## Cover Page Algebra 1

Content Area:
Course(s):
Time Period:
Length:
180-185 days
Status:
Published

## Course Overview

Algebra 1 is designed to give students the requisite skills that provide a foundation for all future mathematics courses. Students will explore writing and solving linear equations, powers and exponents, quadratic equations, polynomials and factoring, graphing, and solving linear inequalities, functions, and geometry. Throughout the course, mathematical concepts will be taught, emphasizing real-world application, technology, and cross-curricular interaction. Questions like "How do you solve for the unknown?" "How do you graph a situation that you encounter in your own life?" and "How can I use math to make my life easier?" will be addressed throughout the course.

To demonstrate a cohesive and complete implementation plan, the following general suggestions are provided:

- The use of various formative assessments is encouraged to provide an ongoing method of determining the students' current level of understanding of the material presented.
- Homework, when assigned, should be relevant and reflect the current teaching in the classroom.
- Organizational strategies should be in place that allow the students to take the information gained in the classroom and apply it in terms that are relevant to them.
- Instruction should be differentiated to allow students the best opportunity to learn.
- Assessments should be varied, and instruction topics should be assessed in class.
- Modifications to the curriculum should be included that address students with Individualized Educational Plans (IEP), Multi Language Learners (ML), and those requiring other modifications (504 plans).


## Course Name, Length, Date of Revision and Curriculum Writer

Algebra 1 Honors
Full Year
Date: 06/07/2024
Curriculum Writer: Kelly Purcell

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Scope and Sequence Pacing Guide for Year
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Unit 4: Graphing Exponential Functions, Quadratic Functions and Sequences

Scope and Sequence


## Unit 1: Linear Equations and Functions

| Content Area: | Mathematics |
| :--- | :--- |
| Course(s): |  |
| Time Period: | 1st Marking Period |
| Length: | $\mathbf{4 0}$ days |
| Status: | Published |

## Summary of the Unit

This unit provides a comprehensive introduction to algebra, starting with the fundamentals of solving simple and multi-step equations, and progressing to more complex equations with variables on both sides. Students then delve into the world of functions, focusing on linear functions and their representation in various forms (standard, slope-intercept). Key concepts include:

- Solving Equations: Mastering the manipulation of equations to isolate variables.
- Literal Equations: Understanding how to solve for specific variables within formulas.
- Functions: Grasping the concept of functions as relationships between inputs and outputs.
- Linear Functions: Exploring the characteristics and graphs of linear functions, including slope and intercepts.
- Function Notation: Learning to use function notation effectively.
- Graphing Linear Functions: Developing skills in graphing linear functions in both standard and slope-intercept form.
- Writing Linear Equations: Practicing writing linear equations given different types of information (slope and intercept, point and slope).
- Parallel \& Perpendicular Lines: Understanding the relationships between slopes of parallel and perpendicular lines.
- Scatter Plots \& Lines of Fit: Analyzing data to determine trends and relationships, and using lines of best fit to make predictions.
- Piecewise Functions: Exploring functions defined by different rules over different parts of their domain.
Throughout the unit, students engage in regular quizzes and graded assignments to assess their understanding and progress. A comprehensive review and test conclude the unit, solidifying their knowledge of algebraic principles and linear functions.


## Enduring Understandings

## Enduring Understandings:

1. Equations represent relationships: Equations are tools used to model and solve problems involving relationships between quantities.
2. Functions are relationships between quantities: Functions are a special type of relation where each input has exactly one output, and they are essential for modeling real-world phenomena.
3. Linear functions have a constant rate of change: The slope of a linear function represents this constant rate of change and reveals how the output changes in relation to the input.
4. Multiple representations of linear functions are interconnected: Equations, graphs, tables, and verbal descriptions all provide different but equivalent ways to represent and analyze linear functions.
5. Linear functions can be used to model and solve real-world problems: By understanding the properties of linear functions and how to manipulate their representations, we can make predictions and solve problems in various fields like science, business, and engineering.
6. Scatterplots and lines of fit can be used to analyze data and make predictions: By understanding the relationship between two variables in a scatterplot and fitting a line to the data, we can make predictions about future values or understand trends.
7. Piecewise functions are used to model situations with changing rules: Different rules apply over different parts of the domain, allowing for more complex modeling of real-world scenarios.
These enduring understandings capture the core concepts and big ideas students should take away from the unit, ensuring that they not only develop the necessary skills but also understand the underlying principles and applications of algebra and linear functions.

## Essential Questions

- How can we use mathematics to model and solve real-world problems involving quantities and
relationships?
- Why is it important to understand the reasoning behind solving equations and inequalities, rather than just memorizing procedures?
- How can different representations (equations, graphs, tables) help us understand and solve linear relationships?


## Summative Assessment and/or Summative Criteria

Required District/State Assessments
Unit Assessments
NJSLA
SGO Assessments

Suggested Formative/Summative Classroom Assessments
Describe Learning Vertically
Identify Key Building Blocks
Make Connections (between and among key building blocks)
Short/Extended Constructed Response Items
Multiple-Choice Items (where multiple answer choices may be correct)
Drag and Drop Items

Use of Equation Editor
Quizzes
Journal Entries/Reflections/Quick-Writes
Accountable talk
Projects
Portfolio
Observation
Graphic Organizers/ Concept Mapping
Presentations
Role Playing
Teacher-Student and Student-Student Conferencing
Homework

## Resources

Khan Academy https://www.khanacademy.org
Achieve the Core http://achievethecore.org
Illustrative Mathematics https://www.illustrativemathematics.org/
Inside Mathematics www.insidemathematics.org

Learn Zillion https://learnzillion.com
National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/vlibrary.html
Big Ideas Math https://www.bigideasmath.com/
Youcubed https://www.youcubed.org/week-of-inspirational-math/
NCTM Illuminations https://illuminations.nctm.org/Search.aspx?view=search\&type=1s\&gr=9-12
Shmoop http://www.shmoop.com/common-core-standards/math.html
Desmos https://www.desmos.com/
Geogebra http://www.geogebra.org/
CPALMS http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14
Partnership for Assessment of Readiness for College and Careers https://parcc.pearson.com/\#
McGraw-Hill ALEKS https://www.aleks.com/

Unit Plan

| Topic/Selection Timeframe | General Objectives | Instructional Activities | Benchmark / Assessments |
| :---: | :---: | :---: | :---: |
| Solving Simple Equations (1.1) | Students will evaluate algebraic expressions and use exponents | Complete chart on meaning of algebraic expressions and there operations <br> Match operation to algebraic expression <br> Complete chart on exponents and its terminology | Check student graphs. Check for correct use of graphing calculator. <br> Check translations of word problems into algebra. |


|  |  | Evaluate expressions with exponents | Classwork assigned. Homework assigned Assess student recall of this topic and review as needed. Check student responses. |
| :---: | :---: | :---: | :---: |
| Solving MultiStep Equations (1.2) | Students will solve multi-step linear equations using inverse operations. <br> Students will use multistep linear equations to solve real-life problems. <br> Students will use unit analysis to model reallife problems. | Review all terminology related to solving equations. <br> Incorporate the algebraic concepts of "Solving Multi-Step Equations" into the geometric concepts of "Solving for Angle Measures of a Polygon." | Check student graphs. Check for correct use of graphing calculator. <br> Check translations of word problems into algebra. <br> Classwork assigned. Homework assigned Assess student recall of this topic, and review as needed. Check student responses. |
| Solving Equations with Variables on Both Sides (1.3) | Students will solve linear equations with variables on both sides and use linear equations to solve real-life problems. Students will identify special solutions of linear equations. | Review all terminology related to solving equations with variables on both sides of the equation. | Check student graphs. Check for correct use of graphing calculator. <br> Check translations of word problems into algebra. <br> Classwork assigned. Homework assigned Assess student recall of this Algebra 1 topic, and review as needed. <br> Check student |


|  |  |  | responses. |
| :---: | :---: | :---: | :---: |
| Solving Absolute Value Equations (1.4) | Solving Linear Systems. | Visualize solutions of systems of linear equations in three variables. Solve systems of linear equations in three variables algebraically. Solve real-life problems. <br> Have students recall methods of solving linear systems (graphing, elimination, and substitution) and how many solutions a system of linear equations can have. <br> Indicate that the same techniques can be used for quadratic systems. <br> Discuss the number of solutions possible in a quadratic system. Solve multiple systems using the above techniques, stressing that solutions can be checked algebraically. | Check student understanding via oral participation. <br> Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned. |
| Rewriting Equations and Formulas (1.5) | Students will solve absolute value equations including equations involving two absolute values. Students will identify special solutions of absolute value equations. | Add lesson vocabulary terms/examples to notes. <br> Define and model the meaning of absolute value. | Check student understanding via oral participation. <br> Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned. |
| Functions (3.1) | Students will determine whether relations are functions, find the domain and range of a | Add lesson vocabulary terms/examples to notes. Have students determine the domain and range of teacher created/chosen | Teacher chosen/ created worksheets/activities with higher level |


|  | function, and identify the independent and dependent variables of a function. | materials. <br> Have students determine the independent/dependent variables of teacher chosen/created materials. | examples that are helpful in mastering the skills in this lesson. <br> Closure activity to assess common misconceptions such as: Make sure students do not confuse inverse with negative when finding inverse relations. Classwork assigned including real world application problems <br> Homework assigned |
| :---: | :---: | :---: | :---: |
| Linear Functions (3.2) | Students will identify linear functions using graphs, tables, and equations and graph linear functions using discrete and continuous data. | Add lesson vocabulary terms/examples to notes. Have students discover ways to determine if a function is linear or not using teacher chosen/created materials. <br> Have students discover the difference between discrete and continuous functions; then have students graph these functions using both graph paper and a graphing calculator | Teacher chosen/ created worksheets/activities with higher level examples that are helpful in mastering the skills in this lesson. <br> Classwork assigned, including real world application problems Homework assigned |
| Function <br> Notation (3.3) | Students will use function notation to evaluate and interpret | Add lesson vocabulary terms/examples to notes. Have students graph and solve | Teacher chosen/ created worksheets/activities |


|  | functions, and use functions notation to solve and graph functions. | functions using teacher chosen/created materials. | with higher level examples that are helpful in mastering the skills in this lesson. <br> Classwork assigned including real-world application problems Closure activity to assess common misconceptions such as: $f(x)$ does not mean the product of $f$ and $x$. |
| :---: | :---: | :---: | :---: |
| Graphing Linear Equations in Standard Form (3.4) | Students will graph equations of horizontal and vertical lines. Students will graph linear equations in standard form using intercepts and solve real-life problems. | Add lesson vocabulary terms/examples to notes. Have students graph linear equations in standard form. Then, have them discover how equations for horizontal and vertical lines differ. | Mini Quiz based on previous material (3.1-3.3). <br> Teacher chose/ created worksheets/activities with higher-level examples that help master the skills in this lesson. <br> Classwork assigned including real-world application problems Homework assigned |
| Graphing Linear Equations in Slope-Intercept Form (3.5) | Students will find the slope of a line, use the slope-intercept form of linear equations, and use slopes and $y$ intercepts to solve reallife problems. | Add lesson vocabulary terms/examples to notes. Graphing Calculator Investigation: Enter equations into the $\mathrm{Y}=$ list to graph in the standard viewing window and introduce other functions, such as the Zoom and Table. | The teacher chose/ created worksheets/activities with higher-level examples that helped master the skills in this lesson. |


|  |  | Discuss Families of Graphs and have students discuss similarities and differences between the parent function and several given linear equations using teacher-chosen/ created worksheets/activities. | Classwork assigned, including real world application problems Homework assigned |
| :---: | :---: | :---: | :---: |
| Writing Slope Intercept Form (4.1) and (4.2) | Students will be able to identify the slope and y-intercept from a linear equation in slopeintercept form. <br> Students will be able to write the equation of a line in slope-intercept form given its slope and $y$-intercept. <br> Students will be able to write the equation of a line in slope-intercept form given two points on the line. <br> Students will be able to graph a linear equation in slope-intercept form. <br> - Students will be able to interpret the meaning of the slope and $y$ intercept in real-world contexts. | Introduction to Slope: <br> Visual Exploration: Use interactive online tools or physical manipulatives (like staircases or ramps) to illustrate the concept of slope as a rate of change (rise over run). <br> Real-World Connections: Discuss examples of slope in everyday life (e.g., the pitch of a roof, the steepness of a hill). <br> Practice: Have students calculate slope from graphs and tables of values. <br> Discovering the Y-Intercept: <br> Graphing: Guide students through graphing linear equations where the $y$ intercept is apparent. <br> Patterns: Have students analyze patterns in equations and graphs to identify the y-intercept as the point where the line crosses the $y$-axis. <br> Slope-Intercept Form ( $\mathbf{y}=\mathbf{m x}+\mathbf{b}$ ): <br> Explicit Instruction: Introduce the | Formative <br> Assessments: <br> Quizzes or short assignments on identifying slope and y-intercept, writing equations, and graphing. <br> Exit tickets or quick checks to gauge understanding during lessons. <br> Peer review or partner work to encourage discussion and collaboration. <br> Summative <br> Assessments: <br> Unit test covering all objectives, including a mix of multiplechoice, short answer, and graphing problems. <br> Performance-based |

formula, explaining the meaning of ' m ' (slope) and 'b' (y-intercept).

Guided Practice: Provide examples where students substitute given values for $m$ and $b$ to write equations.

Problem-Solving: Pose problems where students need to find the equation given a slope and $y$ intercept, or two points.

## Graphing from Slope-Intercept

 Form:Step-by-Step: Teach the process of plotting the $y$-intercept first, then using the slope to find additional points.

Interactive Practice: Use online graphing tools or graphing calculators to reinforce the connection between equations and graphs.

Application: Have students graph real-world scenarios modeled by linear equations.

Interpreting Slope and Y-Intercept in Context:
Word Problems: Present scenarios where the slope and y-intercept represent real-world quantities (e.g., hourly wage, starting fee).
Discussion: Lead a discussion on how changes in slope or y-intercept affect
assessment: Have students create a realworld scenario, write the equation, graph it, and explain the meaning of the slope and y -intercept.
Project presentation:
Have students present their real-world scenarios and explain the math behind them.

|  |  | the graph and the real-world situation. <br> Projects: Have students design their own scenarios and write corresponding equations. |  |
| :---: | :---: | :---: | :---: |
| Writing equations for parallel and perpendicular lines (4.3) | General Objectives: <br> Students will be able to identify parallel and perpendicular lines based on their slopes. <br> Students will be able to write the equation of a line parallel to a given line that passes through a specified point. <br> Students will be able to write the equation of a line perpendicular to a given line that passes through a specified point. <br> Students will be able to graph parallel and perpendicular lines. <br> Students will be able to apply the concept of parallel and perpendicular lines to solve real-world problems. | Conceptual Understanding: <br> Visual Exploration: Use interactive online tools or physical manipulatives (e.g., straws, toothpicks) to demonstrate the relationships between parallel and perpendicular lines. <br> Slope Connection: Emphasize that parallel lines have the same slope, while perpendicular lines have slopes that are negative reciprocals of each other. <br> Real-World Examples: Discuss realworld examples of parallel and perpendicular lines (e.g., railroad tracks, streets intersecting at right angles). <br> Finding Parallel Lines: <br> Given Equation: Provide examples of linear equations and guide students to write the equation of a parallel line passing through a given point. <br> Given Graph: Have students identify parallel lines on a graph and write their equations. <br> Practice: Offer problems with | Formative Assessments: <br> Quizzes or short assignments on identifying parallel and perpendicular lines, writing their equations, and graphing. <br> Exit tickets or quick checks to gauge understanding during lessons. <br> Group work or partner activities to encourage collaboration and discussion. <br> Summative Assessments: <br> Unit test covering all objectives, including a mix of identification, equation writing, graphing, and application problems. <br> Performance-based |

varying difficulty levels for students to practice finding equations of parallel lines.

## Finding Perpendicular Lines:

Given Equation: Explain the concept of negative reciprocal slopes and guide students to write the equation of a perpendicular line passing through a given point.

Given Graph: Have students identify perpendicular lines on a graph and write their equations.

Practice: Provide problems for students to practice finding equations of perpendicular lines.

## Graphing Parallel and Perpendicular Lines:

Step-by-Step: Teach the process of graphing parallel and perpendicular lines using slope-intercept form.

Hands-On Activity: Have students graph pairs of lines on coordinate planes to visually reinforce their understanding.
Application: Provide real-world scenarios and ask students to graph parallel or perpendicular lines to represent the situation.

Application to Real-World
assessment: Have students create a realworld scenario involving parallel or perpendicular lines, solve the problem, and explain their reasoning.

Open-ended problems: Provide scenarios that require students to apply their knowledge creatively to solve problems beyond basic calculations.

|  | $\|$Problems: <br> Geometry Problems: Present <br> problems involving geometric figures <br> where parallel or perpendicular lines <br> are relevant (e.g., finding the height of <br> a triangle, calculating the distance <br> between parallel lines). <br> Distance Formula: Introduce the <br> distance formula and show how it can <br> be used to find the distance between <br> parallel or perpendicular lines. <br> Real-World Scenarios: Have <br> students apply their knowledge to <br> solve problems involving navigation, <br> construction, or design. |
| :--- | :--- | :--- |

## Standards

| MATH.9-12.F.BF.A.1.a | Determine an explicit expression, a recursive process, or steps for calculation from a <br> context. |
| :--- | :--- |
| MATH.9-12.N.Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step <br> problems; choose and interpret units consistently in formulas; choose and interpret the <br> scale and the origin in graphs and data displays. |
| MATH.9-12.S.ID.B.6.a | Fit a function to the data (including with the use of technology); use functions fitted to <br> data to solve problems in the context of the data. Use given functions or choose a function <br> suggested by the context. Emphasize linear and exponential models. <br> MATH.9-12.S.ID.B.6.b |
| Informally assess the fit of a function by plotting and analyzing residuals, including with <br> the use of technology. |  |

dentify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Fit a linear function for a scatter plot that suggests a linear association.
Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Compute (using technology) and interpret the correlation coefficient of a linear fit.
Distinguish between correlation and causation.
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y=f(x)$.

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Graph linear and quadratic functions and show intercepts, maxima, and minima.
Graph square root, cube root, and piecewise-defined functions, including step functions
and absolute value functions.

MATH.9-12.A.REI.D. 10

MATH.9-12.F.IF.C. 9

MATH.9-12.F.LE.A.1.b

MATH.9-12.F.LE.A. 2

MATH.9-12.F.LE.B. 5

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret the parameters in a linear or exponential function in terms of a context.

## Suggested Modifications for Special Education, ELL and Gifted Students

## Special Education Students

- Visual Supports: Use graphic organizers, flowcharts, and diagrams to break down complex concepts like solving multi-step equations or graphing linear functions.
- Manipulatives: Utilize algebra tiles, counters, or other hands-on tools to model equations and function relationships.
- Simplified Instructions: Break down tasks into smaller, manageable steps. Provide clear, concise instructions with visual cues and examples.
- Modified Assessments: Offer alternative assessment formats, such as oral presentations, projects, or modified worksheets with fewer problems or larger font sizes.
- Individualized Support: Work with students one-on-one or in small groups to provide targeted instruction and scaffolding.


## ELL Students

- Vocabulary Support: Provide visual aids and real-world examples to help students connect new mathematical terms to familiar concepts. Use bilingual dictionaries or glossaries to facilitate language acquisition.
- Scaffolding: Break down tasks into smaller steps, providing clear instructions and modeling for each step. Use sentence frames or graphic organizers to help students organize their thoughts and communicate their understanding.
- Culturally Responsive Instruction: Incorporate examples and problems that reflect the students' cultural backgrounds and interests. Encourage collaboration and peer support among ELL students.


## Gifted Students

- Enrichment Activities: Provide opportunities for students to explore advanced topics related to algebra and linear functions, such as systems of equations, linear programming, or matrices.
- Independent Research: Encourage students to conduct independent research projects on real-world applications of algebra and linear functions, such as modeling economic trends or analyzing data from scientific experiments.
- Peer Tutoring: Allow gifted students to share their knowledge and understanding with their peers by leading small-group discussions or tutoring sessions.
- Challenge Problems: Incorporate challenging problems that require students to apply their knowledge in new and creative ways.


## Suggested Technological Innovations/ Use

For Solving Equations and Literal Equations:

- Online Equation Solvers (e.g., Symbolab, Mathway): These tools can be used to check answers, provide step-by-step solutions, and help students identify errors in their own work.
- Digital Manipulatives (e.g., Algebra Tiles): Virtual algebra tiles can help students visualize the process of solving equations and literal equations.
- Interactive Worksheets and Quizzes (e.g., Kahoot, Quizizz): These platforms offer engaging ways for students to practice solving equations and receive immediate feedback.

For Functions, Linear Functions, and Function Notation:

- Desmos or GeoGebra: These graphing calculators allow students to visualize functions, explore transformations, and analyze key features.
- Online Function Machines: Interactive simulations where students input values and observe the corresponding outputs, helping them understand the concept of functions.
- Function Notation Practice Websites or Apps: These resources provide targeted practice with using function notation.

For Graphing and Writing Linear Equations:

- Desmos or GeoGebra: Students can graph equations, explore the effects of changing slope and yintercept, and practice finding equations from graphs.
- Online Graphing Activities: Interactive activities that challenge students to identify key features of graphs, write equations from graphs, and transform linear functions.
- Slope-Intercept Form Calculators: Online tools that help students calculate slope and y-intercept from points or equations.

For Scatter Plots, Lines of Fit, and Piecewise Functions:

- Spreadsheet Software (e.g., Google Sheets, Excel): Students can create scatterplots, add lines of best fit, and analyze data.
- Online Regression Tools: These tools can calculate lines of best fit and provide statistical information about the data.
- Piecewise Function Graphing Tools: Online graphing calculators specifically designed for piecewise functions can help students visualize these complex functions.


## Cross Curricular/Career Readiness, Life Literacies and Key Skills Practice

## Cross-Curricular Connections:

- Science:
- Use linear functions to model the relationship between distance and time for objects moving at a constant speed.
- Analyze data from science experiments (e.g., temperature change over time) using scatter plots and lines of best fit.
- Social Studies:
- Investigate historical trends (e.g., population growth, economic data) using linear functions and graphs.
- Analyze voting patterns or demographic data using scatter plots and lines of fit.
- Physical Education:
- Track athletic performance (e.g., running speed, jump height) over time and model improvements using linear functions.
- Calculate the average speed of athletes using distance and time data.


## Career Readiness:

- Business and Finance:
- Use linear equations to model production costs, revenue, and profit.
- Analyze sales data and predict future sales using linear regression.
- Engineering and Technology:
- Apply linear functions to design ramps, bridges, or other structures with constant slopes.
- Use linear equations to model electrical circuits or fluid flow.


## - Healthcare:

- Analyze medical data (e.g., medication dosage, heart rate) using linear functions and graphs.
- Predict patient outcomes based on trends in medical data.

Life Literacies and Key Skills:

- Critical Thinking and Problem Solving:
- Challenge students to solve real-world problems using algebraic equations and functions.
- Encourage students to analyze different approaches to problem-solving and evaluate their effectiveness.
- Communication and Collaboration:
- Have students work in pairs or groups to solve problems and explain their reasoning.
- Encourage students to present their findings to the class and engage in discussions about different solution strategies.
- Data Literacy and Analysis:
- Teach students how to collect, organize, and analyze data using spreadsheets and graphing tools.
- Help students interpret data and draw conclusions based on their analysis.
- Financial Literacy:
- Teach students how to use linear equations to manage budgets, calculate interest, and make
informed financial decisions.
By incorporating these cross-curricular connections, career readiness activities, and life literacy skills into your Algebra Fundamentals \& Linear Functions unit, you can help students see the relevance of mathematics in their everyday lives and prepare them for future success in a variety of fields.


# Unit 2: Statistics, Linear Inequalities, Solving Systems, Absolute Value Equations and Inequalities 

Content Area: Mathematics
Course(s):
Time Period:
Length:
2nd Marking Period
40 days
Published

## Summary of the Unit

In this unit students will focus on how to solve linear inequalities in one variable and graph linear inequalities in two variables.

## Enduring Understandings

- The characteristics of linear inequalities and their representations help solve real-world problems. Reason quantitatively and use units to solve problems
- Solve [linear] equations and inequalities in one variable.
- Understand solving equations as a process of reasoning and explain the reasoning
- Create equations that describe numbers or relationships
- Interpret the structure of expressions
- Represent and solve equations graphically
- Interpret linear models
- Understand qualities of a function.
- Use, evaluate, and interpret function notation.


## Essential Questions

- Quantitative Reasoning and Units:
- How do units of measurement influence the interpretation and solution of real-world problems involving linear relationships?
- Why is it crucial to maintain consistent units when solving linear equations and inequalities?
- Linear Equations and Inequalities:
- What are the distinguishing features of linear equations and inequalities, and how do these features affect their graphical representations?
- How can we leverage algebraic reasoning and properties to solve linear equations and inequalities accurately?
- In what real-world situations might we encounter linear inequalities, and how do their solutions differ from those of linear equations?
- Creating Equations:
- How can we translate a real-world problem into a linear equation or inequality that captures its essential relationships?
- What role do variables play in representing unknown quantities within linear equations and inequalities?
- Interpreting Expressions and Equations:
- How can we discern the meaning and significance of coefficients, terms, and constants within linear expressions and equations?
- How does the structure of a linear equation or inequality reveal information about the underlying relationship it models?
- Linear Models and Functions:
- What are the defining characteristics of linear models, and how can we identify them in realworld data?
- How can linear functions be used to predict outcomes, analyze trends, and make informed decisions in practical scenarios?
- What is the significance of function notation, and how does it facilitate the evaluation and interpretation of linear functions?


## Summative Assessment and/or Summative Criteria

Required District/State Assessments
Unit Assessments
NJSLA
SGO Assessments

Suggested Formative/Summative Classroom Assessments
Describe Learning Vertically
Identify Key Building Blocks
Make Connections (between and among key building blocks)
Short/Extended Constructed Response Items
Multiple-Choice Items (where multiple answer choices may be correct)
Drag and Drop Items
Use of Equation Editor
Quizzes
Journal Entries/Reflections/Quick-Writes
Accountable talk
Projects
Portfolio

Observation
Graphic Organizers/ Concept Mapping
Presentations
Role Playing
Teacher-Student and Student-Student Conferencing
Homework

## Resources

Khan Academy https://www.khanacademy.org
Achieve the Core http://achievethecore.org
Illustrative Mathematics https://www.illustrativemathematics.org/
Inside Mathematics www.insidemathematics.org
Learn Zillion https://learnzillion.com
National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/vlibrary.html
Big Ideas Math https://www.bigideasmath.com/
Youcubed https://www.youcubed.org/week-of-inspirational-math/
NCTM Illuminations https://illuminations.nctm.org/Search.aspx?view=search\&type=1s\&gr=9-12
Shmoop http://www.shmoop.com/common-core-standards/math.html

Desmos https://www.desmos.com/
Geogebra http://www.geogebra.org/
CPALMS http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14
Partnership for Assessment of Readiness for College and Careers https://parcc.pearson.com/\#
McGraw-Hill ALEKS https://www.aleks.com/

Unit Plan

| Topic/Selection Timeframe | General Objectives | Instructional Activities | Benchmark / Assessments |
| :---: | :---: | :---: | :---: |
|  | Students will be able to construct a scatter plot to represent bivariate data. <br> Students will be able to identify and describe positive, negative, or no correlations within scatter plots. <br> Students will be able to draw a line of fit (trend line) that best represents the data in a scatter plot. <br> Students will be able to use the line of fit to make predictions and | Human Scatter Plot: <br> Prepare a set of data points on index cards (e.g., hours of sleep vs. test scores). <br> Have students act as data points, arranging themselves in a human scatter plot on the floor. <br> Discuss the visual patterns that emerge and what they might represent. <br> Scatter Plot Gallery Walk: <br> Display a variety of scatter plots with different correlation types (positive, negative, no correlation). | Exit Ticket: Provide a scatter plot and have students identify the correlation type and write a brief interpretation. Project: Have students collect their data on a topic of interest, create a scatter plot, and present their findings, including an analysis of the correlation. Quiz: Include questions that assess the ability to construct scatter plots, identify |

extrapolate.
Students will be able to understand the limitations of making predictions with lines of fit.

Have students rotate through the gallery, identifying the type of correlation and writing a sentence to describe the relationship in the data.

## Real-World Data Exploration:

Provide students with real datasets (e.g., ice cream sales vs. temperature, study time vs. grades).

Guide them through creating scatter plots using graph paper or online tools.

Have them analyze the graphs, discuss correlations, and make predictions based on the patterns.

## Line of Best Fit Challenge:

Display a scatter plot without a line of fit.

Provide students with rulers or string, and have them work in pairs to draw what they believe is the best line of fit.

Discuss the different lines, strategies for drawing them, and the concept of minimizing the distance between the line and the data points.

Making Predictions Activity:
correlation types, and interpret real-world scenarios.

|  |  | Use a scatter plot with a drawn line of fit. <br> Ask students to use the line to predict values not shown on the graph (interpolation and extrapolation). <br> Discuss the accuracy of predictions and factors that might influence the accuracy. <br> Investigating Outliers: <br> Present a scatter plot with an outlier. <br> Have students draw a line of fit with and without the outlier. <br> Discuss how outliers can impact the line of fit and the accuracy of predictions. |  |
| :---: | :---: | :---: | :---: |
| 4.7 Piecewise Functions | Evaluate piecewise functions. Graph and write piecewise functions. Describe piecewise functions in terms of real-life circumstances. Graph and write step functions. Describe piecewise functions in terms of the real-life circumstances. Write absolute value | Create examples of piecewise, step and absolute value functions so that students have a reference to use when working with and discussing the similarities and differences of the functions. Develop questions that encourage students to think through their understanding of piecewise, step and absolute value functions | Check student graphs. Check for correct use of graphing calculator. <br> Check translations of word problems into algebra. <br> Classwork assigned. Homework assigned Assess student recall of this topic, and review as needed. Check student responses. |


|  | functions. |  |  |
| :---: | :---: | :---: | :---: |
| 5.1/5.5 Solving Systems by Graphing | Understand and describe the meaning of a system of equations in context. Check solutions of systems of linear equations. Solve systems of linear equations by graphing. Solve systems of linear equations by substitution. Solve systems of linear equations by elimination. Discuss the thinking process behind the choice of method when solving a system of linear equations. Determine the number of solutions of linear systems. Use systems of linear equations to solve reallife problems. | Introduce the topic in a contextual way so that students can visualize how the graphing of a system of a linear equations would apply to real life. Have students graph the functions in different colors to enable students to see the intersection more clearly. Use graphing calculators to enable students to practice the skills needed, to see the intersections clearly and to check their work. | Check student understanding via oral participation. <br> Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned. |
| 5.2 Solving Systems by Substitution | Understand and describe the meaning of a system of equations in context. Check solutions of systems of linear equations. Solve systems of linear equations by graphing. Solve systems of linear equations by substitution. Solve | Introduce the topic in a contextual way so that students can visualize how the graphing of a system of a linear equations would apply to real life. Have students graph the functions in different colors to enable students to see the intersection more clearly. Use graphing calculators to enable students to practice the skills needed, to see the intersections | Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned. |


|  | systems of linear equations by elimination. Discuss the thinking process behind the choice of method when solving a system of linear equations. Determine the number of solutions of linear systems. Use systems of linear equations to solve reallife problems. | clearly and to check their work. |  |
| :---: | :---: | :---: | :---: |
| 5.3 Solving Systems by Elimination | Understand and describe the meaning of a system of equations in context. Check solutions of systems of linear equations. Solve systems of linear equations by graphing. Solve systems of linear equations by substitution. Solve systems of linear equations by elimination. Discuss the thinking process behind the choice of method when solving a system of linear equations. Determine the number of solutions of linear systems. Use systems of linear equations to solve reallife problems. | Introduce the topic in a contextual way so that students can visualize how the graphing of a system of a linear equations would apply to real life. Have students graph the functions in different colors to enable students to see the intersection more clearly. Use graphing calculators to enable students to practice the skills needed, to see the intersections clearly and to check their work. | Teacher chosen/ created worksheets/activities with higher level examples that are helpful in mastering the skills in this lesson. |


| 5.4 Special Solutions (mix within topics) | Understand and describe the meaning of a system of equations in context. Check solutions of systems of linear equations. Solve systems of linear equations by graphing. Solve systems of linear equations by substitution. Solve systems of linear equations by elimination. Discuss the thinking process behind the choice of method when solving a system of linear equations. Determine the number of solutions of linear systems. Use systems of linear equations to solve real-life problems. | Introduce the topic in a contextual way so that students can visualize how the graphing of a system of a linear equations would apply to real life. Have students graph the functions in different colors to enable students to see the intersection more clearly. Use graphing calculators to enable students to practice the skills needed, to see the intersections clearly and to check their work. | The teacher chose/ created worksheets/activities with higher-level examples that help students master the skills in this lesson. Closure activity Classwork assigned including real world application problems <br> Homework assigned |
| :---: | :---: | :---: | :---: |
| 2.1 Writing \& Graphing Inequalities | Students will identify Write linear inequalities from verbal and written description. Write linear inequalities from graphs. Sketch graphs of linear inequalities. Solve inequalities using addition and subtraction. | Expand on the chart for Properties of Inequality Use large number lines to graph solutions | Teacher chose/ created worksheets/activities with higher-level examples that help master the skills in this lesson. <br> Classwork assigned, including real world application problems Homework assigned |


|  |  |  |  |
| :---: | :---: | :---: | :---: |
| 2.2 One-Step Inequalities (+,-) | Students will Solve inequalities using addition and subtraction. | Solve inequalities by multiplication and division using integers. <br> Use inequalities in a contextual situation to solve real-life problems. <br> Solve multi-step inequalities and use multi-step inequalities to solve real-life problems. Write, graph, and solve compound inequalities and use compound inequalities to solve real-life problems. Solve absolute value inequalities and use absolute value inequalities to solve real-life problems. <br> Recognize that the equations and inequalities represent the constraints of the problem. Equations and inequalities describe relationships. | The teacher chose/ created worksheets/activities with higher-level examples that help students master the skills in this lesson. <br> Classwork assigned, including real-world application problems Closure activity |
| 2.3 One-Step Inequalities (x,/) | Students will Solve inequalities using addition and subtraction. | Use a large number line to graph solutions | Mini Quiz <br> The teacher chose/ created worksheets/activities with higher-level examples that helped master the skills in this lesson. <br> Classwork assigned including real-world application problems Homework assigned |
| 2.4 Multi-Step | Solve multi-step | Encourage students to refer to the | The teacher chose/ |


| Inequalities | inequalities and use them to solve real-life problems. Write, graph, and solve compound inequalities and use them to solve real-life problems. | resources provided when constructing algebraic models to solve problems. Support and strengthen students' ability to justify reasoning by modeling, providing resources, praising students, and providing them with sentence stems if needed. | created <br> worksheets/activities with higher-level examples that helped master the skills in this lesson. <br> Classwork assigned, including real-world application problems Homework assigned |
| :---: | :---: | :---: | :---: |
| 2.5 Compound Inequalities | Write, graph and solve compound inequalities and use compound inequalities to solve real-life problems. | Encourage students to refer to the resources provided when constructing algebraic models to solve problems. Support and strengthen students' ability to justify reasoning by modeling, providing resources, praising students, and providing them with sentence stems if needed. | The teacher chose/ created worksheets/activities with higher-level examples that helped master the skills in this lesson. <br> Classwork assigned, including real world application problems |
| 5.6 Graphing Inequalities in 2 Variables | Graph inequalities and solve real-life problems | Provide visual cues, graphic representations, gestures, and pictures. This includes updating the word wall and grounding the work in a relevant context. Design questions and prompts for different proficiency levels, being mindful of linguistic complexity | The teacher chose/ created worksheets/activities with higher-level examples that helped master the skills in this lesson. <br> Classwork assigned, including real world application problems |
| 5.7 Systems of Linear Inequalities | Solve multi-step inequalities and use them to solve real-life problems. Write, graph, and solve compound | Encourage students to refer to the resources provided when constructing algebraic models to solve problems. Support and strengthen students' ability to | Classwork assigned, including real world application problems |


|  | inequalities and use them to solve real-life problems. | justify reasoning by modeling, providing resources, praising students, and providing them with sentence stems if needed. |  |
| :---: | :---: | :---: | :---: |
| 1.4 Absolute Value Equations | Students will understand the concept of absolute value as the distance from zero. <br> Students will be able to solve absolute value equations both algebraically and graphically. <br> Students will be able to interpret solutions to absolute value equations in context. <br> Students will be able to identify and solve absolute value equations with extraneous solutions. | Conceptual Understanding of Absolute Value: <br> Number Line Exploration: Have students plot various numbers on a number line and discuss their distances from zero. Introduce absolute value notation (e.g., $\|-5\|$ $=5$ ). <br> Real-World Examples: Discuss real-world scenarios where absolute value is used (e.g., distances, temperature variations, stock market fluctuations). <br> Folding Activity: Have students fold a number line at zero and observe how numbers on either side "match up." Explain that this symmetry represents absolute value. <br> Solving Absolute Value Equations Algebraically: <br> Two-Case Method: Teach students to solve absolute value equations by setting up two separate equations, one with the expression inside the absolute value equal to the positive value and the other equal to the negative | Whiteboard Practice: <br> Have students solve problems on miniwhiteboards and hold them up for quick checks. Exit Tickets: Provide a few problems at the end of class to assess understanding. <br> Observing Group <br> Work: Monitor student interactions during group activities to gauge their comprehension. <br> Summative <br> Assessments: <br> Quizzes/Tests: Include a mix of multiple-choice, short answer, and problem-solving questions. <br> Projects: Have students research and present realworld applications of absolute value equations. Performance Tasks: Design tasks that require students to apply their knowledge to solve complex problems or analyze scenarios. |


|  |  | value. <br> Practice Problems: Provide a variety of problems, starting with simple ones (e.g., $\|x\|=3$ ) and progressing to more complex ones with variables on both sides (e.g., $\|2 x-5\|=\|x+1\|)$. <br> Extraneous Solutions: Introduce examples where a solution to one of the cases does not satisfy the original equation. Emphasize the importance of checking for extraneous solutions. <br> Graphical Representation of Absolute Value Equations: <br> Graphing Calculator Exploration: Use a graphing calculator or online tool to graph simple absolute value functions (e.g., $y=\|x\|, y=\|x-2\|+1$ ). <br> Discuss how the graphs relate to the solutions of the equations. <br> Hand-Graphing: Have students graph absolute value functions by hand, plotting points and connecting them to form a "V" shape. <br> Connecting Graphs and Equations: Have students solve absolute value equations both algebraically and graphically and | Differentiation: Scaffolding: Provide guided notes, visual aids, and step-by-step instructions for struggling learners. <br> Extension: Offer more challenging problems with multiple absolute values or inequalities. Enrichment: Have students research and present on the use of absolute value in advanced mathematical concepts (e.g., complex numbers, calculus). Technology Integration: Utilize online graphing tools or interactive activities to reinforce concepts. |
| :---: | :---: | :---: | :---: |


|  |  | compare the solutions. <br> Applications and Problem Solving: <br> Word Problems: Provide realworld scenarios that can be modeled with absolute value equations (e.g., distance from a target, acceptable range of manufacturing error). <br> Error Analysis: Present students with worked-out solutions containing errors and have them identify and correct the mistakes. <br> Group Work: Have students work in groups to solve more challenging absolute value problems or create their own word problems. |  |
| :---: | :---: | :---: | :---: |
| 2.6 Absolute Value Inequalities | Instructional Objectives: <br> Students will understand the geometric interpretation of absolute value inequalities as distances on a number line. <br> Students will be able to solve absolute value inequalities of the form: | Conceptual Understanding of Absolute Value Inequalities: <br> Number Line <br> Exploration: Start with a review of absolute value as distance from zero. Have students plot numbers on a number line and discuss inequalities in terms of distances (e.g., "What numbers are less than 3 units away from | Formative Assessments: Whiteboard Practice: Have students solve and graph inequalities on mini-whiteboards. <br> Thumbs Up/Down: Ask conceptual questions and have students indicate agreement or disagreement with thumbs up/down. Exit Tickets: Provide problems for students to |

$$
\begin{aligned}
& |\mathrm{x}|<\mathrm{a} \\
& |\mathrm{x}|>\mathrm{a} \\
& |\mathrm{ax}+\mathrm{b}|<\mathrm{c} \\
& |\mathrm{ax}+\mathrm{b}|>\mathrm{c}
\end{aligned}
$$

Students will be able to graph solutions to absolute value inequalities on a number line.

Students will be able to write absolute value inequalities to represent real-world scenarios.
zero?").
Visualizing "Less Than" and "Greater Than":
Use yarn or string to represent distances on the number line. Have students physically show solutions to inequalities like $|\mathrm{x}|<2$ or $|\mathrm{x}|>4$.

Compound Inequalities:
Connect absolute value inequalities to compound inequalities (e.g., $|x|<2$ is equivalent to $-2<x<2$ ).

## Solving Absolute Value Inequalities Algebraically:

## Two-Case Method:

Teach the "less than" case as a compound inequality and the "greater than" case as two separate
inequalities joined by "or".
Guided Practice: Start with simpler examples (e.g., $|x|<5$ ) and progress to more complex ones with variables on both sides (e.g., $|2 \mathrm{x}-3|>7$ ).
Special Cases: Discuss absolute value inequalities where the absolute value
solve independently at the end of class.
Summative
Assessments:
Quizzes/Tests: Include a mix of solving, graphing, and writing absolute value inequalities.
Projects: Have students research and present realworld applications of absolute value inequalities.

## Performance Tasks:

Design tasks where students must apply their knowledge to solve complex problems or analyze scenarios.
Differentiation:
Scaffolding: Provide graphic organizers, step-by-step instructions, and pre-filled examples.
Extension: Offer more challenging problems with multiple absolute values or combined inequalities.
Enrichment: Explore connections between absolute value inequalities and other mathematical concepts like piecewise functions.

|  |  | expression is less than a negative number (no solution) or greater than or equal to zero (all real numbers). <br> Graphing Solutions on a Number Line: <br> Open and Closed <br> Circles: Emphasize the difference between open circles for strict inequalities ( $\langle,>$ ) and closed circles for inequalities that include equality ( $\leq, \geq$ ). <br> Shading: Show how to shade the appropriate region(s) of the number line to represent the solution set. <br> Checking Solutions: Have students test points within and outside the shaded region to verify their solution. <br> Applications and Problem Solving: <br> Real-World Scenarios: <br> Provide examples where absolute value inequalities are used (e.g., tolerances | Technology Integration: Utilize online graphing tools or interactive activities. |
| :---: | :---: | :---: | :---: |

$\left.\left.\begin{array}{|l|l|l|l|} & \begin{array}{l}\text { in manufacturing, } \\ \text { acceptable temperature } \\ \text { ranges). }\end{array} \\ \text { Writing Inequalities: } \\ \text { Give students word } \\ \text { problems and have them } \\ \text { write absolute value } \\ \text { inequalities to model the } \\ \text { situation. }\end{array}\right] \begin{array}{l}\text { Error Analysis: Present } \\ \text { solutions with errors and } \\ \text { have students identify and } \\ \text { correct the mistakes. }\end{array}\right\}$

|  |  | (dilation), and sliding (translation) the graphs to see how the equation changes. <br> Graphing Calculator Exploration: Use graphing calculators or online tools to graph various absolute value functions. Adjust the parameters (a, h, k) to see how they affect the graph. Summarizing Transformations: Create a chart or graphic organizer to summarize the effects of each parameter on the graph: <br> $a$ - Vertical stretch/compression, reflection over x -axis <br> $h$-Horizontal translation (opposite direction) <br> $k$ - Vertical translation <br> - Graphing Absolute Value <br> Functions: <br> Guided Practice: Provide several examples of functions to graph. Start with simple transformations and gradually increase complexity. <br> Multiple Representations: Have students connect equations, graphs, and verbal descriptions of transformations. <br> Vertex Form: Emphasize the usefulness of vertex form ( $\mathrm{y}=\mathrm{a}\|\mathrm{x}-\mathrm{h}\|$ <br> +k ) for quickly identifying the vertex ( $\mathrm{h}, \mathrm{k}$ ) and transformations. <br> - Writing Equations from Graphs: Reverse Engineering: Show students graphs of absolute value functions and have them work backward to | transformations, and writing equations. <br> Projects: Have students create a presentation or poster explaining absolute value function transformations. Performance Tasks: Design tasks where students apply their knowledge to real-world scenarios involving absolute value functions (e.g., designing a ramp for skateboarding). <br> Differentiation: Scaffolding: Provide graph paper with predrawn coordinate planes and labeled axes. Offer step-by-step instructions and worked examples. <br> Extension: Explore piecewise functions and how they relate to absolute value functions. Enrichment: Have students research applications of absolute value functions in fields like physics or engineering. Technology Integration: Utilize online graphing tools or interactive |
| :---: | :---: | :---: | :---: |

## determine the equation.

## Matching Activity: Create a

matching activity where students pair graphs with their corresponding equations.

## Standards

MATH.9-12.S.ID.A. 1

## MATH.9-12.S.ID.A. 2

MATH.9-12.S.ID.A. 3

MATH.9-12.S.ID.B. 5

MATH.9-12.A.CED.A. 1

MATH.9-12.A.CED.A. 3

MATH.9-12.A.REI.B. 3

MATH.9-12.A.REI.C. 5

MATH.9-12.A.REI.C. 6

Represent data with plots on the real number line (dot plots, histograms, and box plots).
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
Solve systems of linear equations algebraically (include using the elimination method) and graphically, focusing on pairs of linear equations in two variables.

Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## Suggested Modifications for Special Education, ELL and Gifted Students

Consistent with individual plans, when appropriate.
8. Students will be allowed to submit assignments using additional time per IEP modifications.
9. Students will be encouraged to use different size and type of font to avoid print confusion.
10. ML students will be allowed to use an internet translator or language glossary in order to translate vocabulary and assignments properly.

ML students may be allowed to work with another student who is fluent in their native language.

## Suggested Technological Innovations/ Use

- Instructional technology should be used to present and assess lessons, such as a

SmartNotebook, PowerPoint, graphing calculators, and Communicators/individual dry-erase boards.

- Teachers are encouraged to use electronic assessments to determine mastery of the concepts taught.
- The use of kahoot or other type of interactive software is encouraged.

Model interdisciplinary thinking to expose students to other disciplines.
Science Connection: Weather (MS-ESS2-D and HS-ESS2-D) • This task uses weather data and maps to utilize unit rates. Boyle's Law (MS-PS1-4, HS-PS1-3) • Students will explore rational numbers and functions in the context of Boyle's Law. Physical Education Connection: Free Throws (2.2.12.MSC.2) • This task uses basketball data to create scatter plots and the equation of the line of best fit. Throwing Baseballs (2.2.12.MSC.2) • Students will analyze the time, pathway and height of a baseball as the players compete

# Unit 3: Writing Linear Systems, \& Exponential Functions and Polynomials 

Content Area: Mathematics<br>Course(s):<br>Time Period: $\quad$ 3rd Marking Period<br>Length: 40 days<br>Status: Published<br>\section*{Summary of the Unit}

This unit provides a comprehensive exploration of exponential and polynomial functions, with a focus on their applications in solving equations and modeling real-world phenomena. Key concepts covered include:

- Properties of Exponents: Students master the rules of exponents, including multiplication, division, powers of powers, and negative exponents.
- Radical \& Rational Functions: They learn to manipulate and simplify expressions with radicals and rational exponents, laying the groundwork for working with exponential functions.
- Growth \& Decay Exponential Functions: Students explore the characteristics of exponential growth and decay, analyzing their graphs and applying them to model real-life situations like population growth, radioactive decay, and compound interest.
- Solving Exponential Equations: They develop strategies to solve exponential equations, both algebraically and graphically.
- Polynomial Operations: Students learn to add, subtract, and multiply polynomials, mastering the techniques for manipulating these expressions.
- Special Products: They explore special patterns in polynomial multiplication, like the difference of squares and perfect square trinomials.
- Factoring: Students develop a deep understanding of factoring techniques, including factoring out a greatest common factor, factoring trinomials with leading coefficients of 1 and greater than 1 , and factoring special products.
- Solving Equations in Factored Form: They apply factoring to solve polynomial equations.
- Properties of Radicals: Students revisit the properties of radicals and use them to simplify radical expressions.
- Solving Quadratics: They explore multiple methods for solving quadratic equations, including factoring, completing the square, and using the quadratic formula.
- Systems of Linear \& Quadratic Equations: Students learn to solve systems of equations where one or both equations are quadratic.


## Enduring Understandings

- Exponential Functions Model Growth and Decay: Exponential functions capture how quantities change over time when the rate of change is proportional to the current amount, making them essential tools for understanding various natural and social phenomena.
- The Rules of Exponents are Fundamental: The properties of exponents govern how we manipulate and simplify expressions with powers, providing a foundation for working with both exponential and polynomial functions.
- Polynomial Functions Can Model Complex Relationships: Polynomials can represent a wide range of relationships between quantities, capturing curves and patterns that linear functions cannot.
- Factoring is a Key Tool for Solving Polynomial Equations: Factoring allows us to break down complex polynomials into simpler forms, enabling us to find solutions to polynomial equations and gain insights into their behavior.
- Multiple Methods Can Solve Quadratic Equations: Quadratic equations can be solved using a variety of techniques, each with its strengths and weaknesses, providing flexibility and choice in problem-solving.
- Radicals and Rational Exponents Extend the Number System: Radicals and fractional exponents allow us to express roots and powers in new ways, expanding our understanding of the number system and its properties.


## Essential Questions

- How can we use mathematical functions to model and understand real-world phenomena?
- How can we use algebraic techniques to solve equations and inequalities involving exponential and polynomial functions?
- What are the connections between different types of functions, and how can we use those connections to solve problems?


## Summative Assessment and/or Summative Criteria

Required District/State Assessments
Unit Assessments
NJSLA
SGO Assessments

Suggested Formative/Summative Classroom Assessments
Describe Learning Vertically
Identify Key Building Blocks
Make Connections (between and among key building blocks)
Short/Extended Constructed Response Items
Multiple-Choice Items (where multiple answer choices may be correct)
Drag and Drop Items
Use of Equation Editor
Quizzes
Journal Entries/Reflections/Quick-Writes

Accountable talk
Projects
Portfolio
Observation
Graphic Organizers/ Concept Mapping
Presentations
Role Playing
Teacher-Student and Student-Student Conferencing
Homework

## Resources

Khan Academy https://www.khanacademy.org
Achieve the Core http://achievethecore.org
Illustrative Mathematics https://www.illustrativemathematics.org/
Inside Mathematics www.insidemathematics.org
Learn Zillion https://learnzillion.com
National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/vlibrary.html
Big Ideas Math https://www.bigideasmath.com/

Youcubed https://www.youcubed.org/week-of-inspirational-math/
NCTM Illuminations https://illuminations.nctm.org/Search.aspx?view=search\&type=1s\&gr=9-12
Shmoop http://www.shmoop.com/common-core-standards/math.html
Desmos https://www.desmos.com/
Geogebra http://www.geogebra.org/
CPALMS http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14
Partnership for Assessment of Readiness for College and Careers https://parcc.pearson.com/\#
McGraw-Hill ALEKS https://www.aleks.com/

## Unit Plan

| Topic/Selection Timeframe | General Objectives | Instructional Activities | Benchmark / <br> Assessments |
| :---: | :---: | :---: | :---: |
| 6.1 Properties of Exponents | Identify and evaluate exponential functions. Graph exponential functions and describe the key features in a real-life problem. <br> Compare exponential functions and conclude. <br> Use exponential functions to model real-life | Review the properties of exponents and provide clear examples to remind students of this prior learning. <br> Develop mnemonic devices with students to help them remember the properties of exponents and the circumstances under which the rules apply. <br> Create a Google Doc/Anchor Chart/Notes recapping the properties of | Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. <br> Classwork assigned. Homework assigned Assess student recall of this topic and review as needed. <br> Check student |


|  | exponential growth and decay. <br> Use properties of exponents (such as power of a power, product of powers, power of a product, and rational exponents, etc.) to write, an equivalent form. <br> of an exponential function to reveal and explain specific information about its approximate rate of growth or decay. | exponents with students for their reference when problem solving. <br> Provide students with opportunities to relate the mathematical depiction of exponential functions to real life scenarios that can be modeled as exponential functions. | responses. |
| :---: | :---: | :---: | :---: |
| 6.3/6.4 Growth and Decay Exponential Functions | Identify and evaluate exponential functions. Graph exponential functions and describe the key features in a real-life problem. <br> Compare exponential functions and conclude. <br> Use exponential functions to model real-life exponential growth and decay. <br> Use properties of exponents (such as power of a power, product of powers, power of a | Review the properties of exponents and provide clear examples to remind students of this prior learning. <br> Develop mnemonic devices with students to help them remember the properties of exponents and the circumstances under which the rules apply. <br> Create a Google Doc/Anchor Chart/Notes recapping the properties of exponents with students for their reference when problem solving. <br> Provide students with opportunities to relate the mathematical depiction of exponential functions to real life scenarios that can be modeled as | Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. <br> Classwork assigned. Homework assigned Assess student recall of this topic, and review as needed. <br> Check student responses. |


|  | product, and rational exponents, etc.) to write, an equivalent form. <br> of an exponential function to reveal and explain specific information about its approximate rate of growth or decay. | exponential functions. |  |
| :---: | :---: | :---: | :---: |
| 7.1 Adding \& Subtracting Polynomials | Develop an understanding of the degree of polynomials and relate it to previous learning about combining like terms. <br> Classify polynomials and write them in standard form. <br> Use the horizontal and vertical format for adding and subtracting polynomials. <br> Solve real-life problems that use the addition, subtraction, and multiplication of polynomials. | Create anchor chart/notes with students that document the thinking and processes behind adding, subtracting, and multiplying polynomials. <br> Relate multiplication of polynomials to students by using real-life problems such as area problems to help students understand the concept on a different level. | Check student understanding via oral participation. Check student work. Check for correct use of graphing calculator. Classwork assigned. Homework assigned. |
| 7.2 Multiplying Polynomials | Multiply binomials and trinomials and connect the process to the distributive property. | Create anchor chart/notes with students that document the thinking and processes behind adding, subtracting, and multiplying polynomials. | Check student understanding via oral participation. Check student work. Check for correct use of |


|  | Solve real-life problems that use the addition, subtraction, and multiplication of polynomials | Relate multiplication of polynomials to students by using real-life problems such as area problems to help students understand the concept on a different level. | graphing calculator. Classwork assigned. Homework assigned. |
| :---: | :---: | :---: | :---: |
| 7.3 Special Products | Solve real-life problems that use the addition, subtraction, and multiplication of polynomials. | Create anchor chart/notes with students that document the thinking and processes behind adding, subtracting, and multiplying polynomials. <br> Relate multiplication of polynomials to students by using real-life problems, such as area problems, to help them understand the concept on a different level. | Teacher chosen/ created worksheets/activities with higher level examples that help master the skills in this lesson. |
| 7.4 Solving Equations in Factored Form | Use the Zero Product Property. <br> Factor polynomials using the greatest common factor (GCF). <br> Solve polynomial equations by factoring. <br> Solve multi-variable formulas or literal equations, for a specific variable. <br> Factor polynomials with coefficient $=1$ and terms $b$ and $c$ are either positive or | Create a document with students that clearly depict visual and verbal models for factoring using appropriate language level or native language. <br> Use relevant contextual examples to reinforce and extend understanding of polynomials and factoring. <br> Ensure that language supports are provided such as access to word-toword dictionary, notes, anchor charts, linguistically simpler explanations and visual models. | The teacher chose/ created worksheets/activities with higher-level examples that help students master the skills in this lesson. Closure activity Classwork assigned including real world application problems <br> Homework assigned |

$\left.\begin{array}{|l|l|l|l|} & \begin{array}{l}\text { negative. } \\ \text { Factor polynomials with } \\ \text { coefficient }>1 \text { and terms } b \\ \text { and } c \text { are either positive or } \\ \text { negative. } \\ \text { Factor the difference of } \\ \text { two squares. } \\ \text { Factor perfect square } \\ \text { trinomials. } \\ \text { Solve real-life problems }\end{array} & \\ \text { involving factoring to } \\ \text { ground the concept in } \\ \text { contexts that facilitate } \\ \text { understanding. }\end{array} \quad \begin{array}{l}\text { 7.5 Factoring a=1 } \\ \\ \\ \begin{array}{l}\text { Use the Zero Product } \\ \text { Property. } \\ \text { Factor polynomials using } \\ \text { the greatest common factor } \\ \text { (GCF). } \\ \text { Solve polynomial } \\ \text { equations by factoring. } \\ \text { Solve multi-variable } \\ \text { formulas or literal } \\ \text { equations, for a specific } \\ \text { variable. } \\ \text { Factor polynomials with }\end{array} \\ \begin{array}{l}\text { Create a document with students that } \\ \text { clearly depicts visual and verbal } \\ \text { models for factoring using the } \\ \text { appropriate language level or native } \\ \text { language. } \\ \text { Use relevant contextual examples to } \\ \text { reinforce and extend understanding of } \\ \text { polynomials and factoring. } \\ \text { Ensure that language supports are } \\ \text { provided such as access to word-to- } \\ \text { word dictionary, notes, anchor charts, } \\ \text { linguistically simpler explanations and } \\ \text { visual models. }\end{array}\end{array} \begin{array}{l}\text { Teacher chose/ created } \\ \text { worksheets/activities } \\ \text { with higher-level } \\ \text { examples that help } \\ \text { master the skills in this } \\ \text { lesson. } \\ \text { Classwork assigned, } \\ \text { including real world } \\ \text { application problems } \\ \text { Homework assigned }\end{array}\right\}$

|  | coefficient $=1$ and terms $b$ and $c$ are either positive or negative. <br> Factor polynomials with coefficient $>1$ and terms $b$ and $c$ are either positive or negative. <br> Factor the difference of two squares. <br> Factor perfect square trinomials. <br> Solve real life problems involving factoring to ground the concept in contexts that facilitate understanding. |  |  |
| :---: | :---: | :---: | :---: |
| 7.6 Factoring $\mathrm{a}>1$ | Use the Zero Product Property. <br> Factor polynomials using the greatest common factor (GCF). <br> Solve polynomial equations by factoring. <br> Solve multi-variable formulas or literal equations, for a specific variable. | Create a document with students that clearly depict visual and verbal models for factoring using appropriate language level or native language. <br> Use relevant contextual examples to reinforce and extend understanding of polynomials and factoring. <br> Ensure that language supports are provided such as access to word-toword dictionary, notes, anchor charts, linguistically simpler explanations and visual models. | The teacher chose/ created worksheets/activities with higher-level examples that help students master the skills in this lesson. <br> Classwork assigned, including real-world application problems Closure activity |


|  | Factor polynomials with coefficient $=1$ and terms $b$ and $c$ are either positive or negative. <br> Factor polynomials with coefficient $>1$ and terms $b$ and $c$ are either positive or negative. <br> Factor the difference of two squares. <br> Factor perfect square trinomials. <br> Solve real life problems involving factoring to ground the concept in contexts that facilitate understanding. |  |  |
| :---: | :---: | :---: | :---: |
| 7.7 Factoring Special Products | Use the Zero Product Property. <br> Factor polynomials using the greatest common factor (GCF). <br> Solve polynomial equations by factoring. <br> Solve multi-variable formulas or literal | Create a document with students that clearly depict visual and verbal models for factoring using appropriate language level or native language. <br> Use relevant contextual examples to reinforce and extend understanding of polynomials and factoring. <br> Ensure that language supports are provided such as access to word-toword dictionary, notes, anchor charts, | Mini Quiz <br> The teacher chose/ created worksheets/activities with higher-level examples that helped master the skills in this lesson. <br> Classwork assigned including real-world application problems Homework assigned |


|  | equations, for a specific variable. <br> Factor polynomials with coefficient $=1$ and terms $b$ and $c$ are either positive or negative. <br> Factor polynomials with coefficient $>1$ and terms $b$ and $c$ are either positive or negative. <br> Factor the difference of two squares. <br> Factor perfect square trinomials. <br> Solve real life problems involving factoring to ground the concept in contexts that facilitate understanding. | linguistically simpler explanations and visual models. |  |
| :---: | :---: | :---: | :---: |
| 7.8 Factoring Completely | Use the Zero Product Property. <br> Factor polynomials using the greatest common factor (GCF). <br> Solve polynomial equations by factoring. | Create a document with students that clearly depict visual and verbal models for factoring using appropriate language level or native language. <br> Use relevant contextual examples to reinforce and extend understanding of polynomials and factoring. <br> Ensure that language supports are | The teacher chose/ created worksheets/activities with higher-level examples that helped master the skills in this lesson. <br> Classwork assigned, including real-world application problems |


|  | Solve multi-variable formulas or literal equations, for a specific variable. <br> Factor polynomials with coefficient $=1$ and terms $b$ and $c$ are either positive or negative. <br> Factor polynomials with coefficient >1 and terms $b$ and $c$ are either positive or negative. <br> Factor the difference of two squares. <br> Factor perfect square trinomials. <br> Solve real life problems involving factoring to ground the concept in contexts that facilitate understanding. | provided such as access to word-toword dictionary, notes, anchor charts, linguistically simpler explanations and visual models. | Homework assigned |
| :---: | :---: | :---: | :---: |
| 9.1 Properties of Radicals | Solve radical equations and identify extraneous solutions. <br> Solve radical equations involving square roots and cube roots. <br> Solve real life problems involving radical equations | Create a graphic organizer for students to use while engaging in independent work that highlights the how to solve radical equations. <br> Provide students with real life problems that can be modeled using radical equations. This will help students relate to the concept on a more | The teacher chose/ created worksheets/activities with higher-level examples that helped master the skills in this lesson. <br> Classwork assigned, |


|  | to ground conceptual <br> understanding | meaningful level. <br> Use assessing and advancing questions <br> to determine additional strategies and <br> support that may be needed to enhance <br> learning. | including real world <br> application problems |
| :--- | :--- | :--- | :--- |
| 9.3 Solving Using Square <br> Roots | Solve radical equations <br> involving square roots and <br> cube roots. <br> Solve real life problems <br> involving radical equations <br> to ground conceptual <br> understanding | Create a graphic organizer for students <br> to use while engaging in independent <br> work that highlights the how to solve <br> radical equations. <br> Provide students with real life <br> problems that can be modeled using <br> radical equations. This will help <br> students relate to the concept on a more <br> meaningful level. <br> Use assessing and advancing questions <br> to determine additional strategies and <br> support that may be needed to enhance <br> learning. | The teacher chose/ <br> created <br> worksheets/activities <br> with higher-level <br> examples that helped <br> master the skills in this <br> lasson. <br> including real world <br> application problems |
| 9.4 Solving by <br> Completing the Square | Solve radical equations <br> involving square roots and <br> cube roots. <br> Solve real life problems <br> involving radical equations <br> to ground conceptual <br> understanding | Create a graphic organizer for students <br> to use while engaging in independent <br> work that highlights the how to solve <br> radical equations. <br> Provide students with real life <br> problems that can be modeled using <br> radical equations. This will help <br> students relate to the concept on a more <br> meaningful level. <br> Use assessing and advancing questions <br> to determine additional strategies and <br> support that may be needed to enhance <br> learning. | Classwork assigned, <br> including real world <br> application problems |
| 9.5 Solving Using |  | Solve radical equations | Create a graphic organizer for students |$\quad$| The teacher chose/ |
| :--- |

$\left.\begin{array}{|l|l|l|l|}\text { Quadratic Formula } & \begin{array}{l}\text { involving square roots and } \\ \text { cube roots. } \\ \text { Solve real life problems } \\ \text { involving radical equations } \\ \text { to ground conceptual } \\ \text { understanding }\end{array} & \begin{array}{l}\text { to use while engaging in independent } \\ \text { work that highlights the how to solve } \\ \text { radical equations. } \\ \text { Provide students with real life } \\ \text { problems that can be modeled using } \\ \text { radical equations. This will help } \\ \text { students relate to the concept on a more } \\ \text { meaningful level. } \\ \text { Use assessing and advancing questions } \\ \text { to determine additional strategies and } \\ \text { support that may be needed to enhance } \\ \text { learning. }\end{array} & \begin{array}{l}\text { with higher-level } \\ \text { examples that helped } \\ \text { master the skills in this } \\ \text { lesson. } \\ \text { Classwork assigned, } \\ \text { including real world } \\ \text { application problems }\end{array} \\ \hline \begin{array}{ll}\text { 9.6 Systems of } \\ \text { Linear/Quadratic }\end{array} & \begin{array}{l}\text { Understand and describe } \\ \text { the meaning of a system of } \\ \text { equations in context. }\end{array} & \begin{array}{l}\text { Create a graphic organizer for students } \\ \text { to use while engaging in independent } \\ \text { work that highlights the how to solve } \\ \text { radical equations. } \\ \text { Provide students with real life } \\ \text { problems that can be modeled using } \\ \text { radical equations. This will help } \\ \text { students relate to the concept on a more } \\ \text { meaningful level. }\end{array} & \begin{array}{l}\text { The teacher chose/ } \\ \text { created } \\ \text { worksheets/activities } \\ \text { with higher-level } \\ \text { examples that helped } \\ \text { master the skills in this } \\ \text { lesson. } \\ \text { Classwork assigned, }\end{array} \\ \text { including real world } \\ \text { application problems } \\ \text { to assessing and advancing questions }\end{array}\right\}$

|  | functions and the changes that occur as the value of the lead coefficient changes. <br> Sketch graphs of parent functions and the changes that occur as the value of the constant changes. <br> Graph quadratic functions in the form of $\mathrm{fx}=\mathrm{ax} 2+\mathrm{bx}+\mathrm{c}$ and find the maximum and minimum values of the function. <br> Identify odd and even functions. <br> Develop an understanding of vertex form of a quadratic equation and how it impacts graphing. <br> Use real life situations that quadratic equations can model to make the process of graphing them more relevant. | models of quadratic functions. <br> Model the thinking that is needed when creating sketches from verbal models. | including real world application problems |
| :---: | :---: | :---: | :---: |
| 8.2 Graphing ax^2+c | Identify characteristics of quadratic functions to develop an understanding of how the graphs look and what changes in the form | students' understanding of quadratic functions by reviewing and creating a document that highlights the key features of a quadratic function and how to notice them in a graph. | The teacher chose/ created worksheets/activities with higher-level examples that helped |


|  | can tell us. <br> Sketch graphs of parent functions and the changes that occur as the value of the lead coefficient changes. <br> Sketch graphs of parent functions and the changes that occur as the value of the constant changes. <br> Graph quadratic functions in the form of $f x=a x 2+b x+c$ and find the maximum and minimum values of the function. <br> Identify odd and even functions. <br> Develop an understanding of vertex form of a quadratic equation and how it impacts graphing. <br> Use real life situations that quadratic equations can model to make the process of graphing them more relevant. | Provide opportunities to practice sketching graphs based on verbal models of quadratic functions. <br> Model the thinking that is needed when creating sketches from verbal models. | master the skills in this lesson. <br> Classwork assigned, including real world application problems |
| :---: | :---: | :---: | :---: |
| 8.3 Graphing ax^2+bx+c | Identify characteristics of quadratic functions to | students' understanding of quadratic functions by reviewing and creating a | The teacher chose/ created |


|  | develop an understanding of how the graphs look and what changes in the form can tell us. <br> Sketch graphs of parent functions and the changes that occur as the value of the lead coefficient changes. <br> Sketch graphs of parent functions and the changes that occur as the value of the constant changes. <br> Graph quadratic functions in the form of $\mathrm{fx}=\mathrm{ax} 2+\mathrm{bx}+\mathrm{c}$ and find the maximum and minimum values of the function. <br> Identify odd and even functions. <br> Develop an understanding of vertex form of a quadratic equation and how it impacts graphing. <br> Use real life situations that quadratic equations can model to make the process of graphing them more relevant. | document that highlights the key features of a quadratic function and how to notice them in a graph. <br> Provide opportunities to practice sketching graphs based on verbal models of quadratic functions. <br> Model the thinking that is needed when creating sketches from verbal models. | worksheets/activities with higher-level examples that helped master the skills in this lesson. <br> Classwork assigned, including real world application problems |
| :---: | :---: | :---: | :---: |


| 9.2 Solving by Graphing | Identify characteristics of quadratic functions to develop an understanding of how the graphs look and what changes in the form can tell us. <br> Sketch graphs of parent functions and the changes that occur as the value of the lead coefficient changes. <br> Sketch graphs of parent functions and the changes that occur as the value of the constant changes. <br> Graph quadratic functions in the form of $f x=a x 2+b x+c$ and find the maximum and minimum values of the function. <br> Identify odd and even functions. <br> Develop an understanding of vertex form of a quadratic equation and how it impacts graphing. <br> Use real life situations that quadratic equations can | Students' understanding of quadratic functions by reviewing and creating a document that highlights the key features of a quadratic function and how to notice them in a graph. <br> Provide opportunities to practice sketching graphs based on verbal models of quadratic functions. <br> Model the thinking that is needed when creating sketches from verbal models. | The teacher chose/ created worksheets/activities with higher-level examples that helped master the skills in this lesson. <br> Classwork assigned, including real world application problems |
| :---: | :---: | :---: | :---: |

model to make the process of graphing them more relevant.

## Standards

## MATH.9-12.N.RN.A. 1

MATH.9-12.A.APR.A. 1

MATH.9-12.F.BF.A.1.a

MATH.9-12.N.RN.A. 2

MATH.9-12.A.APR.B. 3

MATH.9-12.N.RN.A. 3
MATH.9-12.F.BF.A. 2

MATH.9-12.F.BF.B. 3

MATH.9-12.A.CED.A. 1

MATH.9-12.A.CED.A. 2

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Determine an explicit expression, a recursive process, or steps for calculation from a context.

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Simplify radicals, including algebraic radicals (e.g., $\sqrt[3]{54}=3 \sqrt[3]{2}$, simplify $\sqrt{ } 32 x^{2}$ ).
Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Create equations in two or more variables to represent relationships between quantities;
\(\left.$$
\begin{array}{ll}\text { MATH.9-12.F.IF.A. } 3 & \begin{array}{l}\text { Recognize that sequences are functions, sometimes defined recursively, whose domain is } \\
\text { a subset of the integers. }\end{array} \\
\text { MATH.9-12.A.REI.A. } 1 & \begin{array}{l}\text { Explain each step in solving a simple equation as following from the equality of numbers } \\
\text { asserted at the previous step, starting from the assumption that the original equation has } \\
\text { a solution. Construct a viable argument to justify a solution method. }\end{array}
$$ <br>
MATH.9-12.F.IF.B.4 <br>
For a function that models a relationship between two quantities, interpret key features of <br>
graphs and tables in terms of the quantities, and sketch graphs showing key features given <br>
a verbal description of the relationship. <br>

Use the method of completing the square to transform any quadratic equation in x into an\end{array}\right\}\)| equation of the form $(x-p)^{2}=q$ that has the same solutions. Derive the quadratic formula |
| :--- |
| from this form. |
| MATH.9-12.A.REI.B.4.b |

## Suggested Modifications for Special Education, ELL and Gifted Students

## Special Education Students:

- Visual Supports: Use graphic organizers, flowcharts, and diagrams to break down complex concepts like factoring or solving quadratic equations.
- Manipulatives: Utilize algebra tiles, blocks, or online manipulatives to help students visualize exponent rules, polynomial operations, and factoring.
- Simplified Instructions: Break down tasks into smaller, manageable steps with clear and concise language. Provide examples and model problems step-by-step.
- Modified Assessments: Offer alternative assessment formats, such as oral presentations, projects, or modified worksheets with fewer problems or larger font sizes.
- Extra Time and Support: Provide additional time for assignments and assessments, as well as individualized support or small group instruction to address specific learning needs.


## ELL Students:

- Vocabulary Support: Provide visual aids and real-world examples to help students connect new mathematical terms to familiar concepts. Use bilingual dictionaries or glossaries, and encourage the use of cognates.
- Scaffolding: Break down tasks into smaller steps with clear instructions and modeling. Use sentence frames or graphic organizers to help students organize their thoughts and communicate their understanding.
- Culturally Responsive Instruction: Incorporate examples and problems that reflect the students' cultural backgrounds and interests. Encourage collaboration and peer support among ELL students.
- Simplified Language: Use clear and concise language, avoiding idioms and complex sentence structures. Provide translated materials if available.


## Gifted Students:

- Enrichment Activities: Provide opportunities for students to explore advanced topics related to exponential and polynomial functions, such as complex numbers, higher-degree polynomials, or
applications in calculus.
- Independent Research: Encourage students to conduct independent research projects on real-world applications of exponential and polynomial functions, such as population modeling, financial investments, or engineering problems.
- Challenge Problems: Incorporate challenging problems that require students to apply their knowledge in new and creative ways, promoting critical thinking and problem-solving skills.
- Mentorship Opportunities: Connect gifted students with mentors in STEM fields to foster their interest and provide guidance for future academic and career paths.


## Suggested Technological Innovations/ Use

- Graphing Calculators/Software:
- Desmos: This free online graphing calculator is intuitive and allows students to visualize functions, explore transformations, and find key features (intercepts, vertices, asymptotes) easily. You can also create interactive activities and assignments.
- GeoGebra: Similar to Desmos, GeoGebra offers powerful graphing capabilities, as well as interactive geometry tools that can be used to explore connections between functions and geometric shapes.
- TI-84 or Similar: While not free, graphing calculators like the TI-84 are widely used in classrooms and offer a familiar interface for students. They can be used to perform calculations, graph functions, analyze data, and even program functions.


## Online Simulations and Interactive Activities:

- PhET Interactive Simulations: This free resource from the University of Colorado Boulder offers interactive simulations on exponential growth and decay, making these abstract concepts more tangible for students.
- Polynomial Function Explorer: Online tools that allow students to manipulate coefficients and observe how changes affect the shape and behavior of polynomial graphs.
- Virtual Algebra Tiles: These digital manipulatives can help students visualize the process of factoring and multiplying polynomials.


## Other Technology Tools:

- Wolfram Alpha: This computational knowledge engine can solve equations, simplify expressions, factor polynomials, and even provide step-by-step solutions. It's a valuable resource for students who need additional support or want to check their work.
- Online Whiteboards (Miro, Jamboard): These collaborative tools allow for realtime interaction and brainstorming. Students can work together on problems, share solutions, and visualize concepts using shared digital whiteboards.
- Learning Management Systems (Canvas, Google Classroom): These platforms can be used to organize course materials, assign and collect assignments, provide feedback, and facilitate communication between teachers and students.
- Khan Academy, MathPapa: These websites offer video tutorials, practice problems, and personalized learning paths for students who need extra support or want to review specific topics.


## Cross Curricular/Career Readiness, Life Literacies and Key Skills Practice

## Cross-Curricular Connections:

- Science (NJSLS-S A1-PS1-1):
- Use exponential functions to model radioactive decay and half-life. Students can research different isotopes and their applications in medicine, archaeology, or geology.
- Analyze population growth or the spread of diseases using exponential models.
- Social Studies (NJSLS-S 6.1.12.History.1.a):
- Investigate historical events involving exponential growth or decay, such as compound interest in historical economies or the spread of information through social networks.
- Financial Literacy (NJSLS 9.1.12.FP.1):
- Analyze investment options (e.g., stocks, bonds) using exponential growth models to understand the potential impact of compound interest.
- Calculate loan payments or mortgage amortization using formulas based on exponential functions.


## Career Readiness:

- STEM Fields: Highlight the use of exponential and polynomial functions in various STEM fields:
- Engineering: Design bridges or analyze the trajectory of projectiles using quadratic functions.
- Computer Science: Use polynomials to model data compression algorithms or create smooth curves for animation and graphics.
- Environmental Science: Model the growth of invasive species or the decay of pollutants using exponential functions.


## - Business and Finance:

- Analyze stock market trends and economic forecasts using exponential and polynomial models.
- Calculate depreciation rates for assets or model the growth of investments.


## Life Literacies and Key Skills (NJSLS 9.4):

- Critical Thinking and Problem Solving:
- Challenge students to solve real-world problems involving exponential and polynomial functions, such as calculating compound interest or designing a roller coaster.
- Encourage students to analyze different approaches to problem-solving and justify their reasoning.
- Communication and Collaboration (NJSLS 9.4.12.CT.1):
- Have students work in pairs or groups to explore and solve problems involving exponential and polynomial functions.
- Encourage students to present their findings to the class and engage in discussions about different solution strategies.
- Creativity and Innovation (NJSLS 9.4.12.CI.1):
- Challenge students to create their own real-world scenarios that can be modeled using exponential or polynomial functions.
- Have students design a game or activity that incorporates concepts from the unit.
- Digital Literacy (NJSLS 9.4.12.TL.1):
- Use online graphing tools like Desmos or GeoGebra to visualize exponential and polynomial functions.
- Have students research real-world applications of these functions using online resources.


## Unit 4: Graphing Exponential Functions, Quadratic Functions, \& Sequences

Content Area: Mathematics
Course(s):
Time Period: 4th Marking Period
Length: 40 days
Status: Published

## Summary of the Unit

This unit delves into the world of quadratic and radical functions, exploring their graphical representations, algebraic properties, and applications in real-world scenarios. Additionally, it touches upon the fundamentals of arithmetic and geometric sequences, providing students with a comprehensive understanding of these key mathematical concepts.

Quadratic Functions (NJSLS A1.F.IF.B, A1.F.IF.C, A1.A.REI.B):

- Graphing quadratic functions in various forms: standard ( $\mathrm{ax}^{\wedge} 2$ ), vertex $\left(\mathrm{a}(\mathrm{x}-\mathrm{h})^{\wedge} 2+\mathrm{k}\right)$, and intercept (a(x-p)(x-q)).
- Identifying key features of quadratic graphs: vertex, axis of symmetry, intercepts, and maximum/minimum values.
- Solving quadratic equations by graphing, factoring, completing the square, and using the quadratic formula.
- Comparing linear, quadratic, and exponential functions to understand their distinct characteristics and applications.
Radical Functions (NJSLS A1.F.IF.B, A1.F.IF.C):
- Graphing square root and cube root functions.
- Identifying key features of radical function graphs: domain, range, and intercepts.
- Solving radical equations algebraically and graphically.

Sequences (NJSLS A1.F.BF.A):

- Defining and identifying arithmetic and geometric sequences.
- Finding the nth term and sum of finite arithmetic and geometric series.
- Applying sequences to model real-world situations, such as compound interest and population growth.


## Enduring Understandings

Exponential Functions

- Enduring Understanding 1: Exponential functions model situations where quantities change at a constant percent rate over time. Understanding this fundamental concept enables students to analyze growth and decay phenomena in diverse contexts, from population dynamics to financial investments.
- Enduring Understanding 2: The parameters within an exponential function (base and coefficient) determine the behavior and shape of the graph, highlighting how even small changes in these values can significantly affect the modeled outcome.


## Quadratic Functions

- Enduring Understanding 1: Quadratic functions represent relationships between two quantities where one quantity varies as the square of the other. Recognizing this pattern allows students to model various phenomena, such as projectile motion and optimization problems, and make predictions based on the quadratic model.
- Enduring Understanding 2: The different forms of a quadratic equation (standard, vertex, factored) offer unique insights into the function's graph and behavior. By manipulating these forms, students can extract key information like the vertex, intercepts, and axis of symmetry, enhancing their understanding of the function's properties and applications.


## Essential Questions

- How can we use functions to model and understand real-world relationships and patterns?
- What are the key characteristics of quadratic and radical functions, and how do they differ from linear and exponential functions?
- How can we solve equations involving quadratic and radical expressions to answer questions
about real-world situations?
- What are the patterns that define arithmetic and geometric sequences, and how can we use these patterns to predict future terms and solve problems?


## Topic-Specific Questions:

- Quadratic Functions:
- How can we represent quadratic functions using graphs, tables, and equations?
- What are the key features of a parabola (vertex, axis of symmetry, intercepts), and how do they relate to the different forms of a quadratic equation?
- How can we use the graph of a quadratic function to solve quadratic equations?
- What are the different methods for solving quadratic equations, and how do we choose the most appropriate method?
Radical Functions:
- What are the domain and range restrictions of square root and cube root functions?
- How can we graph square root and cube root functions, and how do they differ from other types of functions?
- How can we solve equations involving radical expressions?
- Arithmetic and Geometric Sequences:
- What are the defining characteristics of arithmetic and geometric sequences?
- How can we find the nth term of an arithmetic or geometric sequence?
- How can we calculate the sum of a finite arithmetic or geometric series?
- How can we use sequences to model real-world situations like compound interest and population growth?


## Summative Assessment and/or Summative Criteria

Required District/State Assessments
Unit Assessments

NJSLA
SGO Assessments

Suggested Formative/Summative Classroom Assessments
Describe Learning Vertically
Identify Key Building Blocks
Make Connections (between and among key building blocks)
Short/Extended Constructed Response Items
Multiple-Choice Items (where multiple answer choices may be correct)
Drag and Drop Items
Use of Equation Editor
Quizzes
Journal Entries/Reflections/Quick-Writes
Accountable talk
Projects
Portfolio
Observation
Graphic Organizers/ Concept Mapping

Presentations

## Role Playing

Teacher-Student and Student-Student Conferencing
Homework

## Resources

Khan Academy https://www.khanacademy.org
Achieve the Core http://achievethecore.org
Illustrative Mathematics https://www.illustrativemathematics.org/
Inside Mathematics www.insidemathematics.org
Learn Zillion https://learnzillion.com
National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/vlibrary.html
Big Ideas Math https://www.bigideasmath.com/
Youcubed https://www.youcubed.org/week-of-inspirational-math/
NCTM Illuminations https://illuminations.nctm.org/Search.aspx?view=search\&type=1s\&gr=9-12
Shmoop http://www.shmoop.com/common-core-standards/math.html
Desmos https://www.desmos.com/
Geogebra http://www.geogebra.org/

CPALMS http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14
Partnership for Assessment of Readiness for College and Careers https://parcc.pearson.com/\#
McGraw-Hill ALEKS https://www.aleks.com/

Unit Plan

| Topic/Selection Timeframe | General Objectives | Instructional Activities | Benchmark / Assessments |
| :---: | :---: | :---: | :---: |
| 8.1 Graphing ax^2 | - Understand the concept of a quadratic function in the form $f(x)=a x^{\wedge} 2$. <br> - Graph quadratic functions of the form $\mathrm{f}(\mathrm{x})=$ $\mathrm{ax}^{\wedge} 2$, identifying key features such as vertex, axis of symmetry, and direction of opening. <br> - Describe the effects of the coefficient 'a' on the graph of the parabola (stretch/compression, reflectio n). <br> - Compare the graph of $f(x)=$ $\mathrm{ax}^{\wedge} 2$ to the parent function $\mathrm{f}(\mathrm{x})=\mathrm{x}^{\wedge} 2$. | - Warm-Up/Review: <br> - Review the concept of a function and how to create a table of values. <br> - Briefly discuss the parent function $f(x)=x^{\wedge} 2$ and its graph. <br> - Exploration and Discovery: <br> - Have students create tables of values and graph several quadratic functions of the form $\mathrm{f}(\mathrm{x})=\mathrm{ax} \wedge 2$, where ' a ' takes on different positive and negative values (e.g., $f(x)=2 x^{\wedge} 2, f(x)=$ $-0.5 x^{\wedge} 2$ ). <br> - Guide them to observe the patterns in the graphs: <br> - How does the value of 'a' affect the width/narrowness of the parabola? | Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this topic and review as |



|  |  | - Review the key takeaways from the lesson and assess student understanding through a quick quiz or exit ticket. <br> - Preview the next lesson, which will build on this foundation by introducing quadratic functions with additional terms (e.g., $a x^{\wedge} 2+c$ or $a x^{\wedge} 2+b x+$ c). <br> Technology Integration: <br> - Utilize graphing calculators or software (e.g., Desmos, GeoGebra) to facilitate graphing and exploration of quadratic functions. <br> - Use online tools or apps to provide interactive practice problems and immediate feedback to students. <br> Differentiation: <br> - Support: Provide pre-filled tables of values or partially completed graphs for struggling students. <br> - Challenge: Offer extension activities, such as exploring the relationship between the coefficient ' $a$ ' and the focus/directrix of a parabola. |  |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { 8.2 Graphing } \\ & \mathrm{ax} \mathrm{\wedge} 2+\mathrm{c} \end{aligned}$ | General Objectives: <br> By the end of this lesson, students should be able to: <br> - Understand the concept of a quadratic function in the form $f(x)=a x^{\wedge} 2+c$. <br> - Graph quadratic functions of | - Warm-Up/Review: <br> - Review the concepts from the previous lesson (graphing f(x) $=a x^{\wedge} 2$ ) by having students graph a few quick examples. <br> - Discuss the vertex, axis of symmetry, and the effects of 'a' | Check student graphs. Check for correct use of graphing calculator. Check |

the form $f(x)=a x^{\wedge} 2+$
c , identifying key features such as vertex, axis of symmetry, and y-intercept.

- Describe the effects of the constant term 'c' on the graph of the parabola (vertical shift).
- Compare the graph of $f(x)=$ $a x^{\wedge} 2+c$ to the parent function $f(x)=x^{\wedge} 2$ and to functions of the form $f(x)=a x^{\wedge} 2$.
on the graph.
- Exploration and Discovery:
- Have students create tables of values and graph several quadratic functions of the form $f(x)=a x^{\wedge} 2+c$, where ' $c$ ' takes on different positive and negative values (e.g., $f(x)=$ $\left.x^{\wedge} 2+3, f(x)=-2 x^{\wedge} 2-1\right)$.
- Guide them to observe the patterns in the graphs:
- How does the value of 'c' affect the position of the parabola on the $y$ axis?
- How does the sign of ' $c$ ' determine whether the vertex is above or below the x -axis?
- Encourage students to articulate their observations and generalizations in their own words.
- Direct Instruction:
- Formalize the concept of vertical shift and explain how the constant term ' c ' determines the $y$-intercept of the parabola.
- Provide explicit instructions on how to graph $f(x)=\mathrm{ax}^{\wedge} 2+$ c, emphasizing the relationship between the vertex of the parent function and the vertex of the transformed function.
translations of word problems into algebra.
Classwork assigned.
Homework assigned Assess student recall of this topic and review as needed. Check
student responses.


|  |  | Differentiation: <br> - Support: Provide scaffolding for struggling students by offering partially completed graphs or tables of values. <br> - Challenge: Encourage advanced learners to explore the relationship between the vertex form of a quadratic function and its standard form. |  |
| :---: | :---: | :---: | :---: |
| 8.3 Graphing $a x^{\wedge} 2+b x+c$ | By the end of this lesson, students should be able to: <br> - Understand the concept of a quadratic function in standard form: $f(x)=a x^{\wedge} 2+b x+c$. <br> - Graph quadratic functions in standard form, identifying key features such as vertex, axis of symmetry, y-intercept, and xintercepts (if any). <br> - Determine the vertex and axis of symmetry using the formula $\mathrm{x}=-\mathrm{b} / 2 \mathrm{a}$ and by completing the square. <br> - Connect the graph of a quadratic function to its algebraic representation in standard form. | - Warm-Up/Review: <br> - Review graphing quadratic functions in the forms $\mathrm{f}(\mathrm{x})=$ $a x^{\wedge} 2$ and $f(x)=a x^{\wedge} 2+c$. <br> - Have students identify the vertex, axis of symmetry, and y-intercept of a few examples. <br> - Exploration and Discovery: <br> - Present students with a few quadratic functions in standard form (e.g., $f(x)=x^{\wedge} 2+4 x+$ $\left.3, f(x)=-2 x^{\wedge} 2-6 x+5\right)$. <br> - Ask them to try graphing these functions using the techniques they learned in previous lessons. <br> - Discuss the challenges they encounter and lead them to realize that a new approach is needed to find the vertex and axis of symmetry. <br> - Direct Instruction: <br> - Introduce the formula $x=-$ | Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this topic and review as needed. Check student |



|  |  | student understanding through a quick quiz or exit ticket. <br> Discuss the advantages and disadvantages of different methods for graphing quadratic functions in standard form. <br> Technology Integration: <br> - Utilize graphing calculators or software (e.g., Desmos, GeoGebra) to verify graphs, explore transformations, and find key features efficiently. <br> - Use online tools or apps to provide interactive practice problems, step-bystep guidance, and immediate feedback. <br> Differentiation: <br> - Support: Provide step-by-step instructions for completing the square and offer scaffolding for struggling students. <br> - Challenge: Encourage advanced learners to explore the relationship between the discriminant and the nature of the roots of a quadratic equation. |  |
| :---: | :---: | :---: | :---: |
| 9.2 Solving by Graphing | By the end of this lesson, students should be able to: <br> - Understand the concept of solving quadratic equations by graphing. | - Warm-Up/Review: <br> - Review the concepts of quadratic functions, their graphs (parabolas), and key features (vertex, axis of | Check <br> student <br> graphs. <br> Check for correct use of graphing |

- Graph quadratic functions and identify their x -intercepts as the solutions to the corresponding quadratic equations.
- Determine the number and nature of solutions (real or imaginary) of a quadratic equation by analyzing its graph.
- Connect the graphical representation of a quadratic function to the algebraic process of solving the related quadratic equation.
symmetry, intercepts).
- Have students graph a few quadratic functions and identify their x -intercepts.
- Exploration and Discovery:
- Present students with a few quadratic equations (e.g., $x^{\wedge} 2-$ $4=0,2 x^{\wedge} 2+5 \mathrm{x}-3=0$ ).
$\circ$ Ask them to try solving these equations algebraically (factoring, quadratic formula) if they have already learned these methods.
- Then, have them graph the corresponding quadratic functions ( $\mathrm{y}=\mathrm{x}^{\wedge} 2-4, \mathrm{y}=$ $2 x^{\wedge} 2+5 x-3$ ) using graphing calculators or software.
- Guide them to observe the connection between the x intercepts of the graph and the solutions to the equations.
- Direct Instruction:
- Explain that the x-intercepts of a quadratic function's graph represent the solutions (roots) of the corresponding quadratic equation.
- Discuss the three possible scenarios:
- Two distinct real solutions (two xintercepts)
- One repeated real solution (one x-
calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this topic and review as needed. Check student responses.

|  |  | intercept, where the vertex touches the x axis) <br> - No real solutions (no xintercepts) <br> - Introduce the concept of imaginary solutions for quadratic equations with no real solutions. <br> - Practice and Application: <br> - Provide a variety of quadratic equations for students to solve by graphing. <br> - Include equations with different numbers and types of solutions. <br> - Have students practice using graphing calculators or software to find the x intercepts accurately. <br> Incorporate real-world problems that can be modeled by quadratic equations, such as projectile motion or optimization problems. <br> - Closure: <br> - Review the key takeaways from the lesson and assess student understanding through a quick quiz or exit ticket. <br> - Discuss the advantages and limitations of solving quadratic equations by graphing (e.g., visual representation but potential for inaccuracy if not |
| :---: | :---: | :---: |


|  |  | graphed precisely). <br> Technology Integration: <br> - Utilize graphing calculators or software (e.g., Desmos, GeoGebra) extensively to graph quadratic functions and find their x -intercepts. <br> - Use online tools or apps to provide interactive practice problems and immediate feedback to students. <br> Differentiation: <br> - Support: Provide step-by-step instructions for using graphing calculators/software. Offer pregraphed examples to help struggling students identify x-intercepts. <br> - Challenge: Encourage advanced learners to explore how changing the coefficients of a quadratic equation affects the number and nature of its solutions. |  |
| :---: | :---: | :---: | :---: |
| 8.4 Vertex Form | By the end of this lesson, students should be able to: <br> - Understand the vertex form of a quadratic function: $\mathrm{f}(\mathrm{x})=$ $a(x-h)^{\wedge} 2+k$ <br> - Identify the vertex ( $\mathrm{h}, \mathrm{k}$ ) and axis of symmetry ( $\mathrm{x}=\mathrm{h}$ ) directly from the vertex form. <br> - Graph quadratic functions in vertex form, recognizing the effects of the parameters $\mathrm{a}, \mathrm{h}$, and k on the | - Warm-Up/Review: <br> - Review graphing quadratic functions in standard form $\left(a x^{\wedge} 2+b x+c\right)$. <br> - Have students identify the vertex and axis of symmetry of a few examples, either by graphing or using the formula $x=-b / 2 a$. <br> - Introducing Vertex Form: <br> - Present students with a few quadratic functions in vertex | Check <br> student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork |

shape and position of the parabola.

- Convert between vertex form and standard form of a quadratic function by completing the square.
form (e.g., $f(x)=2(x-3)^{\wedge} 2+$ $\left.1, g(x)=-0.5(x+1)^{\wedge} 2-4\right)$.
- Ask them to identify any patterns they notice in the equations and how they might relate to the graphs of the functions.
- Introduce the vertex form equation and explain the roles of $\mathrm{a}, \mathrm{h}$, and k :
- a determines the direction of opening and the vertical stretch/compression of the parabola.
- (h, k) represents the coordinates of the vertex.
- $\mathrm{x}=\mathrm{h}$ is the equation of the axis of symmetry.
- Graphing from Vertex Form:
- Demonstrate how to graph a quadratic function in vertex form by starting with the vertex, then plotting points symmetrically around the axis of symmetry.
- Emphasize the connection between the values of $\mathrm{a}, \mathrm{h}$, and k and the transformations applied to the parent function $f(x)=x^{\wedge} 2$.
- Use graphing calculators or software
(e.g., Desmos, GeoGebra) to
assigned. Homework assigned Assess student recall of this topic and review as needed. Check student responses.

|  |  | visually reinforce the connection between the equation and its graph. <br> - Converting Between Forms: <br> - Show students how to convert from standard form to vertex form by completing the square. <br> Explain the reverse process of expanding vertex form to obtain standard form. <br> - Practice and Application: <br> - Provide a variety of practice problems where students graph quadratic functions given in vertex form. <br> Include problems that require converting between vertex form and standard form. <br> Incorporate real-world applications of quadratic functions, such as projectile motion, emphasizing how the vertex form can be used to easily determine maximum or minimum values. <br> - Closure: <br> - Summarize the key takeaways from the lesson and assess student understanding through a quick quiz or exit ticket. <br> Discuss the advantages of using vertex form for graphing and analyzing quadratic functions. <br> Technology Integration: |
| :---: | :---: | :---: |


|  |  | - Utilize graphing calculators or software for visualizing graphs, exploring transformations, and verifying solutions. <br> - Use online tools or apps to provide interactive practice problems and immediate feedback to students. <br> Differentiation: <br> - Support: Provide step-by-step instructions and examples for completing the square. Offer scaffolding with partially completed graphs. <br> - Challenge: Pose problems that involve complex transformations or require deriving vertex form from non-standard quadratic equations. |  |
| :---: | :---: | :---: | :---: |
| 8.5 Intercept Form | By the end of this lesson, students should be able to: <br> - Understand the intercept form of a quadratic function: $f(x)=a(x-p)(x-q)$ <br> - Identify the x -intercepts $(\mathrm{p}, 0)$ and $(\mathrm{q}, 0)$ directly from the intercept form. <br> - Graph quadratic functions in intercept form, recognizing the effects of the parameters $\mathrm{a}, \mathrm{p}$, and q on the shape and position of the parabola. <br> - Convert between intercept | - Warm-Up/Review: <br> - Review graphing quadratic functions in standard form and vertex form. <br> - Have students identify the xintercepts (if any) from the graphs of a few examples. <br> - Introducing Intercept Form: <br> - Present students with a few quadratic functions in intercept form (e.g., $f(x)=(x-2)(x+$ 3), $g(x)=-3(x+1)(x-5))$. <br> - Ask them to identify any patterns they notice in the equations and how they might | Check <br> student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned |

form and standard form of a quadratic function by expanding.

- Relate the x -intercepts to the factored form of the quadratic expression.
relate to the graphs of the functions.
- Introduce the intercept form equation and explain the roles of $\mathrm{a}, \mathrm{p}$, and q :
- a determines the direction of opening and the vertical stretch/compression of the parabola.
- p and q represent the x coordinates of the x intercepts.
- The axis of symmetry is located halfway between the x intercepts, at $\mathrm{x}=(\mathrm{p}+$ q) $/ 2$
- Graphing from Intercept Form:
- Demonstrate how to graph a quadratic function in intercept form by starting with the x intercepts, then finding the axis of symmetry and the vertex.
- Emphasize the connection between the values of $\mathrm{a}, \mathrm{p}$, and q and the graph of the parabola.
$\circ$
Use graphing calculators or software
(e.g., Desmos, GeoGebra) to visually reinforce the connection between the equation and its graph.

Assess student recall of this topic and review as needed. Check student responses.

## - Converting Between Forms:

- Show students how to convert from intercept form to standard form by expanding the expression.
- Explain how factoring a quadratic expression in standard form can lead to intercept form.
- Practice and Application:
- Provide a variety of practice problems where students graph quadratic functions given in intercept form.
- Include problems that require converting between intercept form and standard form.
- Incorporate real-world applications of quadratic functions, such as projectile motion, emphasizing how the intercept form can be used to find when an object hits the ground (x-intercepts represent time).
- Closure:
- Summarize the key takeaways from the lesson and assess student understanding through a quick quiz or exit ticket.
- Discuss the advantages of using intercept form for graphing and analyzing quadratic functions.
Technology Integration:

|  |  | - Utilize graphing calculators or software for visualizing graphs, exploring transformations, and verifying solutions. <br> - Use online tools like Desmos or GeoGebra to create interactive activities where students can manipulate parameters and observe the effects on the graph. <br> Differentiation: <br> - Support: Provide step-by-step instructions and examples for expanding and factoring quadratic expressions. Offer scaffolding with partially completed graphs. <br> - Challenge: Pose problems that involve finding intercept form from non-standard quadratic equations or situations where the x -intercepts are irrational numbers. |  |
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| 8.6 Comparing Linear/Quad/Exp | By the end of this lesson, students should be able to: <br> - Distinguish between linear, quadratic, and exponential functions based on their equations, graphs, and tables of values. <br> - Identify key features of each type of function (rate of change, constant differences/ratios, intercepts, e nd behavior). <br> - Compare and contrast the | - Warm-Up/Review: <br> - Briefly review the characteristics of linear, quadratic, and exponential functions covered in previous lessons. <br> Have students recall key features of each function type, such as slope for linear functions, vertex for quadratic functions, and growth/decay factor for exponential functions. | Check <br> student <br> graphs. <br> Check for <br> correct use <br> of graphing <br> calculator. <br> Check <br> translations <br> of word <br> problems <br> into algebra. <br> Classwork <br> assigned. |



|  |  | (constant for linear, increasing/decreasing for quadratic, exponential for exponential). <br> - Real-World Applications: <br> - Present real-world scenarios and have students determine which function type best models the situation. For example: <br> - Linear: the cost of buying multiple items at the same price. <br> - Quadratic: the trajectory of a projectile. <br> - Exponential: the growth of bacteria in a petri dish. <br> - Have students justify their choices based on the characteristics of each function type. <br> - Practice and Application: <br> - Provide a mix of problems where students identify function types from equations, graphs, and tables. - Include problems that require students to match scenarios with appropriate function types. <br> - Challenge students to create their own examples of each function type. |  |
| :---: | :---: | :---: | :---: |


|  |  | - Closure: <br> - Review the key takeaways from the lesson and assess student understanding through a quick quiz or exit ticket. <br> - Have students summarize the differences between linear, quadratic, and exponential functions in their own words. <br> Technology Integration: <br> - Utilize graphing calculators or software (e.g., Desmos, GeoGebra) to compare and contrast graphs of different function types. <br> - Use online tools or apps to provide interactive practice problems and simulations of real-world scenarios. <br> Differentiation: <br> - Support: Provide graphic organizers or tables to help students organize information about each function type. <br> - Challenge: Offer extension activities, such as exploring piecewise functions that combine different function types. |  |
| :---: | :---: | :---: | :---: |
| 10.1 Graphing Square Roots Functions | By the end of this lesson, students should be able to: <br> - Understand the concept of a square root function and its relationship to the squaring function. <br> - Graph square root functions, | - Warm-Up/Review: <br> - Review the concept of inverse functions and how to find the inverse of a function algebraically. <br> - Briefly discuss the squaring function $\mathrm{f}(\mathrm{x})=\mathrm{x}^{\wedge} 2$ and its | Check student graphs. Check for correct use of graphing calculator. |




|  |  | learners to explore the relationship between square root functions and other types of functions (e.g., quadratic, exponential). |  |
| :---: | :---: | :---: | :---: |
| 10.2 Graphing Cube Root Functions | By the end of this lesson, students should be able to: <br> - Understand the concept of a cube root function and its relationship to the cubing function. <br> - Graph cube root functions, identifying key features such as domain, range, and intercepts. <br> - Describe the transformations of the parent function $f(x)=$ $\sqrt[3]{\mathrm{x}}$ (translations, reflections, stretches, compressions). <br> - Connect the graphical representation of a cube root function to its algebraic expression. | - Warm-Up/Review: <br> - Review the concepts of square root functions and their graphs from the previous lesson. <br> - Briefly discuss the cubing function $\mathrm{f}(\mathrm{x})=\mathrm{x}^{\wedge} 3$ and its graph. <br> - Exploration and Discovery: <br> - Introduce the cube root function as the inverse of the cubing function. <br> - Have students create tables of values and graph the parent function $\mathrm{f}(\mathrm{x})=\sqrt[3]{\mathrm{x}}$. <br> - Guide them to observe the key features of the graph: <br> - Domain: All real numbers <br> - Range: All real numbers <br> - x-intercept: $(0,0)$ <br> - y-intercept: $(0,0)$ <br> - Point of inflection at $(0,0)$ <br> - Direct Instruction: <br> - Explain the concept of cube roots and how to evaluate them. <br> - Introduce the general form of a cube root function: $f(x)=a \sqrt[3]{(x}$ | Check <br> student <br> graphs. <br> Check for <br> correct use <br> of graphing <br> calculator. <br> Check <br> translations <br> of word <br> problems <br> into algebra. <br> Classwork <br> assigned. <br> Homework <br> assigned <br> Assess <br> student <br> recall of this <br> topic and <br> review as <br> needed. <br> Check <br> student <br> responses. |



|  |  | functions. <br> Technology Integration: <br> - Utilize graphing calculators or software (e.g., Desmos, GeoGebra) to visualize and explore the graphs of cube root functions. <br> - Use online tools or apps to provide interactive practice problems and immediate feedback to students. <br> Differentiation: <br> - Support: Provide scaffolding for struggling students by offering partially completed graphs or tables of values. <br> - Challenge: Encourage advanced learners to explore the relationship between cube root functions and other types of functions (e.g., polynomials, rational functions). |  |
| :---: | :---: | :---: | :---: |
| 10.3 Solving Radical Equations | By the end of this lesson, students should be able to: <br> - Understand the concept of a radical equation and the potential for extraneous solutions. <br> - Solve radical equations involving square roots and cube roots, both algebraically and graphically. <br> - Check their solutions for extraneous roots by substituting them back into the | - Warm-Up/Review: <br> - Review the concepts of square root and cube root functions, their graphs, and domain/range restrictions. <br> Have students evaluate radical expressions and solve simple equations involving radicals (e.g., $\sqrt{x}=4, \sqrt[3]{x}=-2$ ). <br> - Exploration and Discovery: <br> - Present students with a few radical equations (e.g., $\sqrt{ }(x+$ $5)=3, \sqrt[3]{(2 x-1)}=2)$. | Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. |

original equation.

- Apply their knowledge of radical equations to solve realworld problems.
- Ask them to brainstorm strategies for solving these equations, drawing upon their knowledge of solving other types of equations.
- Guide them to realize the importance of isolating the radical term and then squaring/cubing both sides.


## - Direct Instruction:

- Explain the steps involved in solving radical equations:
- Isolate the radical term.
- Square (for square roots) or cube (for cube roots) both sides of the equation.
- Solve the resulting equation.
- Check for extraneous solutions by substituting the solutions back into the original equation.
- Emphasize the importance of checking for extraneous solutions, as squaring or cubing both sides of an equation can introduce solutions that do not satisfy the original equation.
- Demonstrate how to solve radical equations both algebraically and graphically (by finding the intersection

Homework assigned Assess student recall of this topic and review as needed. Check student responses.


|  |  | - Support: Provide step-by-step instructions and scaffolding for struggling students. Offer partially completed solutions to guide their thinking. <br> - Challenge: Encourage advanced learners to explore radical equations with higher-index radicals or equations with multiple radical terms. |  |
| :---: | :---: | :---: | :---: |
| 4.6 Arithmetic Sequences | By the end of this lesson, students should be able to: <br> - Understand the concept of an arithmetic sequence and common difference. <br> - Identify arithmetic sequences and find their common difference. <br> - Determine any term of an arithmetic sequence using the explicit formula. <br> - Write an explicit formula for an arithmetic sequence. <br> - Apply arithmetic sequences to solve real-world problems. | 1. Warm-Up/Review: <br> - Review the concept of a sequence as an ordered list of numbers. <br> - Present examples of sequences and have students determine the pattern. <br> - Discuss the concept of common difference as the constant amount added to each term to get the next term in an arithmetic sequence. <br> 2. Exploration and Discovery: <br> - Present students with a few arithmetic sequences (e.g., 3 , $7,11,15, \ldots . ;-5,-1,3,7, \ldots$ ). <br> - Have them identify the common difference and predict the next few terms. <br> - Guide them to discover the recursive pattern (each term is the previous term plus the common difference) and the explicit formula (a_n = a_1 + ( $\mathrm{n}-1$ ) d ). | Check student graphs. Check for correct use of graphing calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this topic and review as needed. Check student responses. |



|  |  | from the lesson and assess student understanding through a quick quiz or exit ticket. <br> Discuss the importance of arithmetic sequences in modeling patterns in the real world. <br> Technology Integration: <br> - Utilize online graphing calculators (e.g., Desmos) or spreadsheet software (e.g., Excel) to visualize arithmetic sequences and explore patterns. <br> - Use interactive tools or apps to provide practice problems and immediate feedback to students. <br> Differentiation: <br> - Support: Provide scaffolding for struggling students by offering partially completed formulas or examples. <br> - Challenge: Encourage advanced learners to explore more complex patterns involving arithmetic sequences, such as finding the sum of an arithmetic series or solving application problems with multiple steps. |  |
| :---: | :---: | :---: | :---: |
| 6.6/6.7 <br> Geometric <br> Sequences | By the end of this lesson, students should be able to: <br> - Understand the concept of a geometric sequence and common ratio. <br> - Identify geometric sequences | Instructional Activities: <br> 1. Warm-Up/Review: <br> - Review the concept of a sequence as an ordered list of numbers. | Check student graphs. Check for correct use of graphing |

and find their common ratio.

- Determine any term of a geometric sequence using the explicit formula.
- Write an explicit formula for a geometric sequence.
- Apply geometric sequences to solve real-world problems.
- Present examples of sequences and have students determine the pattern.
- Discuss the concept of common ratio as the constant factor multiplied to each term to get the next term in a geometric sequence.

2. Exploration and Discovery:

- Present students with a few geometric sequences (e.g., 2 , $6,18,54, \ldots ; 100,50,25$, $12.5, \ldots$ ).
- Have them identify the common ratio and predict the next few terms.
- Guide them to discover the recursive pattern (each term is the previous term times the common ratio) and the explicit formula ( $\mathrm{a} \_\mathrm{n}=\mathrm{a} \_1 * \mathrm{r}^{\wedge}(\mathrm{n}-1)$ ).

3. Direct Instruction:

- Formally define geometric sequences and common ratio.
- Introduce the explicit formula $\mathrm{a} \_\mathrm{n}=\mathrm{a} \_1 * \mathrm{r} \wedge(\mathrm{n}-1)$ and explain its components:
- a_n: the nth term
- a_1: the first term
- r: the common ratio
- Demonstrate how to use the formula to find any term of a geometric sequence.
- Show students how to write an
calculator. Check translations of word problems into algebra. Classwork assigned. Homework assigned Assess student recall of this topic and review as needed. Check student responses.
$\left.\left.\begin{array}{|c|c|} & \begin{array}{l}\text { explicit formula given the first } \\ \text { term and common ratio of a } \\ \text { sequence. }\end{array} \\ \text { 4. Practice and Application: } \\ \text { o Provide a variety of practice } \\ \text { problems where students } \\ \text { identify geometric sequences, } \\ \text { find the common ratio, } \\ \text { determine specific terms, and } \\ \text { write explicit formulas. } \\ \text { o Include word problems where } \\ \text { geometric sequences model } \\ \text { real-world situations, such as } \\ \text { the growth of an investment } \\ \text { with compound interest or the } \\ \text { spread of a virus. }\end{array}\right\} \begin{array}{l}\text { Closure: } \\ \text { o Review the key takeaways } \\ \text { from the lesson and assess } \\ \text { student understanding through } \\ \text { a quick quiz or exit ticket. } \\ \text { o Discuss the importance of } \\ \text { geometric sequences in } \\ \text { modeling exponential growth } \\ \text { and decay in the real world. }\end{array}\right\}$

|  |  | - Show simulations or videos demonstrating real-world applications of geometric sequences, such as compound interest or population growth. <br> Differentiation: <br> - Support: Provide scaffolding for struggling students by offering partially completed formulas or examples. Use visual aids like diagrams or manipulatives to help students understand the concept of a common ratio. <br> - Challenge: Encourage advanced learners to explore more complex patterns involving geometric sequences, such as finding the sum of an infinite geometric series or solving application problems with multiple steps. Introduce the concept of exponential functions and connect them to geometric sequences. |
| :---: | :---: | :---: |

## Standards

Determine an explicit expression, a recursive process, or steps for calculation from a context.

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Write arithmetic and geometric sequences both recursively and with an explicit formula,

| MATH.9-12.F.BF.B.3 | Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for <br> specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. <br> Experiment with cases and illustrate an explanation of the effects on the graph using <br> technology. |
| :--- | :--- |
| MATH.9-12.A.CED.A. 1 | Create equations and inequalities in one variable and use them to solve problems. Include <br> equations arising from linear and quadratic functions, and simple rational and exponential <br> functions. |
| Create equations in two or more variables to represent relationships between quantities; |  |
| graph equations on coordinate axes with labels and scales. |  |

## Suggested Modifications for Special Education, ELL and Gifted Students

## Special Education Students

- Visual Supports: Use graphic organizers, diagrams, and manipulatives to reinforce concepts like exponential growth/decay and the shape of parabolas.
- Simplified Language: Break down complex instructions and vocabulary into smaller, more manageable chunks. Provide clear examples and model problem-solving steps explicitly.
- Modified Assessments: Offer alternative assessment formats, such as oral presentations, projects, or modified worksheets with fewer problems or larger font sizes.
- Individualized Support: Work with students one-on-one or in small groups to provide targeted instruction and address specific learning needs.


## ELL Students

- Vocabulary Support: Provide visual aids and real-world examples to help students connect new mathematical terms to familiar concepts. Use bilingual dictionaries or glossaries to facilitate language acquisition.
- Scaffolding: Break down tasks into smaller steps, providing clear instructions and modeling for each step. Use sentence frames or graphic organizers to help students organize their thoughts and communicate their understanding.
- Culturally Responsive Instruction: Incorporate examples and problems that reflect the students' cultural backgrounds and interests. Encourage collaboration and peer support among ELL students.


## Gifted Students

- Enrichment Activities: Provide opportunities for students to explore advanced topics related to exponential and quadratic functions, such as exponential regression, complex roots of quadratic equations, or applications in calculus.
- Independent Research: Encourage students to conduct independent research projects on real-world applications of exponential and quadratic functions, such as population modeling, financial investments, or engineering problems.
- Peer Tutoring: Allow gifted students to share their knowledge and understanding with their peers by leading small-group discussions or tutoring sessions.


## Suggested Technological Innovations/ Use

## Graphing Calculators or Software:

- Desmos: This free online graphing calculator is intuitive and allows students to visualize functions, explore transformations, and find key features (intercepts, vertices, asymptotes) easily. You can also create interactive activities for students.
- GeoGebra: Similar to Desmos, GeoGebra offers powerful graphing capabilities, as well as interactive geometry tools that can be used to explore connections between functions and geometric shapes.
- TI-84 or Similar: While not free, graphing calculators like the TI-84 are widely used in classrooms and offer a familiar interface for students. They can be used to perform calculations, graph functions, and analyze data.
Online Simulations and Interactive Activities:
- PhET Interactive Simulations: This free resource from the University of Colorado Boulder offers interactive simulations on exponential growth and decay, quadratic functions, and projectile motion, allowing students to manipulate variables and observe the effects in real-time.
- Math Playground: This website offers a variety of games and activities related to algebra, including those focused on exponential and quadratic functions. These can be used for reinforcement or enrichment.
- Virtual Nerd: This platform provides video tutorials, practice problems, and quizzes on various math topics, including exponential and quadratic functions. It's a great resource for independent learning and review.


## Other Technology Tools:

- Google Sheets or Excel: These spreadsheet programs can be used to create tables of values, graph functions, and perform calculations, helping students understand the numerical patterns associated with exponential and quadratic functions.
- Online Whiteboards (Miro, Jamboard): These collaborative tools allow for real-time interaction and brainstorming. Students can work together on problems, share solutions, and visualize concepts using shared digital whiteboards.
- Learning Management Systems (Canvas, Google Classroom): These platforms can be used to organize course materials, assign and collect assignments, provide feedback, and facilitate communication between teachers and students.


## Cross Curricular/Career Readiness, Life Literacies and Key Skills Practice

. Financial Literacy and Career Exploration:

- Activity: Have students research and compare different types of investments (e.g., stocks, bonds, savings accounts) and model their potential growth using exponential functions.
- Career Connection: Discuss the role of financial analysts, actuaries, and investment bankers in using exponential functions to make informed financial decisions.
- Life Literacy \& Key Skills: Students practice critical thinking and problem-solving as they evaluate different investment options, and communication skills as they present their findings to the class. They also gain an understanding of financial responsibility and the importance of long-term planning.

2. Scientific Inquiry and Data Analysis:

- Activity: Have students design and conduct an experiment to investigate exponential decay (e.g., the cooling of a hot liquid, the decay of a radioactive substance). They will collect data, analyze it using exponential regression, and create a model to predict future behavior.
- Career Connection: Highlight the use of exponential functions in various scientific fields, such as physics, chemistry, and biology, to model natural phenomena and predict outcomes.
- Life Literacy \& Key Skills: Students develop scientific inquiry skills as they formulate hypotheses, design experiments, and analyze data. They also practice critical thinking and problemsolving as they interpret results and refine their models. Additionally, they gain an appreciation for the importance of evidence-based decision-making.

