line tangent to a function through the limit process.



## Essential Questions

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- What is the relationship between the graph of a function and its limit?
- How are limits used to describe the end behavior of functions?
- Why do we define continuity using limits?

## Summative Assessment and/or Summative Criteria

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Describe Learning Vertically  
Identify Key Building Blocks  
Make Connections (between and among key building blocks)  
Short/Extended Constructed Response Items  
Multiple-Choice Items (where multiple answer choices may be correct)  
Drag and Drop Items  
Use of Equation Editor Quizzes  
Journal Entries/Reflections/Quick-Writes  
Accountable talk  
Projects Portfolio  
Observation  
Graphic Organizers/Concept Mapping  
Presentations  
Role Playing  
Teacher-Student and Student-Student Conferencing Homework

## Resources

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- [http://www.mathwords.com/index\\_adv\\_alg\\_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- <http://www.hershey.k12.pa.us/Page/3608>
- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/precalculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>
- [https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured\\_content](https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content)
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- The Data and Story Library (DASL). <http://lib.stat.cmu.edu/DASL/>

## Unit Plan

| Topic/Selection<br>Timeframe                    | General Objectives   | Instructional Activities   | B  |
|---|--|--|--|
| Rational Functions and Asymptotes<br><br>8 days | SWBAT find the domain of rational functions.<br><br>SWBAT find vertical and horizontal asymptotes of graphs of rational functions.   | Graph rational functions, factor numerator and denominator to find equations of vertical asymptotes and open points, find equation of horizontal asymptote, if one exists, and discuss domain and range.   | Group a<br>daily, fi<br><br>Homew<br>textboo<br><br>Introdu<br>will be<br>student<br>underst<br><br>Activity<br><br>Student<br>and wil<br>having<br>provide<br>their jus |
| Introduction to Limits<br><br>6 days            | SWBAT understand the limit concept.<br><br>SWBAT use the definition of a limit to estimate limits.<br><br>SWBAT determine a limit by looking at a graph.<br><br>SWBAT determine whether limit of a function exist. | Activity: Investigate sums of series such as $1 + 1/2 + 1/3 + \dots$ or $1/1! + 1/2! + 1/3! \dots$<br><br>Use the graphing calculator to analyze the limit of a function, as $x$ approaches different values on the graph. Includes a situation where the limit for the function will fail to exist. | Group a<br>daily, fi<br><br>Homew<br>textboo<br><br>Introdu<br>will be<br>student<br>underst<br><br>Quiz on  |
| Techniques for Evaluating Limits<br><br>7 days  | SWBAT use different techniques to evaluate limits of functions.  | Model when and why to use divide out or rationalization technique.<br><br>Describe the process of using divide out and rationalization techniques to find the limit.<br><br>Illustrate the concept of one-sided limits with tables and graphs.   | Group a<br>daily, fi<br><br>Homew<br>textboo<br><br>Introdu<br>will be<br>student<br>underst<br><br>Mid un   |

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|---|--|--|---|
|   |  |  |   |
| <p>Limits at Infinity</p> <p>3 days</p>       | <p>SWBAT evaluate limits at infinity of rational functions.</p>  | <p>Students will study behavior of different parts of a function as <math>x</math> increases or decreases without bound.</p>   | <p>Group a daily, fi</p> <p>Homev textboo</p> <p>Introdu will be student underst</p> <p>Quiz or</p> |
| <p>The tangent Line Problem</p> <p>2 days</p> | <p>SWBAT evaluate limits of difference quotients from calculus.</p> <p>SWBAT find the formula of the line tangent to a function through the limit process.</p>                     | <p>Model definition of slope using limit process.</p> <p>Demonstrate few examples how to find slope of a tangent line that intersect graphs at a given point using limit process.</p>  | <p>Group a daily, fi</p> <p>Homev textboo</p> <p>Introdu will be student underst</p> <p>Quiz or</p> |
| <p>Continuity</p> <p>4 days</p>               | <p>SWBAT distinguish between continuous and discontinuous functions.</p> <p>SWBAT graph discontinuous functions both removable and non-removable by hand and using technology.</p> | <p>Recognize that a rational function is continuous at every number, except any at which it is not defined.</p> <p>Demonstrate examples of rational functions by factor numerator and denominator to find removable and non-removable discontinuities.</p> | <p>Group a daily, fi</p> <p>Homev textboo</p> <p>Introdu will be student underst</p> <p>Quiz or</p> |

## **Suggested Modifications for Special Education, ELL and Gifted Students**

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- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
- Introduce vocabulary before lessons, so that ELLs have access to and can build understand of the math language.
- Support ELL students by modeling, creating visuals, describing with words, symbols, or numbers. Chart key vocabulary and examples with students. Leave these anchor-charts up so you and the students can refer back to them.
- Supports such as sentence stems, think-pair-shares, intentional groupings for cooperative learning activities, and guided math groups so ELLs can be successful.
- The ability to synthesize concepts and make real world and cross- curricular connections through examples for gifted students
- Elevated contextual complexity for gifted students
- Sometimes independent activities, sometimes direct instruction for gifted students
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- Using supplementary materials in addition to the normal range of resources for gifted students
- Choices for students for gifted students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly) for gifted students

## **Suggested Technological Innovations/Use**

---

TECH.8.1.12

Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

## **Cross Curricular/21st Century Connections**

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Science Connection:

Name of Task: Rabbits Population NJSL: HS-LS1-2;

HS-LS1-4 The population of rabbits over a 2 year period in a certain county is given below

- 1- Draw a scatter plot of the data
- 2- Find a logistic regression model for the data. Find the limit of that model as time approaches infinity
- 3- What can you conclude about the limit of the rabbit population growth in the county?
- 4- Provide a reasonable explanation for the population growth limit.

CRP.K-12.CRP2

Apply appropriate academic and technical skills.

CRP.K-12.CRP4

Communicate clearly and effectively and with reason.

CRP.K-12.CRP7

Employ valid and reliable research strategies.

# Unit 09 Logarithmic Functions

Content Area: **Mathematics**  
Course(s): **Pre Calculus Honors**  
Time Period: **4th Marking Period**  
Length: **8-10 Days**  
Status: **Not Published**

## Summary of the Unit

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This unit will focus on the study of exponential and logarithmic functions. Exponential functions describe relationships that have a variable rate of change. Students are introduced to the natural base,  $e$ . Logarithmic functions are the inverses of exponential functions. Students will learn to analyze both types of functions using analytical, numerical and graphical approaches.

|              |  |
|--------------|--|
| MA.F-BF.B.5  | Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.                                    |
| MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. |

## Enduring Understandings

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Students will be able to

- Recognize, evaluate, and graph exponential and logarithmic functions.
- Use exponential and logarithmic functions to model and solve real life problems.

## Essential Questions

---

How do we model quantities that change at a constant rate over time?

What is a logarithm? How do mathematicians use them?

What is the relationship between exponential and logarithmic functions?

## Summative Assessment and/or Summative Criteria

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Describe Learning Vertically  
Identify Key Building Blocks  
Make Connections (between and among key building blocks)  
Short/Extended Constructed Response Items  
Multiple-Choice Items (where multiple answer choices may be correct)  
Drag and Drop Items  
Use of Equation Editor Quizzes  
Journal Entries/Reflections/Quick-Writes  
Accountable talk  
Projects Portfolio  
Observation  
Graphic Organizers/Concept Mapping

Presentations  
 Role Playing  
 Teacher-Student and Student-Student Conferencing Homework

## Resources

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- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/prec calculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>
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## Unit Plan

| Topic/Selection<br>Timeframe                 | General Objectives  | Instructional Activities   | Be  |
|--|---|--|---|
| Logarithmic<br>Functions<br><br>5 days       | SWBAT review graphing logarithmic functions, functions involving $e$ , and evaluating logarithms. | Provide review problems on graphing logarithmic functions, functions involving $e$ , and evaluating logarithms.  | Group<br>assigne<br><br>Homew<br>textbo<br><br>Introdu<br>will be<br>student<br>underst |
| Solve logarithmic<br>equations<br><br>6 days | SWBAT solve logarithmic equations   | Provide notes and examples on how to solve logarithmic equations, in one of the two following cases:<br>1) in problems where you have (or can obtain by using log properties) "log" with the same base on BOTH sides, or<br>2) in problems where you have (or can obtain by using log properties) one "log" on one side, but not on the other. Use the graphing calculator to "check" solutions. | Group<br>daily, f<br><br>Homew<br>textbo  |

|  |  |   |  |
|--|--|---|--|
|  |  | As time permits, include word problems. | Introductory<br>will be<br>student<br>understanding<br><br>Quiz/Assessment |
|--|--|---|--|

## **Suggested Modifications for Special Education, ELL and Gifted Students**

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
- Introduce vocabulary before lessons, so that ELLs have access to and can build understanding of the math language.
- Support ELL students by modeling, creating visuals, describing with words, symbols, or numbers. Chart key vocabulary and examples with students. Leave these anchor-charts up so you and the students can refer back to them.
- Supports such as sentence stems, think-pair-shares, intentional groupings for cooperative learning activities, and guided math groups so ELLs can be successful.
- The ability to synthesize concepts and make real world and cross- curricular connections through examples for gifted students
- Elevated contextual complexity for gifted students
- Sometimes independent activities, sometimes direct instruction for gifted students



- Inquiry based or open ended assignments and projects for gifted students
- Using supplementary materials in addition to the normal range of resources for gifted students
- Choices for students for gifted students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly) for gifted students

## **Suggested Technological Innovations/Use**

---

TECH.8.1.12

Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

## **Cross Curricular/21st Century Connections**

---

CRP.K-12.CRP2

Apply appropriate academic and technical skills.

CRP.K-12.CRP4

Communicate clearly and effectively and with reason.

CRP.K-12.CRP7

Employ valid and reliable research strategies.

# Unit 10 Sequences and Series

Content Area: **Mathematics**  
Course(s): **Pre Calculus Honors**  
Time Period: **1st Marking Period**  
Length: **10 -12 Days**  
Status: **Not Published**

## Summary of the Unit

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In this unit, students will study sequences and series, which are used for approximating functions.

MA.F-BF.A.2

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

## Essential Questions

---

- How do mathematicians model arithmetic and geometric sequences and series? What purpose do these models serve?
- What are different methods which can be used when determining the sum of a sequence? What is the  $n$ th term of a sequence and how is it useful in making predictions?
- How are patterns used to find the sum of arithmetic sequences?
- Is it possible for an infinite series to have a finite sum?
- What is the role of the interval of convergence in a geometric series?
- How are sequences and series used to model real life problems?

## Summative Assessment and/or Summative Criteria

---

Describe Learning Vertically  
Identify Key Building Blocks  
Make Connections (between and among key building blocks)  
Short/Extended Constructed Response Items  
Multiple-Choice Items (where multiple answer choices may be correct)  
Drag and Drop Items  
Use of Equation Editor Quizzes  
Journal Entries/Reflections/Quick-Writes  
Accountable talk  
Projects Portfolio  
Observation  
Graphic Organizers/Concept Mapping  
Presentations  
Role Playing  
Teacher-Student and Student-Student Conferencing Homework

## Resources

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## Unit Plan

| Topic/Selection<br>Timeframe        | General Objectives   | Instructional Activities  | Be   |
|-------------------------------------|--|---|--|
| Sequences and Series<br><br>12 days | <p>SWBAT model arithmetic and geometric sequences and series? and determine what purpose do these models serve?</p> <p>SWBAT to determine the sum of a sequence using different methods.</p> <p>SWBAT find the nth term of a sequence and how is it useful in making predictions?</p> <p>SWBAT to determine how are patterns used to find the sum of arithmetic sequences?</p> <p>SWBAT determine if it is possible for an infinite series to have a finite sum?</p> <p>SWBAT to determine the role of the interval of convergence in a geometric series?</p> <p>SWBAT to determine how are sequences and series used to model real life problems?</p> | <p>Use the formula to find the sum of a finite arithmetic and geometric series, given a problem written in summation notation form.</p> <p>Derive the Sum of an Infinite Geometric Series formula, and discuss when a sum does not exist.</p> | <p>Group daily, f</p> <p>Homev textbo</p> <p>Introdu will be studen unders</p> <p>Quiz/A</p> |

## **Suggested Modifications for Special Education, ELL and Gifted Students**

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- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
- Introduce vocabulary before lessons, so that ELLs have access to and can build understand of the math language.
- Support ELL students by modeling, creating visuals, describing with words, symbols, or numbers. Chart key vocabulary and examples with students. Leave these anchor-charts up so you and the students can refer back to them.
- Supports such as sentence stems, think-pair-shares, intentional groupings for cooperative learning activities, and guided math groups so ELLs can be successful.
- The ability to synthesize concepts and make real world and cross- curricular connections through examples for gifted students
- Elevated contextual complexity for gifted students
- Sometimes independent activities, sometimes direct instruction for gifted students
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## **Suggested Technological Innovations/Use**

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TECH.8.1.12

Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to

create and communicate knowledge.

## **Cross Curricular/21st Century Connections**

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|               |  |
|---------------|--|
| CRP.K-12.CRP2 | Apply appropriate academic and technical skills.     |
| CRP.K-12.CRP4 | Communicate clearly and effectively and with reason. |
| CRP.K-12.CRP7 | Employ valid and reliable research strategies.       |

## **Enduring Understandings**

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Students will be able to:

- Variety of ways to represent a sequence of terms.
- Find sum of infinite series.
- Use sequences and series to model and solve real-life problems.
- Recognize, write, and find the  $n$ th terms of arithmetic and geometric sequences.
- Find  $n$ th partial sums of arithmetic and geometric sequences.
- Use arithmetic and geometric sequences to model and solve real-life problems.

# Unit 11 Probability

Content Area: **Mathematics**  
Course(s): **Pre Calculus Honors**  
Time Period: **4th Marking Period**  
Length: **7-9 Days**  
Status: **Not Published**

## Summary of the Unit

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In this unit, students will study probability, a field used for analyzing data and decision making. Students will learn how to find conditional probabilities, permutations and combinations.

|             |  |
|-------------|--|
| MA.S-CP.B.6 | Find the conditional probability of $B$ given $A$ as the fraction of $A$ 's outcomes that also belong to $B$ , and interpret the answer in terms of the model.   |
| MA.S-CP.B.7 | Apply the Addition Rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ , and interpret the answer in terms of the model.   |
| MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, $P(A \cap B) = [P(A)] \times [P(B A)] = [P(B)] \times [P(A B)]$ , and interpret the answer in terms of the model.  |
| MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems.  |
| MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").  |
| MA.S-CP.A.2 | Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.   |
| MA.S-CP.A.3 | Understand the conditional probability of $B$ given $A$ as $P(A \cap B)/P(A)$ , and interpret independence of $A$ and $B$ as saying that the conditional probability of $B$ given $A$ is the same as the probability of $B$ , and the conditional probability of $A$ given $B$ is the same as the probability of $A$ . |
| MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.   |

## Enduring Understandings

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Students will be able to:

- Use the Fundamental Counting Principle to solve counting problems.
- Use Permutation and Combination to solve counting problems.
- Use Probability to solve a variety of problems.

## Essential Questions

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- How do you count the number of ways in which an event can occur?
- When do I use a permutation vs. a combination?
- How do you find probability that a series of events will occur?

- How is probability related to real world events?
- How can experimental and theoretical probabilities be used to make predictions or draw conclusions?

## Summative Assessment and/or Summative Criteria

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Describe Learning Vertically  
 Identify Key Building Blocks  
 Make Connections (between and among key building blocks)  
 Short/Extended Constructed Response Items  
 Multiple-Choice Items (where multiple answer choices may be correct)  
 Drag and Drop Items  
 Use of Equation Editor Quizzes  
 Journal Entries/Reflections/Quick-Writes  
 Accountable talk  
 Projects Portfolio  
 Observation  
 Graphic Organizers/Concept Mapping  
 Presentations  
 Role Playing  
 Teacher-Student and Student-Student Conferencing Homework

## Resources

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- Cengage Learning: Precalculus with Limits - A Graphing Approach, Sixth Edition
- <https://www.ixl.com/standards/new-jersey/math/high-school>
- [http://www.mathwords.com/index\\_adv\\_alg\\_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- <http://www.hershey.k12.pa.us/Page/3608>
- <https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates>
- <https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets>
- <https://www.ixl.com/math/prec calculus>
- <https://www.illustrativemathematics.org/>
- <http://map.mathshell.org/materials/lessons.php?gradeid=24>
- <http://www.achieve.org/ccss-cte-classroom-tasks>
- [https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured\\_content](https://www.ck12.org/algebra/Applications-of-Function-Models/lesson/Applications-of-Function-Models-BSC-ALG/?referrer=featured_content)
- Statistics Education Web (STEW). <http://www.amstat.org/education/STEW/>
- The Data and Story Library (DASL). <http://lib.stat.cmu.edu/DASL/>

## Unit Plan

| Topic/Selection | General Objectives | Instructional Activities | B |
|-----------------|--------------------|--------------------------|---|
|-----------------|--------------------|--------------------------|---|

| Timeframe                                    |   |  |   |
|--|---|--|---|
| <p>The Counting Principle<br/>2 days</p>     | <p>SWBAT use the Fundamental Counting Principle to solve counting problems.</p> | <p>Introduce the Fundamental Counting Principle, and provide practice problems using it (both independent and dependent events).</p>   | <p>Group daily,<br/>Homev textbo<br/>Introdu will be studen unders<br/>Quiz/A</p> |
| <p>Premutaion and Combination<br/>3 days</p> | <p>SWBAT use Permutation and Combination to solve counting problems.</p>        | <p>Introduce the permutation and combination formula, and show how to use it in the graphing calculator.</p> <p>Show the formula for permutation with repetition, and provide practice problems to use it.</p> <p>Provide practice problems with simple events, and multiple events.</p>   | <p>Group daily,<br/>Homev textbo<br/>Introdu will be studen unders<br/>Quiz/A</p> |
| <p>Probability<br/>4 days</p>                | <p>SWBAT use Probability to solve a variety of problems.</p>                    | <p>Provide notes that: If 2 events are independent, then the probability of both events happening is the product of the two independent probabilities.</p> <p>Practice problems together, and extend the above to more than two independent events.</p> <p>Define simple event, compound event, mutually exclusive, and inclusive.</p> <p>Provide notes that: If two events are mutually exclusive, then the probability of either the first or the second event occurs is the sum of their probabilities.</p> <p>Provide notes on conditional probability. Model problems before providing practice problems.</p> | <p>Group daily,<br/>Homev textbo<br/>Introdu will be studen unders<br/>Quiz/A</p> |



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## **Cross Curricular/21st Century Connections**

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Social Studies Connection:

Name of Task: Security System at Quail Creek NJSLS: 9.1.12.A.4 Ten homes in the Quail Creek area were identified by a surveyor; you were told that 4 of the homes have security system.

1- If you are to pick 3 homes randomly:

a. What is the probability that all 3 homes have security system? b. What is the probability that only 1 has a security system?

2- If 40% of the homes constructed in the Quail creek area include a security system.

Three homes are selected at random:

a. What is the probability all three of the selected homes have a security system?

b. What is the probability none of the selected homes have a security system?

c. What is the probability that at least one has a security system?

3- Create probability distribution tables for part 1 and part 2

CRP.K-12.CRP2

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CRP.K-12.CRP4

Communicate clearly and effectively and with reason.

CRP.K-12.CRP7

Employ valid and reliable research strategies.