Algebra 2 CP/Core Cover Page

Content Area: **Mathematics**
Course(s):
Time Period:
Length: **180-184 days**
Status: **Published**

Course Overview

Algebra 2 builds upon the foundational concepts learned in Algebra 1 and Geometry, delving deeper into advanced algebraic techniques and their applications in various fields. This course emphasizes problem-solving, critical thinking, and mathematical modeling to prepare students for college-level mathematics and future STEM careers.

Key Themes and Focus Areas:

- **Polynomials:** Students will explore in-depth operations with polynomials, including factoring, graphing, and solving polynomial equations. The fundamental theorem of algebra will be introduced, along with its implications for finding roots of polynomials.
- **Rational Expressions and Equations:** Students will learn to manipulate and simplify rational expressions, solve rational equations, and graph rational functions. Real-world applications of rational functions, such as inverse variation and modeling with asymptotes, will be explored.
- **Radical Expressions and Equations:** Students will deepen their understanding of radical expressions and equations, including simplifying radicals, solving radical equations, and graphing radical functions.
- **Exponential and Logarithmic Functions:** This unit will focus on the properties of exponential and logarithmic functions, their graphs, and applications such as compound interest and exponential growth/decay models.
- **Trigonometry:** Building upon basic trigonometry concepts from Geometry, students will explore trigonometric identities, functions, and equations. They will also learn about the unit circle and its relationship to trigonometric functions.
- **Quadratic Functions and Equations:** Students will review and extend their knowledge of quadratic functions, including vertex form, standard form, and factored form. They will also learn about complex numbers and their role in solving quadratic equations.
- **Sequences and Series:** This unit will introduce students to arithmetic and geometric sequences and series, along with their formulas and applications in real-world scenarios.
- **Probability and Statistics:** Students will explore basic probability concepts, such as sample spaces, events, and probability calculations. They will also learn about measures of central tendency and spread, and their application in data analysis.

Course Name, Length, Date of Revision and Curriculum Writer

Course Name: Algebra 2 CP/ core

Length : Full Year

Date of Revision: June 2024

Curriculum Writer: Sarah Rodis

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Marking Period 1				
#	**Student Learning Objective**	**Big Ideas Math Correlation**	**CP/Core**	**Number of Days**
1	Modeling with Linear Functions.	1-3	Yes	5
2	Transformations of Quadratic Functions.	2-1	Yes	5
3	Characteristics of Quadratic Functions.	2-2	Yes	5
4	Focus of a Parabola.	2-3	Yes	3
5	Modeling with Quadratic Functions	2-4	Yes	5
6	Solving Quadratic Equations	3-1	Yes	6
7	Solving Nonlinear Systems	3-5	Yes	8
8	Quadratic Inequalities	3-6	Yes	5
	Total			42

Marking Period 2				
#	**Student Learning Objective**	**Big Ideas Math Correlation**	**CP/ Core**	**Number of days**
1	Graphing Polynomial Functions	4-1	Yes	4
2	Adding, Subtracting, and Multiplying Polynomials	4-2	Yes	4
3	Dividing Polynomials	4-3	Yes	4
4	Factoring Polynomials	4-4	Yes	4
5	Solving Polynomial Equations	4-5	Yes	5
6	Analyzing Graphs of Polynomial Functions	4-8	Yes	5
7	Modeling with Polynomial Functions	4-9	Yes	5
8	*nth Roots and Rational Exponents*	5-1	Yes	4
9	Properties of Rational Exponents and Radicals	5-2	Yes	5
10	Solving Radical Equations and Inequalities	5-4	Yes	3
11	Performing Function Operations	5-5	Yes	4

Marking Period 3				
#	**Student Learning Objective**	**Big Ideas Math Correlation**	**CP/ Core**	**Number of Days**
1	Exponential Growth and Decay Functions	6-1	Yes	4
2	Properties of Logarithms	6-5	Yes	3
3	Solving Exponential and Logarithmic Equations	6-6	Yes	4
4	Modeling with Exponential and Logarithmic Functions	6-7	Yes	4
5	Graphing Rational Functions	7-2	Yes	3
6	Multiplying and Dividing Rational Expressions	7-3	Yes	4
7	Adding and Subtracting Rational Expressions	7-4	Yes	4
8	Solving Rational Equations	7-5	Yes	4
9	Defining and Using Sequences and Series	8-1	Yes	3
10	Analyzing Arithmetic Sequences and Series	8-2	Yes	2
11	Analyzing Geometric Sequences and Series	8-3	Yes	2
12	Using Recursive Rules with Sequences	8-5	Yes	3
				40

Marking Period 4				
#	**Student Learning Objective**	**Big Ideas Math Correlation**	**Core/CP**	**Number of Days**
1	Data Analysis: Making Sense of Data Analyzing Categorical Data Represent data with plots on the real number line (dot plots, histograms, and box plots). Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	11-1	Yes	10

2	Describing Quantitative Data with Graphs Describing Quantitative Data with Numbers Use dot plot, stem plot, or histograms to show the distribution of a quantitative variable by looking for an overall pattern and for notable departures. Describe a numerical summary of a distribution using its center, spread or variability.	11-2	Yes	10
				20

Unit 1 Linear and Quadratic Functions, Quadratic Equations and Complex Numbers

Content Area: **Mathematics**
Course(s):
Time Period:
Length: **40-45 days**
Status: **Published**

Summary of the Unit

This unit delves into the foundations of functions, focusing on linear, quadratic, and complex number systems. It begins with an exploration of parent functions and transformations, emphasizing how changes in parameters affect the graph's shape, position, and orientation.

Students then investigate linear functions, applying their understanding of slope, intercepts, and equations to model real-world scenarios. They learn to solve linear systems using various methods such as substitution, elimination, and graphing.

Moving on to quadratic functions, students discover the characteristics of parabolas, including vertex, axis of symmetry, and direction of opening. They explore transformations of quadratic functions and learn how to identify the focus of a parabola. Additionally, students apply quadratic functions to model real-world phenomena.

The unit also covers solving quadratic equations using a variety of techniques, including factoring, completing the square, and the quadratic formula. Students are introduced to complex numbers and learn how to perform operations with them.

Finally, students investigate nonlinear systems and quadratic inequalities, utilizing their knowledge of quadratic functions and equations to solve these more complex problems.

Enduring Understandings

- Functions as Relationships: Students will understand that functions represent relationships between quantities, where each input has a unique output. They will be able to analyze these relationships using tables, graphs, and equations.

- Parent Functions and Transformations: Students will grasp the concept of parent functions as basic building blocks for various families of functions (linear, absolute value, quadratic). They will understand how transformations (translations, reflections, stretches, and compressions) modify the graphs and equations of these parent functions.

- Modeling Real-World Phenomena: Students will recognize the power of functions to model and analyze real-world situations. They will learn to use linear and quadratic functions to represent and solve problems involving various scenarios.

- Solving Equations and Systems: Students will develop proficiency in solving linear systems using various methods (graphing, substitution, elimination). They will also extend their equation-solving skills to quadratic equations, including factoring, completing the square, and using the quadratic formula.

- Quadratic Functions and Their Properties: Students will understand the key characteristics of quadratic functions, including vertex, axis of symmetry, intercepts, and maximum or minimum values. They will connect these properties to the graph and equation of a quadratic function.

- Focus of a Parabola: Students will explore the concept of the focus of a parabola and its relationship to the directrix. They will understand how this geometric property relates to the reflective properties of parabolas.

- Complex Numbers: Students will expand their understanding of numbers to include complex numbers. They will learn to perform operations with complex numbers and understand their role in solving quadratic equations.

- Inequalities and Their Solutions: Students will be able to solve quadratic inequalities and represent their solutions graphically. They will understand how the graph of a quadratic function relates to the solution set of a quadratic inequality.

- Nonlinear Systems: Students will extend their understanding of systems to include nonlinear systems involving quadratic equations. They will learn to solve these systems using various methods, including substitution and elimination.

Essential Questions

- Identify parent functions and transformations.
- Describe transformations of parent functions.
- Model with linear functions and solve linear systems.
- Describe transformations of quadratic functions.
- Identify characteristics of quadratic functions.
- Write equations of parabolas.
- Model with quadratic functions.
- Perform operations with complex numbers.
- Solve quadratic equations by completing the square.
- Describe how to use the quadratic formula.
- Solve nonlinear systems and quadratic inequalities.

Summative Assessment and/or Summative Criteria

The summative assessment for this unit will consist of two parts:

1. Individual Assessment:
    - A written test covering a range of question types (multiple-choice, short answer, problem-solving) assessing students' understanding of the enduring understandings outlined above.
    - Questions will require students to:
        - Identify and graph parent functions
        - Perform transformations on functions given equations or graphs
        - Solve linear and quadratic equations and systems
        - Interpret and analyze real-world scenarios using linear and quadratic models
        - Identify key characteristics of quadratic functions
        - Solve quadratic inequalities
        - Perform operations with complex numbers
        - Solve nonlinear systems
2. Performance Task:
    - A real-world problem-solving task where students will apply their knowledge of functions and transformations to model and analyze a given situation.
    - Students will be assessed on their ability to:
        - Choose appropriate functions to model the problem
        - Apply transformations to the functions as needed
        - Solve relevant equations or inequalities
        - Interpret their solutions in the context of the problem
        - Communicate their findings effectively

Resources

Internet4Classrooms www.internet4classrooms.com

Desmos https://www.desmos.com/

Math Open Reference www.mathopenref.com

National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/index.html

Georgia Department of Education https://www.georgiastandards.org/Georgia-Standards/Pages/Math-9-12.aspx

Illustrative Mathematics www.illustrativemathematics.org/

Khan Academy https://www.khanacademy.org/math/algebra-home/algebra2

Math Planet http://www.mathplanet.com/education/algebra-2

IXL Learning https://www.ixl.com/math/algebra-2

Math Is Fun Advanced http://www.mathsisfun.com/algebra/index-2.html

Partnership for Assessment of Readiness for College and Careers https://parcc.pearson.com/practice-tests/math/

Mathematics Assessment Project http://map.mathshell.org/materials/lessons.php?gradeid=24

Achieve the Core http://www.achieve.org/ccss-cte-classroom-tasks

NYLearns http://www.nylearns.org/module/Standards/Tools/Browse?linkStandardId=0&standardId=97817

Learning Progression Framework K-12 http://www.nciea.org/publications/Math_LPF_KH11.pdf

PARCC Mathematics Evidence Tables. https://parcc-assessment.org/mathematics/

Unit Plan

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmark / Assessments
1-1 Parent Functions and Transformations.	How do the graphs of $y = f(x) + k$, $y = f(x - h)$, and $y = -f(x)$ compare to the graph of the parent function f ? What are the characteristics of some of the basic parent functions? Is it possible to use more than one transformation on a function?	Function notation representation of transformations Performs transformations on graphs of polynomial, exponential, logarithmic, or trigonometric functions. Identify the effect on the graph of replacing f(x) by: o f(x) + k; o k f(x); o f(kx); o and f(x + k) for specific values of k (both positive and negative). Identify the effect of combinations of transformations on the graph. Given the graph, find the value of k.	Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this Algebra 1 topic, and review as needed. Check student responses.
1-2 Transformations of Linear and Absolute Value Functions.	Transformations of Linear and Absolute Value Functions.	Explain the term parent function if it is unfamiliar. Write the *Core Concept*, which shows the graphs of linear functions transformed. Make the connection to the results found in translating the absolute value function in the explorations.	Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this Algebra 1 topic, and review as needed. Check student responses.
1-3 Modeling with Linear Functions.	Modeling with Linear Functions.	Using a Given Input and Output to Build a **Model**. Identify the input and output values. Convert the data to two coordinate pairs. Find the slope. Write the **linear model**. Use the **model** to make a prediction	Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into

		by evaluating the **function** at a given x-value. Use the **model** to identify an x-value that results in a given y-value. Write equations of linear functions using points and slopes. Find lines of fit and lines of best fit. Ask probing questions to gauge students' recollection of what information is necessary to write a linear function and the two forms.	algebra. Classwork assigned. Homework assigned Assess student recall of this Algebra 1 topic, and review as needed. Check student responses.
1-4 Solving Linear Systems.	Solving Linear Systems.	Visualize solutions of systems of linear equations in three variables. Solve systems of linear equations in three variables algebraically. Solve real-life problems. Have students recall methods of solving linear systems (graphing, elimination, and substitution) and how many solutions a system of linear equations can have. Indicate that the same techniques can be used for quadratic systems. Discuss the number of solutions possible in a quadratic system. Solve multiple systems using the above techniques, stressing that solutions can be checked algebraically.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
2-1 Transformations of Quadratic Functions.	Transformations of Quadratic Functions.	Describe transformations of quadratic functions. Write transformations of quadratic functions. Model how the function notation of transformations correlates to changes in the values and graph of a function**.**	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.

		Provide students with a reference document that verbally and pictorially illustrates a function's features and how they are changed due to transformation.	
2-2 Characteristics of Quadratic Functions.	Characteristics of Quadratic Functions.	Explore properties of parabolas. Find maximum and minimum values of quadratic functions—graph quadratic functions using *x*-intercepts. Solve real-life problems. Students should be able to state the vertex in addition to describing the symmetry.	Check student understanding via oral participation. Check student work and graphs. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
2-3 Focus of a Parabola.	Focus of a Parabola.	Explore the focus and the directrix of a parabola. Write equations of parabolas. Solve real-life problems. Pre-teach vocabulary using visual and verbal models that are connected to real-life situations and ensure that students include these definitions in their reference notebooks. Model how to derive the equation of a circle given the center and radius using the Pythagorean Theorem. Ensure that students include this information in their reference notebook. Model how to use the equation of a circle to determine the radius and center. Provide students with hands-on opportunities to explore and extend their understanding by working in small groups, see the application to real life.	Check student understanding via oral participation. Check student work and graphs. Classwork assigned. Homework assigned.

2-4 Modeling with Quadratic Functions	Modeling with Quadratic Functions	Write equations of quadratic functions using vertices, points, and *x*-intercepts. Write quadratic equations to model data sets. Students should be familiar with identifying a linear function from a table of values where the constant rate of change was found in the first differences. Introduce the idea that when second differences are equal, the data is quadratic. Model the thinking behind determining when and how to use the graphing calculator to graph complicated polynomials.	Check student understanding via oral participation. Check student work and graphs. Classwork assigned. Homework assigned.
3-1 Solving Quadratic Equations	Solving Quadratic Equations	The goal is for students to use their understanding of the standard form of a quadratic equation to match the equations and graphs. Expect students to look at the leading coefficient to determine whether the graph opens upward or downward. Students should also look at the constant term to determine the *y*-intercept. Students should make the connection between an *x*-intercept and the value of a function being 0 and, therefore, the solution to the quadratic equation. The real number solutions are all integers so that students may find the solutions by eyesight. Have students recall methods of solving linear systems (graphing, elimination, and substitution). Have students recall how many	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.

		solutions a systems of linear equations can have. Indicate that the same techniques can be used for quadratic systems. Discuss the number of solutions possible in a quadratic system. Solve multiple systems using the above techniques, stressing that solutions can be checked algebraically.	
3-2 Complex Numbers	Complex Numbers	Define and use the imaginary unit *i*. Add, subtract, and multiply complex numbers. Find complex solutions and zeros.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
3-3 Completing the Square	Completing the Square	Provide details in the process of completing the square (both when $a=1$ and when $a\neq1$). Model practice problems of both types. Model practice problems of converting a quadratic function in standard form to vertex form using this process. Include word	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
3-4 Using the Quadratic Formula	Using the Quadratic Formula	Instead of using the technique of completing the square each time we want to solve a quadratic, the technique is applied to the standard form of a quadratic equation, $0 = ax2 + bx + c$, and the result is called the Quadratic Formula. Solve the general case	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.

		and then use the result to solve other quadratics. Help students see the connection between the Quadratic Formula and the equation for the vertex of a parabola. Explain to students that at the end of the lesson they will be introduced to a new model that takes the initial velocity into account. Write the *Core Concept*. Remind students that the quadratic equation must be in standard form that is, set equal to 0.	
3-5 Solving Nonlinear Systems	Solving Nonlinear Systems	How can you solve a nonlinear system of equations? What are the intersection possibilities for a parabola and a line?" two, one, or no intersections "two parabolas?" two, one, or no intersections Can you solve an equation by rewriting it as a system of equations and then solving the system by graphing?	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
3-6 Quadratic Inequalities	How can you solve a quadratic inequality? what would an ordered pair inside the parabola represent?	To graph a quadratic inequality in one of the forms above, follow these steps. **Step 1** Graph the parabola with the equation $y = ax2 + bx + c$. Make the parabola *dashed* for inequalities with < or > and *solid* for inequalities with ≤ or ≥ . **Step 2** Test a point (x, y) inside the parabola to determine whether the point is a solution of the inequality.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned

		Step 3 Shade the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution. Have students write an equation for perimeter and an inequality for area and then discuss an approach with their partners.	

Standards

MATH.9-12.F.BF.A	Build a function that models a relationship between two quantities
MATH.9-12.A.APR.B	Understand the relationship between zeros and factors of polynomials
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.N.CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
MATH.9-12.N.CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
MATH.9-12.A.CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MATH.9-12.N.CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
MATH.9-12.A.REI.B.4	Solve quadratic equations in one variable.
MATH.9-12.A.REI.B.4.b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.

Suggested Modifications for Special Education, ELL and Gifted Students

Special Education Students:

- Visual Aids and Manipulatives: Provide visual aids such as graphic organizers, anchor charts, and manipulatives like algebra tiles or graphing boards to help students visualize concepts and relationships.
- Simplified Instructions: Break down complex concepts into smaller, more manageable steps with clear and concise instructions.
- Extra Practice and Scaffolding: Offer additional practice problems with varying difficulty levels, along with scaffolding (hints, prompts, worked examples) to support students as they progress.
- Alternative Assessments: Provide alternative assessment options, such as oral presentations, projects, or portfolios, to demonstrate understanding.
- Individualized Instruction: Tailor instruction to meet individual needs, considering learning styles, strengths, and weaknesses.

English Language Learners (ELLs):

- Visual Support: Incorporate visuals, such as diagrams, graphs, and real-world examples, to enhance understanding of vocabulary and concepts.
- Simplified Language: Use clear and concise language, avoiding jargon and idioms. Provide a glossary of key terms with definitions and translations.

- Bilingual Resources: Offer bilingual dictionaries, textbooks, or online resources to support students' understanding.
- Collaborative Learning: Encourage peer-to-peer interaction and group work to provide opportunities for language practice and collaborative problem-solving.
- Differentiated Instruction: This method tailors instruction to meet individual language proficiency levels, providing additional support for students who need it.

Gifted Students:

- Extension Activities: Provide opportunities for enrichment and extension beyond the standard curriculum, such as exploring advanced topics, independent research projects, or problem-solving challenges.
- Inquiry-Based Learning: Encourage students to ask questions, explore concepts independently, and develop their solutions.
- Higher-Order Thinking Skills: Challenge students with complex problems that require critical thinking, analysis, and synthesis.
- Acceleration: Consider accelerating gifted students to higher-level math courses if they demonstrate readiness.
- Mentorship: Pair gifted students with mentors who can guide and support their academic pursuits.

By implementing these modifications, teachers can create a more inclusive and equitable learning environment where all students, regardless of their diverse needs and abilities, can succeed in Algebra 2

Suggested Technological Innovations/Use

Graphing Calculators or Software:

- Visualizing Transformations: Students can use graphing calculators or software like Desmos or GeoGebra to visualize the effects of different transformations on parent functions. They can easily manipulate parameters and observe the resulting changes in the graph in real time.
- Solving Equations and Systems: Graphing calculators can solve linear and quadratic equations graphically, reinforcing the connection between algebraic solutions and their visual representations.

They can also be utilized to solve systems of equations, both linear and nonlinear.

Online Simulations and Interactive Activities:

- Manipulating Functions: Online simulations allow students to manipulate functions dynamically, experimenting with different transformations and observing the impact on the graph and equation.
- Real-World Applications: Interactive activities can present real-world scenarios where functions are used for modeling. Students can explore these scenarios, adjust parameters, and see how the model responds, deepening their understanding of function applications.
- Gamification: Incorporating game elements into online activities can make learning about functions more engaging and motivating for students.

Virtual Manipulatives:

- Transforming Functions: Virtual manipulatives can provide a hands-on approach to exploring function transformations. Students can drag and drop functions, change parameters, and see the resulting changes visually.
- Building Functions: Students can use virtual manipulatives to build complex functions by combining different transformations. This can help them understand the composition of functions.

Online Assessment Tools:

- Formative Assessment: Online tools like Kahoot! or Quizlet can be used for quick formative assessments to gauge student understanding of concepts. This can help teachers identify areas where students need additional support.
- Summative Assessment: Online platforms can be used to create and administer summative assessments, such as quizzes or tests, with automatic grading and feedback.

Collaboration Tools:

- Group Projects: Students can use online collaboration tools like Google Docs or Microsoft Teams to work together on projects involving functions and their transformations. They can share ideas, collaborate on problem-solving, and present their findings.

Cross Curricular/Career Readiness, Life Literacies and Key Skills Practice

Social Studies Connection:

Name of Task: Carbon 14 Dating (6.2.12.GeoPP.1.a)

The task requires the student to use logarithms to solve an exponential equation in the realistic context of carbon dating, important in archaeology and geology, among other places

Science Connection:

Name of Task: Course of Antibiotics (HS-PS-1-6)

This task presents a real-world application of finite geometric series. The context can lead to several interesting follow-up questions and projects. Many drugs only become effective after the amount in the body builds up to a certain level, which can be modeled very well with geometric series.

Unit 2 Polynomial Functions, Rational Exponents and Radical Functions

Content Area: **Mathematics**
Course(s):
Time Period:
Length: **40-45 days**
Status: **Published**

Summary of the Unit

This Algebra 2 unit explores polynomials, delving into their graphs, operations (addition, subtraction, multiplication, and division), factoring, and equation solving. Students will learn to apply the Fundamental Theorem of Algebra and analyze polynomial graphs to identify key features. The unit emphasizes real-world applications, using polynomials to model various scenarios.

Additionally, the unit covers radical functions and operations with functions. Students will extend their understanding of exponents to include rational exponents and explore the connection between radical expressions and rational exponents. They will learn to graph radical functions and solve equations and inequalities involving radicals. The unit concludes with an exploration of function operations, including composition and finding inverses, providing students with a comprehensive understanding of algebraic functions and their applications.

Enduring Understandings

- Polynomials as Models: Students will understand that polynomial functions can model a wide range of real-world phenomena, from projectile motion to economic trends, and that analyzing their graphs and equations can provide valuable insights into these situations.

- Structure of Polynomial Expressions: Students will grasp the importance of factoring and operations with polynomials as tools for simplifying expressions, solving equations, and understanding the

behavior of polynomial functions.

- Relationship Between Roots and Factors: Students will recognize the Fundamental Theorem of Algebra as a unifying principle connecting the degree of a polynomial to the number of its roots, both real and complex, and understand how factors of a polynomial relate to its zeros.

- Transformations and Function Families: Students will comprehend that transformations (translations, reflections, stretches, compressions) can be applied to any function, including polynomials and radical functions, to create families of related functions with predictable changes in their graphs and equations.

- Rational Exponents and Radicals: Students will understand the equivalence between radical expressions and expressions with rational exponents, allowing them to manipulate and simplify complex expressions involving roots.

- Function Operations and Inverses: Students will recognize that functions can be combined through operations (addition, subtraction, multiplication, division, composition) and that some functions have inverses that "undo" their effects, providing powerful tools for solving equations and modeling relationships.

Essential Questions

Polynomials:

- How can polynomial functions be used to model real-world situations?
- What information can be gathered from the graph of a polynomial function?
- How are the roots of a polynomial equation related to its factors and graph?
- What are the different methods for solving polynomial equations, and when is each method most appropriate?
- How can transformations be used to manipulate the graph of a polynomial function?
- What is the Fundamental Theorem of Algebra, and how does it apply to polynomial equations?

Radical Functions:

3. How are radicals and rational exponents related, and how can we use this relationship to simplify expressions?
4. What are the key characteristics of radical functions, and how can we identify them from graphs and equations?
5. How do we solve equations and inequalities involving radical expressions, and why is it important to check for extraneous solutions?
6. How can we apply transformations to radical functions, and what effect do these transformations have on their graphs and equations?

Function Operations:

- How can we combine functions through operations such as addition, subtraction, multiplication, division, and composition?
- What is the inverse of a function, and how can we find it algebraically and graphically?
- What is the relationship between a function and its inverse, and how can we use this relationship to solve problems?
- How can we use function operations and inverses to model and analyze real-world situations?

Summative Assessment and/or Summative Criteria

- Unit Test: A comprehensive test covering all major topics in the unit, including:
    - Graphing and analyzing polynomial functions
    - Performing operations with polynomials (addition, subtraction, multiplication, division)
    - Factoring polynomials
    - Solving polynomial equations
    - Applying the Fundamental Theorem of Algebra
    - Transforming polynomial functions
    - Working with rational exponents and radicals
    - Graphing and solving radical equations and inequalities

    - Performing function operations (addition, subtraction, multiplication, division, composition)
    - Finding inverse functions
    - Modeling real-world scenarios with polynomials and radical functions

- Project/Presentation: A project or presentation where students apply their knowledge to a real-world scenario, such as modeling a physical phenomenon with a polynomial function or analyzing a data set using radical functions. The project should demonstrate understanding of the concepts and ability to apply them meaningfully.

- Formative Assessments: Throughout the unit, incorporate formative assessments (quizzes, exit tickets, homework assignments) to monitor student progress and identify areas for additional support.
- Differentiation: Tailor the assessments to meet the diverse needs of learners by providing modifications or accommodations as needed.
- Feedback: Provide timely and constructive feedback to students on their performance, highlighting strengths and areas for improvement.

Resources

Textbooks:

- Algebra 2 Common Core Textbook: Any standard Algebra 2 textbook aligned with Common Core State Standards will cover the topics in this unit. Look for chapters on polynomials, rational exponents, radical functions, and operations with functions.
- OpenStax Algebra 2: A free, open-source textbook available online that covers all the necessary content in a comprehensive and accessible manner.

Websites and Online Resources:

- Khan Academy: Offers a wide range of instructional videos, practice exercises, and articles on all Algebra 2 topics, including polynomials, radical functions, and function operations.
- Math Planet: Provides comprehensive explanations, examples, and practice problems on polynomials and radical expressions, as well as interactive tools and calculators.

- Desmos: A free online graphing calculator that allows students to visualize functions, explore transformations, and solve equations graphically.
- GeoGebra: Similar to Desmos, GeoGebra offers a free online graphing calculator with additional features for geometry and statistics.
- CK-12: Provides free, customizable textbooks and interactive activities on various math topics, including Algebra 2.

Additional Resources:

- Manipulatives: Algebra tiles and graphing boards can be used to help students visualize operations with polynomials and transformations of functions.
- Graphic Organizers: Flowcharts, concept maps, and other graphic organizers can help students organize their understanding of complex concepts and relationships.
- Online Quizzes and Games: Websites like Kahoot! and Quizlet offer interactive quizzes and games that can be used for formative assessment and review.
- Real-World Applications: Look for examples of how polynomials and radical functions are used in fields like physics, engineering, and finance to demonstrate the relevance of the concepts.

Unit Plan

Topic/Selection Timeframe	**General Objectives**	**Instructional Activities**	**Benchmark / Assessments**
4-1Graphing Polynomial Functions	Graphing polynomials	Identify polynomial functions and explain what the end behavior of a polynomial function means. Graph polynomial functions using tables and show them end behavior. Use graphing calculators and ask students to graph f(x) = x3, f(x) = −x3, f(x) = x4, and f(x) = −x4. After each graph, have	Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of

		students stand up and use their arms to describe the end behavior of the function. Share the polynomial model with students and ask them to determine a good viewing window.	this Algebra 1 topic and review as needed. Check student responses.
4-2 Adding, Subtracting, and Multiplying Polynomials	Adding, Subtracting, and Multiplying Polynomials	Add, subtract and multiply the polynomials Use Pascal's Triangle to expand binomials. Write the expression $(5x2 + 7x + 1) + (3x2 - 4x + 2)$ on the board as a warm-up problem. If students are unfamiliar with Pascal's Triangle, give an introduction.	Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this topic and review as needed. Check student responses.
4-3 Dividing Polynomials	Dividing polynomials	Use long division to divide polynomials by other polynomials. Use synthetic division to divide polynomials by binomials of the form $x - k$. Use the Remainder Theorem. Use a completed polynomial long division problem as an example. Circle the coefficients throughout the example. Explain to students that the numeric portion	Check student understanding via oral participation. Check student work and graphs. Check for correct use of graphing calculator.

		is the key part of the problem. Writing variables and exponents is extraneous. The like terms are aligned for a reason. • Explain and model the steps of synthetic division. Compare the results with the same problem done as long division so that students see the parallel work and the same numbers. It may take several examples for students to understand why synthetic division works.	Classwork assigned. Homework assigned.
4-4 Factoring Polynomials	Factoring Polynomials	Factor polynomials and use the **Factor Theorem**. Students may think that factors must be binomials. Show them whether there is a common monomial factor. A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$. Use technology to make a table of values to verify the positive zeros and heights between the zeros.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
4-5 Solving Polynomial Equations	Solving Polynomial Equations	Find solutions of polynomial equations and zeros of polynomial functions. Use the Rational Root Theorem and the Irrational Conjugates Theorem. Give an example of a cubic with only one solution. Use the word *multiplicity* to	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned.

		describe the repeated roots. Discuss the difference between a graph crossing at a root versus touching the axis at a root. Explain why synthetic division is more efficient for finding the roots than substituting the possible roots into the equation. Explain that from the Rational Root Theorem a list of possible roots can be written, but the actual roots still need to be found. Explain why the Irrational Conjugates Theorem should make sense.	Homework assigned.
4-6 The Fundamental Theorem of Algebra	The Fundamental Theorem of Algebra	Use the Fundamental Theorem of Algebra. Find conjugate pairs of complex zeros of polynomial functions. Use Descartes's Rule of Signs. Make sure students to understand, If *f is* a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of *f*, then $a - bi$ is also a zero of *f* and also remind them that if the polynomial has three solutions does not mean they are all real solutions.	Check student understanding via oral participation. Check student work and graphs. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
4-7 Transformations of Polynomial Functions	Transformations of Polynomial Functions	Describe transformations of polynomial functions. Write transformations of polynomial functions.	Check student understanding via oral participation. Check student work and graphs.

		Draw the table and fill in the second column, *f*(*x*) *Notation*. Have partners fill in the first column. Write the examples and have students describe the transformation.	Classwork assigned. Homework assigned.
4-8 Analyzing Graphs of Polynomial Functions	Analyzing Graphs of Polynomial Functions	Use *x*-intercepts to graph polynomial functions. Use the Location Principle to identify zeros of polynomial functions. Find turning points and identify local maximums and local minimums of graphs of polynomial functions. Identify even and odd functions. Make sure students to know, If f is a polynomial function, and a and b are two real numbers such that $f(a) < 0$ and $f(b) > 0$, then f has at least one real zero between a and b. Students should know, the graph of every polynomial function of degree n has *at most* $n - 1$ turning points and if a polynomial function of degree n has n distinct real zeros, then its graph has *exactly* $n - 1$ turning points. Even and odd functions.	Check student understanding via oral participation. Check student work and graphs. Classwork assigned. Homework assigned.
4-9 Modeling with Polynomial Functions	Modeling with Polynomial Functions	Write polynomial functions for sets of points.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator.

			Classwork assigned. Homework assigned.
5-1 *n*th Roots and Rational Exponents	Complex Numbers	Write polynomial functions using finite differences.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
5-2 Properties of Rational Exponents and Radicals	Completing the Square	Provide details in the process of completing the square (both when $a=1$ and when $a\neq1$). Model practice problems of both types. Model practice problems of converting a quadratic function in standard form to vertex form using this process. Include word	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
5-3 Graphing Radical Functions	*n*th Roots and Rational Exponents	Find the *n*th roots of numbers. Evaluate expressions with rational exponents. Solve equations using *n*th roots. Please be careful of common mistakes among students. For example Students sometimes think that to solve a problem involving taking the cube the root of an expression, you divide by 3.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
5-4 Solving Radical Equations and Inequalities	Solving Radical Equations and Inequalities	Solve equations containing radicals and rational exponents. Solve radical inequalities	Check student understanding via oral participation. Check student work.

			Check for correct use of graphing calculator. Classwork assigned. Homework assigned
5-5 Performing Function Operations	Performing Function Operations	Students should know the notation: $(f + g)(x)$, meaning they want to add f and g functions. The notation $f(x) + g(x)$ shows the actual functions being added together. Discuss the domain and range of the two functions' sum, difference, product, and quotient.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
5-6 Inverse of a Function	Inverse of a Function	Explore inverses of functions and verify inverses of nonlinear functions. Solve real-life problems using inverse functions.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned

Standards

MATH.9-12.N.RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MATH.9-12.A.APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MATH.9-12.F.BF.A.1.a	Determine an explicit expression, a recursive process, or steps for calculation from a context.

MATH.9-12.N.RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MATH.9-12.F.BF.A.1.b	Combine standard function types using arithmetic operations.
MATH.9-12.A.APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$.
MATH.9-12.A.APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MATH.9-12.F.BF.B.4.b	Verify by composition that one function is the inverse of another.
MATH.9-12.F.BF.B.4.c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MATH.9-12.F.BF.B.4.d	Produce an invertible function from a non-invertible function by restricting the domain.
MATH.9-12.A.REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MATH.9-12.A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MATH.9-12.F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.

Suggested Modifications for Special Education, ELL and Gifted Students

Special Education Students:

- Visual Aids and Manipulatives: Provide visual aids such as graphic organizers, anchor charts, color-coded notes, and manipulatives like algebra tiles or graphing boards to help students visualize concepts like factoring, polynomial division, and function transformations.
- Simplified Instructions and Chunking: Break down complex concepts into smaller, more manageable

steps with clear and concise instructions. Use chunking to present information in smaller portions, allowing for better comprehension and retention.

- Multi-Sensory Learning: Incorporate multi-sensory approaches, such as using manipulatives, drawing diagrams, or creating songs/mnemonics to help students grasp abstract concepts like the Fundamental Theorem of Algebra or rational exponents.
- Extended Time and Alternative Assessments: Offer additional time for completing assignments and assessments. Provide alternative assessment options, such as oral presentations, projects, or portfolios, to demonstrate understanding.
- Individualized Instruction and Scaffolding: Tailor instruction to meet individual needs, considering learning styles, strengths, and weaknesses. Provide scaffolding (hints, prompts, worked examples) to support students as they progress.

English Language Learners (ELLs):

- Visual Support and Real-World Connections: Utilize visuals, such as diagrams, graphs, charts, and real-world examples to enhance understanding of vocabulary and concepts. Connect mathematical concepts to familiar scenarios to make them more relatable.
- Simplified Language and Bilingual Resources: Use clear and concise language, avoiding jargon and idioms. Provide a glossary of key terms with definitions and translations in the student's native language. Offer bilingual dictionaries, textbooks, or online resources for additional support.
- Pre-Teaching Vocabulary: Introduce key vocabulary terms before each lesson, using visual aids and examples to reinforce understanding.
- Collaborative Learning and Peer Tutoring: Encourage peer-to-peer interaction and group work to provide opportunities for language practice and collaborative problem-solving. Pair ELL students with native English speakers for additional support.
- Differentiated Instruction: Tailor instruction to meet individual language proficiency levels, providing additional support, such as simplified texts or visual aids, for students who need it.

Gifted Students:

- Extension Activities and Enrichment: Offer opportunities for enrichment and extension beyond the standard curriculum, such as exploring advanced topics like complex polynomials, fractional exponents, or connections between polynomial and trigonometric functions.
- Inquiry-Based Learning and Problem-Solving Challenges: Encourage students to ask questions, explore concepts independently, and develop their own solutions. Challenge them with

complex, open-ended problems that require critical thinking, analysis, and creativity.

- Independent Research Projects: Provide opportunities for students to conduct independent research projects on topics of interest related to polynomials, radicals, or functions.
- Mentorship and Competitions: Pair gifted students with mentors who can provide guidance and support in their academic pursuits. Encourage participation in math competitions to challenge their skills and foster a love for mathematics.
- Acceleration: Consider accelerating gifted students to higher-level math courses if they demonstrate readiness and a strong interest in the subject.

Suggested Technological Innovations/Use

Graphing Calculators/Software (Desmos, GeoGebra):

- Visualizing Functions: Students can easily graph polynomial and radical functions, adjust parameters to observe transformations, and find key features like zeros, intercepts, and turning points.
- Solving Equations and Inequalities: Graphing calculators can be used to solve polynomial equations and inequalities graphically, reinforcing the connection between algebraic and visual representations.
- Exploring Transformations: Students can manipulate functions in real-time to observe the effects of translations, reflections, stretches, and compressions.
- Analyzing Real-World Data: Graphing calculators can be used to analyze data sets and model them with polynomial or radical functions.

Cross Curricular/Career Readiness, Life Literacies and Key Skills Practice

Cross-Curricular Connections:

- Physics: Polynomial and radical functions can be used to model projectile motion, acceleration, and other physical phenomena. Students can analyze graphs to interpret the motion of objects and solve

real-world physics problems.

- Chemistry: Polynomial functions can be used to model chemical reactions and rates of reaction. Students can analyze graphs to understand how different variables affect the outcome of a reaction.
- Economics: Polynomial functions can be used to model economic trends, such as supply and demand curves or cost functions. Students can use these models to make predictions and analyze economic behavior.
- Engineering: Polynomial and radical functions are used in various engineering fields, such as designing bridges, analyzing structures, and modeling fluid flow. Students can explore how these functions are applied to solve engineering problems.
- Finance: Polynomial and radical functions can be used to model investment growth, compound interest, and other financial concepts. Students can learn how to use these functions to make informed financial decisions.

Career Readiness:

- Data Analysis and Modeling: Students will develop skills in analyzing data and creating mathematical models using polynomial and radical functions. These skills are valuable in various careers, such as data science, economics, finance, and engineering.
- Problem-Solving: Students will practice solving complex problems involving polynomials, radicals, and function operations, developing critical thinking and analytical skills essential in many professions.
- Communication and Collaboration: Through group projects and presentations, students will enhance their communication and collaboration skills, which are crucial in the workplace.
- Technology Literacy: Students will use graphing calculators, software, and online resources to explore and analyze functions, developing digital literacy skills that are increasingly important in today's workforce.

Unit 3 Exponential, Logarithmic and Rational Functions, Sequences and Series

Content Area: **Mathematics**
Course(s):
Time Period:
Length: **40-45 days**
Status: **Published**

Summary of the Unit

This comprehensive Algebra 2 unit explores the fascinating world of exponential and logarithmic functions, aligning with the New Jersey Student Learning Standards (NJSLS). Students will delve into the concepts of exponential growth and decay, discovering how these functions model real-world phenomena like population growth, radioactive decay, and compound interest. The unit also introduces the natural base e and explores the properties of logarithms, enabling students to solve exponential and logarithmic equations. Through graphing and transformations, students gain a deeper understanding of the behavior of these functions. The unit extends to rational functions, where students explore inverse variation, graphing techniques, and operations like multiplication, division, addition, and subtraction. Solving rational equations and inequalities further strengthens their problem-solving skills. Finally, the unit introduces sequences and series, focusing on arithmetic and geometric sequences, their applications, and the use of recursive rules. This unit equips students with essential algebraic tools for modeling, analyzing, and solving problems involving exponential, logarithmic, and rational functions, fostering a deeper understanding of mathematical relationships and their real-world applications.

Enduring Understandings

- Exponential and Logarithmic Relationships: Students will understand that exponential and logarithmic functions are inverse operations, representing two sides of the same relationship between quantities. They will recognize how these functions model real-world phenomena involving growth, decay, and change.

- Properties of Exponents and Logarithms: Students will grasp the fundamental properties of exponents

and logarithms, using them to simplify expressions, solve equations, and manipulate functions.

- Transformations of Functions: Students will understand how transformations (translations, reflections, stretches, and compressions) affect the graphs and equations of exponential and logarithmic functions, enabling them to analyze and interpret them in different contexts.

- Rational Functions and Asymptotes: Students will recognize the unique characteristics of rational functions, including their asymptotes and how they behave as the input approaches infinity or negative infinity.

- Inverse Variation and Modeling: Students will understand the concept of inverse variation and how it can be modeled using rational functions. They will apply this knowledge to real-world scenarios, such as the relationship between distance, speed, and time.

- Sequences and Series as Patterns: Students will recognize that sequences and series represent patterns in numbers and can be used to model various phenomena. They will learn to identify, analyze, and generate arithmetic and geometric sequences and series.

Essential Questions

Exponential and Logarithmic Functions:

- How do exponential functions model real-world situations involving growth and decay?
- What is the natural base "e," and how does it relate to exponential growth and decay?
- How are logarithms related to exponents, and how can we use them to solve exponential equations?
- How do transformations affect the graphs and equations of exponential and logarithmic functions?
- What are the key properties of logarithms, and how can we use them to simplify expressions and solve equations?

Rational Functions:

7. What are the characteristics of rational functions, including domain, range, asymptotes, and end behavior?
8. How do we graph rational functions, and what information can we glean from their graphs?
9. How do we perform operations with rational expressions, including multiplication, division, addition, and subtraction?
10. How do we solve rational equations and inequalities, and why is it important to check for extraneous solutions?
11. What is inverse variation, and how can we model it using rational functions?

Sequences and Series:

- What are arithmetic and geometric sequences and series, and how do they differ?
- How can we use formulas and recursive rules to generate terms in sequences and find sums of series?
- How can we apply sequences and series to model real-world situations, such as population growth or investment returns?

These essential questions are designed to stimulate critical thinking, inquiry, and deeper understanding of the concepts covered in this unit. They encourage students to explore the connections between different mathematical ideas and to apply their knowledge to solve problems and model real-world situations.

Summative Assessment and/or Summative Criteria

- Unit Test: A comprehensive test covering all major topics in the unit, including:

- Graphing and analyzing exponential and logarithmic functions
- Properties of exponents and logarithms
- Solving exponential and logarithmic equations
- Modeling real-world scenarios with exponential and logarithmic functions
- Graphing and analyzing rational functions
- Operations with rational expressions (multiplication, division, addition, subtraction)

- Solving rational equations and inequalities
- Identifying and analyzing arithmetic and geometric sequences and series
- Using recursive rules with sequences

- Project/Presentation: A project or presentation where students apply their knowledge of exponential, logarithmic, or rational functions to a real-world scenario. Students could choose from options like:

- Modeling population growth or decay
- Analyzing compound interest or investment growth
- Investigating radioactive decay
- Examining real-world data using rational functions
- Creating a presentation comparing and contrasting different types of sequences and series

Resources

Internet4Classrooms www.internet4classrooms.com

Desmos https://www.desmos.com/

Math Open Reference www.mathopenref.com

National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/index.html

Georgia Department of Education https://www.georgiastandards.org/Georgia-Standards/Pages/Math-9-12.aspx

Illustrative Mathematics www.illustrativemathematics.org/

Khan Academy https://www.khanacademy.org/math/algebra-home/algebra2

Math Planet http://www.mathplanet.com/education/algebra-2

IXL Learning https://www.ixl.com/math/algebra-2

Math Is Fun Advanced http://www.mathsisfun.com/algebra/index-2.html

Partnership for Assessment of Readiness for College and Careers https://parcc.pearson.com/practice-tests/math/

Mathematics Assessment Project http://map.mathshell.org/materials/lessons.php?gradeid=24

Achieve the Core http://www.achieve.org/ccss-cte-classroom-tasks

NYLearns http://www.nylearns.org/module/Standards/Tools/Browse?linkStandardId=0&standardId=97817

Learning Progression Framework K-12 http://www.nciea.org/publications/Math_LPF_KH11.pdf

PARCC Mathematics Evidence Tables. https://parcc-assessment.org/mathematics/

Unit Plan

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmark / Assessments
6-1 Exponential Growth and Decay Functions	Exponential Growth and Decay Functions	Graph exponential growth functions and their transformations, and identify their growth factors, domains, ranges, intercepts and asymptotes. Use the exponential growth formula and compound interest formula to solve real-world problems Students will graph a simple exponential	Check student graphs. Check for correct use of the graphing calculator. Check translations of word problems into algebra. Classwork assigned.

		growth function, such as $y = 2^x$ by hand or by calculator, and as a class will explore domain, range, asymptotes and intercepts. Formalize notes on the graph of $f(x) = ab^x$ to include definition of a growth factor, and a restriction that $b>1$. Graph exponential decay functions and their transformations, and identify their growth factors, domains, ranges, intercepts and asymptotes.	Homework assigned Assess student recall of this topic and review as needed.
6-2 The Natural Base *e*	The Natural Base *e*	Define and use the natural base *e*. Graph natural base functions. Solve real-life problems. Understand where the natural base *e* comes from. Simplify natural base expressions, graph the natural base function and translations of it. Use the formula for interest compounded continuously. Write the natural base exponential function and explore the affect *r* has on the graph of y=aerx	Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this topic and review as needed. Check student responses.
6-3 Logarithms and Logarithmic Functions	Logarithms and Logarithmic Functions	Rewrite logarithmic and exponential forms and evaluate logarithms. Use inverse properties of logarithms to find inverse functions. Explore what a logarithm is by posing the questions: "10^0 =?, 10^1 =?, 10^2 =?, so then what is *x* in $10^x = 50$?" Use the graphing calculator to make guesses and to discover that a logarithm is an exponent.	Check student understanding via oral participation. Check student work and graphs. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.

		Graph an exponential function such as $y = 2^x$. Have students recall all they can about inverse functions and use that knowledge to graph the inverse of that exponential function (the result will be a logarithmic function). Discuss domain, range, asymptotes and intercept. Graph translations of logarithmic functions. Explain how to convert from exponential form to logarithmic form and vice versa, and use this knowledge to evaluate logarithms.	
6-4 Transformations of Exponential and Logarithmic Functions	Transformations of Exponential and Logarithmic Functions	Transform graphs of exponential functions. Transform graphs of logarithmic functions. Write transformations of graphs of exponential and logarithmic functions. Discuss the *Core Concept* with students. Again, the issue for students is to be able to distinguish whether the transformation affects the graph horizontally or vertically. Students also need to be aware of what the parent logarithmic function looks like, as well as its domain and its range.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
6-5 Properties of Logarithms	Properties of Logarithms	Use the properties of logarithms to evaluate logarithms. Use the properties of logarithms to expand or condense logarithmic expressions. Use the change-of-base formula to evaluate logarithms. Have students estimate an answer for log6	Check student understanding via oral participation. Check student work. Check for the correct use of graphing calculator. Classwork assigned.

		24 before they change the base.	Homework assigned.
6-6 Solving Exponential and Logarithmic Equations	Solving Exponential and Logarithmic Equations	Solve exponential and logarithmic equations. Solve exponential and logarithmic inequalities. Explain what it means to exponentiate each side of an equation and why a base of 2 was chosen.	Check student understanding via oral participation. Check student work and graphs. Check for the correct use of graphing calculator. Classwork assigned. Homework assigned
6-7 Modeling with Exponential and Logarithmic Functions**.**	Modeling with Exponential and Logarithmic Functions**.**	How can you recognize polynomial, exponential, and logarithmic models? Classify data sets. Write exponential functions. Use technology to find exponential and logarithmic models. The first step with any data set is to make a scatter plot to determine what the graph of the data looks like. Is there some theory or past experience that we have with the data that tells us what type of function would make the most sense?	Check student understanding via oral participation. Check student work and graphs. Classwork assigned. Homework assigned.
7-1 Inverse Variation	Inverse Variation	How can you recognize when two quantities vary directly or inversely? how can you distinguish between direct and inverse variation? Inverse variation is a nice bridge to graphs of rational functions, which is the next	Check student understanding via oral participation. Check student work and graphs. Classwork assigned. Homework assigned.

		lesson. Remind students that before they can add or subtract fractions, they must first find a common denominator. Students can confuse inverse variation with exponential decay. Contexts for inverse variation are generally restricted to the first quadrant and do not have a *y*-intercept. It is the decreasing function and curve of the graph that mislead students.	
7-2 Graphing Rational Functions	Graphing Rational Functions	Graph simple rational functions. Translate simple rational functions. Graph other rational functions. Students should know, what information is known, and how should they calculate an average cost? Students should learn how to find vertical and horizontal asymptote.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
7-3 Multiplying and Dividing Rational Expressions	Multiplying and Dividing Rational Expressions	Simplify rational expressions. Multiply and divide rational expressions. In this lesson, students will add and subtract rational expressions, some with like denominators and others without like denominators. Because the rational expressions themselves could be a single term or a polynomial, working slowly through the various cases is necessary.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.

		Remind students that this is *not* the Cross Products Property! They are multiplying rational expressions. Students can confuse multiplying rational expressions that are monomials and those that are binomials. Keep reminding students that when they simplify first before performing the multiplication, they must be dividing out common factors.	
7-4 Adding and Subtracting Rational Expressions	Adding and Subtracting Rational Expressions	Add or subtract rational expressions. Rewrite rational expressions and graph the related function. Simplify complex fractions. Go over LCM one more time.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
7-5 Solving Rational Equations	Solving Rational Equations	Solve rational equations by cross multiplying. Solve rational equations by using the least common denominator. Use inverses of functions. Have students graph the function and inverse to see symmetry about $y = x$.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
7-6 Defining and Using Sequences and Series	Defining and Using Sequences and Series	Use sequence notation to write terms of sequences. Write a rule for the nth term of a sequence. Sum the terms of a sequence to obtain a series and use summation notation.	Check student understanding via oral participation. Check student work. Check for correct use

		A sequence is an ordered list of numbers. A finite sequence is a function that has a limited number of terms and whose domain is the finite set {1, 2, 3, . . . , n}. The values in the range are called the terms of the sequence. When the terms of a sequence are added together, the resulting expression is a series. A series can be finite or infinite. If time permits, derive the three formulas. This may indeed help students remember the formulas.	of graphing calculator. Classwork assigned. Homework assigned
7-7Analyzing Arithmetic Sequences and Series	Analyzing Arithmetic Sequences and Series	Identify arithmetic sequences. Write rules for arithmetic sequences. Find sums of finite arithmetic series. How can you recognize an arithmetic sequence from its graph? In an arithmetic sequence, the difference of consecutive terms is constant. This constant difference is called the common difference and is denoted by d.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
7-8 Analyzing Geometric Sequences and Series	Multiplying and Dividing Rational Expressions	Identify geometric sequences. Write rules for geometric sequences. Find sums of finite geometric series.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned

Standards

MATH.9-12.F.BF.A	Build a function that models a relationship between two quantities
MATH.9-12.F.BF.A.1.a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MATH.9-12.F.BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MATH.9-12.A.APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x)$ + $r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MATH.9-12.A.CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
MATH.9-12.F.IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MATH.9-12.A.REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MATH.9-12.A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
MATH.9-12.F.IF.C.8.b	Use the properties of exponents to interpret expressions for exponential functions.
	For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
MATH.9-12.A.SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MATH.9-12.F.LE.A	Construct and compare linear and exponential models and solve problems
MATH.9-12.F.LE.A.1	Distinguish between situations that can be modeled with linear functions and with

	exponential functions.
MATH.9-12.F.LE.A.1.a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.
MATH.9-12.F.LE.A.1.b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MATH.9-12.F.LE.A.1.c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MATH.9-12.F.LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MATH.9-12.F.LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
MATH.9-12.F.LE.B	Interpret expressions for functions in terms of the situation they model
MATH.9-12.F.LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Suggested Modifications for Special Education, ELL and Gifted Students

Students will be allowed to submit assignments using additional time per IEP modifications. Students will be encouraged to use different sizes and types of font to avoid print confusion.
ML students will be allowed to use an internet translator or language glossary to translate vocabulary and assignments correctly.

ML students may be allowed to work with another student who is fluent in their native language.

Suggested Technological Innovations/Use

- Graphing Calculators/Software (Desmos, GeoGebra):

- Visualizing Functions: Students can easily graph exponential, logarithmic, and rational functions, adjust parameters to observe transformations, and find key features like asymptotes, intercepts, and points of intersection.
- Solving Equations and Inequalities: Graphing calculators can be used to solve exponential and logarithmic equations graphically, reinforcing the connection between algebraic and visual representations.
- Exploring Transformations: Students can manipulate functions in real-time to observe the effects of translations, reflections, stretches, and compressions.
- Analyzing Real-World Data: Graphing calculators can be used to analyze data sets and model them with exponential, logarithmic, or rational functions to make predictions and draw conclusions.

- Online Simulations and Interactive Activities:

- Virtual Manipulatives: Students can use online tools to manipulate exponential, logarithmic, and rational functions, building their understanding of properties and transformations through interactive exploration.
- Gamification: Incorporate online games and quizzes to make learning about these functions more engaging and motivating for students.
- Real-World Applications: Explore online simulations and interactive activities that showcase how exponential, logarithmic, and rational functions are used in real-world scenarios, such as population growth, radioactive decay, compound interest, and pH calculations.

Online Collaboration Tools:

- Virtual Whiteboards and Shared Documents: Tools like Google Jamboard or Microsoft Whiteboard can be used for collaborative problem-solving, allowing students to work together on problems involving exponential, logarithmic, and rational functions in real time.
- Discussion Forums and Chat Platforms: Create online spaces for students to discuss concepts, ask questions, and share ideas.Online Assessment Tools:

- Formative Assessment: Online tools like Kahoot! or Quizlet can be used for quick formative assessments to gauge student understanding of concepts.
- Summative Assessment: Online platforms can create and administer quizzes or tests with automatic

grading and feedback.

Additional Technologies:

- Spreadsheets: Students can use spreadsheets to create tables of values for functions, perform calculations, and generate graphs.
- Coding: Students can use coding platforms like Python to create programs that generate graphs, solve equations, and model real-world scenarios.

Cross Curricular/Career Readiness, Life Literacies and Key Skills Practice

Social Studies Connection:

Name of Task: Carbon 14 Dating (6.2.12.GeoPP.1.a)

The task requires the student to use logarithms to solve an exponential equation in the realistic context of carbon dating, important in archaeology and geology, among other places

Science Connection:

Name of Task: Course of Antibiotics (HS-PS-1-6)

This task presents a real-world application of finite geometric series. The context can lead to several interesting follow-up questions and projects. Many drugs only become effective after the amount in the body builds up to a certain level, which can be modeled very well with geometric series.

Unit 4: Trigonometric Ratios and Functions, Probability, Data Analysis and Statistics

Content Area: **Mathematics**
Course(s):
Time Period:
Length: **40-45 days**
Status: **Published**

Summary of the Unit

This unit delves into the world of trigonometric functions and their applications in modeling real-world phenomena, while also exploring the fundamentals of data analysis. Students will begin by graphing sine and cosine functions, understanding their periodic nature and key characteristics like amplitude, period, phase shift, and midline. Building on this foundation, they will explore other trigonometric functions (tangent, cotangent, secant, cosecant) and their graphs.

The unit emphasizes the use of trigonometric functions to model periodic phenomena, such as sound waves, tides, and Ferris wheel motion. Students will learn to analyze data and create trigonometric models that accurately represent real-world scenarios. In addition, the unit introduces essential data analysis skills, focusing on categorical data. Students will learn to create and interpret various graphical representations, including dot plots, histograms, and box plots. They will also analyze two-way frequency tables, calculate relative frequencies, and identify potential associations and trends in data. By the end of the unit, students will have a solid understanding of trigonometric functions and their applications, along with fundamental data analysis skills, preparing them for further studies in mathematics and related fields.

Enduring Understandings

- Trigonometric Functions as Models of Cyclical Phenomena: Students will understand that trigonometric functions (sine, cosine, tangent, etc.) model periodic behavior found in various real-

world phenomena, such as sound waves, tides, and planetary motion.

- Unit Circle and Radian Measure: Students will grasp the significance of the unit circle as a tool for defining trigonometric functions and understanding their relationships. They will also recognize the importance of radian measure in working with these functions.

- Transformations of Trigonometric Functions: Students will understand how transformations (amplitude, period, phase shift, vertical shift) affect the graphs and equations of trigonometric functions, enabling them to interpret and manipulate these functions in different contexts.

- Trigonometric Identities: Students will recognize fundamental trigonometric identities and use them to simplify expressions, solve equations, and prove relationships.

- Data Analysis and Representation: Students will understand how to collect, organize, and represent categorical data using various graphical displays (dot plots, histograms, box plots).

- Relative Frequencies and Association: Students will be able to calculate and interpret relative frequencies (joint, marginal, and conditional) in two-way frequency tables to identify potential associations and trends in categorical data.

Essential Questions

Trigonometric Functions:

- How do trigonometric functions (sine, cosine, tangent, etc.) model periodic phenomena in the real world?
- How can we use the unit circle to understand the relationships between trigonometric functions and angles?
- What are the key characteristics of trigonometric functions (amplitude, period, phase shift, vertical shift) and how do they affect the graphs and equations of these functions?

- How can we use trigonometric identities to simplify expressions, solve equations, and prove relationships?
- How can we apply our knowledge of trigonometric functions to solve real-world problems involving angles, distances, and periodic phenomena?

Data Analysis:

1. How can we collect, organize, and display categorical data to gain insights and draw conclusions?
2. What are dot plots, histograms, and box plots, and how can we use them to represent and analyze categorical data?
3. How can we calculate and interpret relative frequencies (joint, marginal, and conditional) in two-way frequency tables?
4. What are some common types of associations and trends in categorical data, and how can we identify them?
5. How can we use data analysis to make informed decisions and draw valid conclusions about real-world situations?

Summative Assessment and/or Summative Criteria

Unit Test: A comprehensive test covering all major topics in the unit, including:

- Graphing and analyzing sine, cosine, and other trigonometric functions.
- Understanding the unit circle and radian measure.
- Applying transformations to trigonometric functions.
- Utilizing trigonometric identities to simplify expressions and solve equations.
- Modeling real-world scenarios with trigonometric functions.
- Representing categorical data using dot plots, histograms, and box plots.
- Analyzing two-way frequency tables and calculating relative frequencies.
- Identifying associations and trends in categorical data.
- Project/Presentation: A project or presentation where students apply their knowledge of trigonometric functions and data analysis to a real-world scenario. Students could choose from options like:

- Modeling a periodic phenomenon (e.g., tides, sound waves, Ferris wheel motion) using trigonometric functions.
- Analyzing a data set involving categorical variables and presenting findings using appropriate graphs and statistical measures.
- Creating a presentation comparing and contrasting different trigonometric functions and their applications.

Resources

Internet4Classrooms www.internet4classrooms.com

Desmos https://www.desmos.com/

Math Open Reference www.mathopenref.com

National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/index.html

Georgia Department of Education https://www.georgiastandards.org/Georgia-Standards/Pages/Math-9-12.aspx

Illustrative Mathematics www.illustrativemathematics.org/

Khan Academy https://www.khanacademy.org/math/algebra-home/algebra2

Math Planet http://www.mathplanet.com/education/algebra-2

IXL Learning https://www.ixl.com/math/algebra-2

Math Is Fun Advanced http://www.mathsisfun.com/algebra/index-2.html

Partnership for Assessment of Readiness for College and Careers https://parcc.pearson.com/practice-tests/math/

Mathematics Assessment Project http://map.mathshell.org/materials/lessons.php?gradeid=24

Achieve the Core http://www.achieve.org/ccss-cte-classroom-tasks

NYLearns http://www.nylearns.org/module/Standards/Tools/Browse?linkStandardId=0&standardId=97817

Learning Progression Framework K-12 http://www.nciea.org/publications/Math_LPF_KH11.pdf

PARCC Mathematics Evidence Tables. https://parcc-assessment.org/mathematics/

Unit Plan

Topic/Selection Timeframe	General Objectives	Instructional Activities	Benchmark / Assessments
Right Triangle Trigonometry	Right Triangle Trigonometry	Evaluate trigonometric functions of acute angles. Find unknown side lengths and angle measures of right triangles. Use trigonometric functions to solve real-life problems. Share with students that 30°, 45°, and 60° are benchmark angles that they really need to be familiar with. Angles with measures 0° and 90° will be added soon. Ask students for examples that involve the location of a point on a circle. They may think of examples from everyday objects, such as the motion of a point on the edge of	Check student graphs. Check for correct use of the graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this topic and review as needed.

		a compact disk, or from sports, geography, or astronomy. • Tell students that some of the formulas used to describe circular motion involve angles	
Angles and Radian Measure	Angles and Radian Measure	Draw angles in standard position. Find coterminal angles. Use radian measure. In a coordinate plane, an angle can be formed by fixing one ray, called the **initial side**, and rotating the other ray, called the **terminal side**, about the vertex. An angle is in **standard position** when its vertex is at the origin and its initial side lies on the positive *x*-axis. Write the *Core Concept*. Students should note that the direction of the rotation matters in terms of the definition of an angle in standard position. Perhaps they have recognized this when using dynamic geometry software and angles are named.	Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this topic and review as needed. Check student responses.
Trigonometric Functions of Any Angle	Trigonometric Functions of Any Angle	Evaluate trigonometric functions of any angle. Find and use reference angles to evaluate trigonometric functions. The circle $x2 + y2 = 1$, which has center (0, 0)	Check student understanding via oral participation. Check student work and graphs. Check for correct use of graphing calculator.

		and radius 1, is called the **unit circle**. The values of sin θ and cos θ are simply the *y*-coordinate and *x*-coordinate, respectively, of the point where the terminal side of θ intersects the unit circle. Write the *Core Concept* and explain that sometimes it is convenient to label the ordered pair on the unit circle as (cos θ, sin θ) instead of (*x*, *y*).	Classwork assigned. Homework assigned.
Graphing Sine and Cosine Functions	Graphing Sine and Cosine Functions	Explore characteristics of sine and cosine functions. Stretch and shrink graphs of sine and cosine functions. Translate graphs of sine and cosine functions. Reflect graphs of sine and cosine functions. To graph a sine or cosine function, locate five key points—the endpoints of one cycle, the midpoint of the cycle, and the maximum and minimum in the cycle. Give students time to work independently with their partners. Trust that they can reason about the transformations to the cosine function.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
Graphing Other Trigonometric Functions	Graphing Other Trigonometric Functions	Explore characteristics of tangent and cotangent functions. Graph tangent and cotangent functions. Graph secant and cosecant functions.	Check student understanding via oral participation. Check student work. Check for the correct use of graphing calculator. Classwork assigned. Homework assigned.
Modeling with Trigonometric	Modeling with Trigonometric Functions	Interpret and use frequency. Write trigonometric functions. Use technology to	Check student understanding via oral

Functions		find trigonometric models. Write the general equation of a sinusoid (sine and cosine) and review the parameters *a*, *b*, *h*, and *k*. From a graph or context, students will determine these parameters, knowing some may be 0 (*h* and *k*) or 1 (*a* and *b*).	participation. Check student work and graphs. Check for the correct use of graphing calculator. Classwork assigned. Homework assigned
Using Trigonometric Identities	Use trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions. Verify trigonometric identities.	How can you verify a trigonometric identity? Is $\sin\theta = \cos\theta$ a trigonometric identity? Explain your reasoning.	Check student understanding via oral participation. Check student work and graphs. Classwork assigned. Homework assigned.
Using Sum and Difference Formulas	Using Sum and Difference Formulas	Use sum and difference formulas to evaluate and simplify trigonometric expressions. Use sum and difference formulas to solve trigonometric equations and rewrite real-life formulas. Instead of properties of equality, you use trigonometric formulas and properties to rewrite expressions. Once simplified, the answer is found by inspection. How can you evaluate trigonometric functions of the sum or difference of two angles?	Check student understanding via oral participation. Check student work and graphs. Classwork assigned. Homework assigned.
Sample Spaces and Probability	Sample Spaces and Probability	Find sample spaces. Find theoretical probabilities. Find experimental probabilities.	Check student understanding via oral participation.

		The **probability of an event** is a measure of the likelihood that the event will occur. Probability is a number from 0 to 1, including 0 and 1. The diagram relates *likelihoods* (described in words) and probabilities. Probability of the Complement of an Event The probability of the complement of event A is $P(\overline{A}) = 1 - P(A)$. The graphing calculator can be used to simulate, and hence estimate, experimental probabilities. This strategy uses the random number generator to simulate events such as spinning a spinner.	Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
Independent and Dependent Events	Independent and Dependent Events	Determine whether events are independent events. Find probabilities of independent and dependent events. Find conditional probabilities If two events A and B are dependent events, then the probability that both events occur is the product of the probability of the first event and the conditional probability of the second event given the first event. Symbols $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ Write the *Core Concept*. Be careful with language and symbols, as students start to confuse both about now.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.

Two-Way Tables and Probability	Two-Way Tables and Probability	Make two-way tables. Find relative and conditional relative frequencies. Use conditional relative frequencies to find conditional probabilities. Relative and Conditional Relative Frequencies A **joint relative frequency** is the ratio of a frequency that is not in the total row or the total column to the total number of values or observations. A **marginal relative frequency** is the sum of the joint relative frequencies in a row or a column. A **conditional relative frequency** is the ratio of a joint relative frequency to the marginal relative frequency. You can find a conditional relative frequency using a row total or a column total of a two-way table.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.
Probability of Disjoint and Overlapping Events	Probability of Disjoint and Overlapping Events	Find probabilities of compound events. Use more than one probability rule to solve real-life problems. Students should read the introduction, which describes new vocabulary about events: disjoint, mutually exclusive, and overlapping. Explain to students that the two Venn diagrams represent the types of events they will study in this lesson.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned.

Permutations and Combinations	Permutations and Combinations	Use the formula for the number of permutations. Use the formula for the number of combinations. Use combinations and the Binomial Theorem to expand binomials. Introduce *n* factorial and the notation. Say when you have four objects taken four at a time and order matters, this is a permutation and can be found by multiplying $4 \cdot 3 \cdot 2 \cdot 1 = 4!$	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
Binomial Distributions	Binomial Distributions	Construct and interpret probability distributions. Construct and interpret binomial distributions. Use a context, such as the outcomes of a six-sided die, to discuss random variables. A probability distribution is a function. What are the possible sums when two six-sided dice are rolled?	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
Using Normal Distributions	Using Normal Distributions	Calculate probabilities using normal distributions. Use *z*-scores and the standard normal table to find probabilities. Recognize data sets that are normal. A normal distribution is one type of probability distribution. It is symmetric about the mean. • Write the *Core Concept*. Spend time interpreting each bullet and how to interpret the area under the curve.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned

		Describe how to use a standard normal table when you know the mean, the standard deviation, and an *x*-value.	
Populations, Samples, and Hypotheses	Populations, Samples, and Hypotheses	Distinguish between populations and samples. Analyze hypotheses. Define *parameter* and *statistic* and give examples of each. For instance, the population of heights of high school boys in New York City is too large to measure, so the sample mean (statistic) is used to estimate the population mean (parameter). Common Misconception: A population can be a subset of a larger population. For instance, the heights of junior boys in New York City are a subset of heights of junior boys in the United States. A sample would be 500 heights of junior boys in New York City.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
Collecting Data	Collecting Data	Identify types of sampling methods in statistical studies. Recognize bias in sampling. Analyze methods of collecting data. For each bulleted description, think of a question that fits the description, meaning it is a biased question." Having to provide examples of each will help clarify the nature of the bias.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned

		Describe the difference between a stratified sample and a cluster sample.	
Experimental Design	Experimental Design	Describe experiments. Recognize how randomization applies to experiments and observational studies. Analyze experimental designs. A rigorous randomized comparative experiment, by eliminating sources of variation other than the controlled variable, can make valid cause-and-effect conclusions possible. • An observational study can identify *correlation* between variables, but not *causality*. Variables, other than what is being measured, may be affecting the results. The discussion of experiments and observational studies will hopefully give students a sense of the many issues to consider when conducting research and gathering data. Having this background should improve students' critical thinking as they read about a study. • Write the *Core Concept*, focusing on cause and effect. Understanding that correlation is not the same as causality is important.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
Making Inferences from Sample	Making Inferences from Sample Surveys	Estimate population parameters. Analyze estimated population parameters. Find	Check student understanding via oral

Surveys		margins of error for surveys. Explain that an estimated population parameter is a hypothesis Circulate and listen to observations from each group. Wait a sufficient time before gathering students together for a whole-class discussion.	participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned
Making Inferences from Experiments	Making Inferences from Experiments	Organize data from an experiment with two samples. Resample data using a simulation to analyze a hypothesis. Make inferences about a treatment. Explain to students missing from class today why and how resampling is done for an experiment with two samples.	Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned

Standards

MATH.9-12.S.ID.A.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).
MATH.9-12.F.BF.B	Build new functions from existing functions
MATH.9-12.S.ID.B.6.a	Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
MATH.9-12.S.ID.B.6.b	Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.

MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.

Suggested Modifications for Special Education, ELL and Gifted Students

Special Education Students:

- Visual Aids and Manipulatives: Provide visual aids like unit circles, angle diagrams, and graphs to illustrate concepts. Use manipulatives like protractors and compasses to help students visualize and measure angles.
- Simplified Instructions and Chunking: Break down complex concepts like trigonometric identities or radian measure into smaller, more manageable steps. Present information in chunks, focusing on one concept at a time before moving on.
- Hands-on Activities: Incorporate hands-on activities like measuring angles in real-world objects or creating models of periodic phenomena to reinforce understanding.
- Alternative Assessments: Offer alternative assessment options, such as oral presentations, projects, or portfolios, to demonstrate understanding beyond traditional tests.
- Individualized Instruction and Scaffolding: Provide individualized support based on each student's needs. Scaffold learning by providing hints, prompts, and partially worked examples to guide students through challenging problems.

English Language Learners (ELLs):

- Visual Support and Real-World Connections: Use visuals like diagrams, graphs, and real-world examples (e.g., Ferris wheel, sound waves) to illustrate concepts and vocabulary. Connect mathematical concepts to familiar scenarios to make them more relatable.
- Simplified Language and Bilingual Resources: Use clear and concise language, avoiding jargon and idioms. Provide a glossary of key terms with definitions and translations in the student's native language. Offer bilingual dictionaries or online resources.
- Pre-Teaching Vocabulary: Introduce key vocabulary terms before each lesson, using visual aids and examples to reinforce understanding.

- Collaborative Learning and Peer Tutoring: Encourage peer-to-peer interaction and group work to provide opportunities for language practice and collaborative problem-solving.
- Scaffolding and Differentiation: Provide additional support, such as sentence frames for explanations or graphic organizers for organizing information. Differentiate instruction to meet individual language proficiency levels.

Gifted Students:

- Extension Activities and Enrichment: Offer opportunities for enrichment, such as exploring advanced topics like polar coordinates, complex numbers in trigonometry, or Fourier series.
- Inquiry-Based Learning and Problem-Solving Challenges: Encourage students to ask questions, explore concepts independently, and derive trigonometric identities. Challenge them with complex, open-ended problems that require critical thinking and creativity.
- Independent Research Projects: Allow students to conduct independent research on topics related to trigonometry and data analysis, such as the history of trigonometry, applications in different fields, or advanced statistical techniques.
- Mentorship and Competitions: Pair gifted students with mentors who can provide guidance and support in their academic pursuits. Encourage participation in math competitions to challenge their skills.
- Acceleration: Consider accelerating gifted students to higher-level math courses or allowing them to pursue independent study in related topics.

By implementing these modifications, teachers can create a more inclusive and engaging learning environment where all students, regardless of their diverse needs and abilities, can thrive and reach their full potential in this Algebra 2 unit.

Suggested Technological Innovations/Use

- Graphing Calculators/Software (Desmos, GeoGebra):
    - Visualizing Functions: Students can easily graph sine, cosine, and other trigonometric functions, adjust parameters to observe transformations (amplitude, period, phase shift), and visualize the relationship between the unit circle and the graphs.

- Solving Equations: Graphically solve trigonometric equations to find solutions and understand their periodic nature.
- Modeling with Functions: Explore real-world scenarios by manipulating trigonometric functions to fit data and make predictions.
- Interactive Activities: Engage in interactive activities that demonstrate the applications of trigonometric functions in fields like music, architecture, and engineering.

- Online Simulations and Applet:

  - Unit Circle Exploration: Interactive unit circle applets allow students to visualize angles in standard position, their corresponding coordinates, and the values of trigonometric functions.
  - Transformations of Functions: Dynamically manipulate trigonometric functions to see the effects of changes in amplitude, period, phase shift, and vertical shift on the graph.
  - Sound Wave Visualization: Simulate sound waves using trigonometric functions to understand their connection to frequency, amplitude, and pitch.

- Data Analysis Tools (Excel, Google Sheets, CODAP):

  - Data Collection and Organization: Collect and organize categorical data using spreadsheets.
  - Creating Graphs: Generate dot plots, histograms, and box plots to visualize the distribution of categorical data.
  - Calculating Statistics: Calculate measures of central tendency (mean, median, mode) and dispersion (range, interquartile range) for categorical data.
  - Two-Way Frequency Tables: Construct and analyze two-way frequency tables to investigate relationships between categorical variables.
  - Simulations: Conduct simulations to explore probability and sampling distributions in the context of data analysis.

- Online Collaboration Tools:

  - Shared Documents and Whiteboards: Collaborate on projects, share data analysis findings, and brainstorm ideas using online platforms like Google Docs, Sheets, or Jamboard.
  - Discussion Forums: Engage in online discussions about trigonometric concepts and data analysis techniques, fostering peer-to-peer learning and interaction.

- Other Technology Integrations:
    - Videos and Animations: Utilize educational videos and animations to reinforce concepts like the unit circle, graphing trigonometric functions, and data analysis techniques.
    - Online Quizzes and Games: Use online platforms like Kahoot! or Quizlet to create interactive quizzes and games that assess understanding of trigonometric functions and data analysis.

Cross Curricular/Career Readiness, Life Literacies and Key Skills Practice

Social Studies Connection:

Name of Task: Margin of Error for Estimating a Population Mean (6.1.12.HistoryUP.2.b)

- This task illustrates the development of the margin of error when estimating a population mean (S.IC.4). The results from several simulations are used to develop the margin of error, and then a way of estimating the margin of error from a single sample is introduced. This is a challenging task, but it is well aligned with standard S.IC.4, one of the more complex statistics standards.

Science Connection: Science Standard

Name of Task: Rain and Lightning (MS-ESS3-2)

- This task uses the concept of weather prediction to explore different concepts of probability theory.