Unit 01 - Linear and Inverse Variation

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **approximately 26 days**
Status: **Published**

General Overview, Course Description or Course Philosophy

Algebra 1, the first of a sequential academic series of mathematics courses, introduces the formal theories that underlie many of the techniques learned in arithmetic. Topics include: real and rational numbers and their relationships, algebraic expressions and operations, application problems and modeling, linear equations and expressions, simultaneous equations, operations with polynomials, quadratic equations and expressions, and an introduction to rational expressions.

In this unit, students will review, extend their understanding of, and improve their skills in working with linear functions and equations. This unit also introduces concepts associated with nonlinear functions. Students will learn that Algebraic functions that represent patterns in experimental data are called mathematical models. Students will use these functions to estimate answers to questions about relationships in the data. This unit also introduces inverse variation. Students work with inverse variations in several realworld contexts. The unit also develops student understanding of associations between variables using basic ideas of correlation and two-way tables. Students will recognize and model linear and inverse variation relationships as they occur in bivariate data and will also measure variation in data, determine the strength of its assocation to the bivariate data and use their findings to make predictions with respect to what the data represents.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Essential Questions:

- How can a line of best fit be drawn with data that show a linear trend and the measure of closeness of fit can be found?
- How can scatter plots of bivariate data can be analyzed to determine the strength of the linear association between the two variables?
- How does knowing the algebraic properties of a function help to graph that function?
- How can you define a function given a set of ordered pairs, an equation or a graph of the function?
- How can you represent data patterns using graphs, tables, word descriptions, and algebraic expressions?
- How does the nature of linear functions appear in the context of a situation?
- How can mathematical models be used to answer questions about linear relationships?
- How do you write linear functions from verbal, numerical, or graphical information?
- How can data be used to make predictions?

Enduring Understandings:

- Data can be analyzed and used to make predictions.
- Correlation coefficients can describe the strength of the linear association illustrated by the scatter plot.
- Linear and inverse functions can be represented in 4 different ways
  - algebraically
  - graphically
  - numerically in tables
  - by verbal descriptions

CONTENT AREA STANDARDS

MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
MA.S-ID.B.6a	Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data.
MA.S-ID.B.6b	Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.
MA.S-ID.B.6c	Fit a linear function for a scatter plot that suggests a linear association.
MA.S-ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
MA.S-ID.C.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.
MA.S-ID.C.9	Distinguish between correlation and causation.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

MA.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

CS.6-8.8.1.8.DA.1	Organize and transform data collected using computational tools to make it usable for a specific purpose.
CS.K-12.3	Recognizing and Defining Computational Problems
CS.K-12.5	Creating Computational Artifacts
CS.K-12.6	Testing and Refining Computational Artifacts
LA.K-12.NJSLSA.L1	Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.R1	Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
WRK.K-12.P.4	Demonstrate creativity and innovation.
WRK.K-12.P.5	Utilize critical thinking to make sense of problems and persevere in solving them.
WRK.K-12.P.8	Use technology to enhance productivity increase collaboration and communicate effectively.
TECH.9.4.8.IML.4	Ask insightful questions to organize different types of data and create meaningful visualizations.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- A formula is required to describe a relationship or highlight a quantity of interest in a contextual situation.
- The graph of a function is the set of ordered pairs consisting of an input and the corresponding output
- Linear equations can contain Numerical coefficients and coefficients represented by letters (e.g., Ax + By = C)
- Functions can have multiple representations, such as verbal, recursive, or explicit symbolic rules from contextual problems.
- An efficient function exists based on the representation to fit a data set or describe the relationship between two quantities.
- A function can have different representations (e.g., graph, table, symbolic notation, verbal description)
- Graphs and tables of functions have vital features, such as intercepts, intervals (increasing, decreasing, positive, or negative, relative maximums and minimums, symmetries, end behavior, and periodicity.

- Key features given a verbal description of the relationship or a contextual situation can be used to sketch the graph.
- Various representations, such as table, graph, and symbols can be used to calculate and explain the average rate of change of a function.
- When appropriate, the rate of change from a graph of a function can be estimated.
- Categorical data for two categories in two-way frequency tables can be summarized and interpreted.
- Possible associations and trends in data on two categorical and quantitative variables by using relative frequencies can be identified.
- Variables can be related in terms of the context of the problem (e.g., using parameters and y-intercepts of the model) and associated in terms of strength and direction (e.g., strongly related, moderately related, weakly related, not related, increase, decrease)
- A function of fit can be found that represents a scatter plot
- A linear, quadratic, or exponential function can be found to the data that best models the relationship between the variables
- Limits of extrapolation for prediction in contextual situations exist.
- Technology (e.g., software, applets) can be used to illustrate relationships, determine the effects of the model and residual, and deduce interdependence
- Data sets can be reviewed to determine whether or not a linear association exists.
- Mathematical reasoning is used to make conclusions about associations.
- I can explain the slope (rate of change) and intercept (constant term) of a linear model in the context of the data can be used to make practical conclusions in contextual situations.
- The correlation coefficient (r) of a linear fit can be found using technology and can be used to make generalizations about the strength of the linear relationship between the two variables.
- There is a difference between correlation and causation and correlation does not imply causation.

Linear and Nonlinear Relationships: Recognize and model patterns in bivariate data

• Represent data patterns using graphs, tables, word descriptions, and algebraic expressions

• Investigate the nature of linear functions in contexts

• Use mathematical models to answer questions about linear relationships

• Write linear functions from verbal, numerical, or graphical information

• Analyze and solve linear equations

• Model situations with inequalities expressed as "at most" and "at least" situations

• Investigate the nature of inverse variation in contexts

• Use mathematical models to answer questions about inverse variation relationships

• Compare inverse variation relationships with linear relationships

Data Analysis: Measure variation in data and strength of association in bivariate data

• Use data to make predictions

• Fit a line to data that show a linear trend and measure closeness of fit

• Analyze scatter plots of bivariate data to determine the strength of the linear association

between the two variables

• Use correlation coefficients informal

ly to describe the strength of the linear association

il

lustrated by scatter plots

• Use standard deviation to measure variability in univariate distributions

• Distinguish between categorical and numerical variables

• Use two-way tables and analysis of cel

l frequencies and relative frequencies to decide whether

two variables are related

Looking for Pythagoras

Procedural Knowledge

Students will be able to:

- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- Determine an explicit expression, a recursive process, or steps for calculation from a context.
- Interpret key features of graphs and tables of a function that models a relationship between two quantities.
- Sketch graphs showing key features given a verbal description of the relationship.
- Calculate and interpret the average rate of change of a function.
- Summarize and interpret data on two categorical variables and recognize possible associations and trends in data.
- Use functions fitted to data to solve problems in the context of the data.
- Informally assess the fit of a function by plotting and analyzing residuals.
- Fit a linear function for a scatter plot that suggests a linear association.
- Interpret the slope and the intercept of a linear model.
- Interpret the correlation coefficient of a linear fit using technology.
- Distinguish between correlation and causation.

EVIDENCE OF LEARNING

Formative Assessments

Mathematical Reflections

Check Up 1

Check Up 2

Self Assessment Questions

Delta Math Assignments

Summative Assessments

Partner Quiz

Teacher created assessments (both test generator and teacher generated questions)

OnCourse generated assessments

Delta Math teacher generated assessments

RESOURCES (Instructional, Supplemental, Intervention Materials)

Instructional Materials

- Thinking With Math Models (All Investigations)
- Pearson-Algebra 1 Common Core
- https://www.savvasrealize.com/ (teacher and student recourses)
- Delta Math
- Illustrative Mathematics

Supplemental/Intervention Materials

- https://www.khanacademy.org/
  - Linear Equations and Graphs
  - Forms of Linear Equations
  - Intro to inverse functions
- https://illuminations.nctm.org/
  - Walk the Plank
  - Barbie Bungie

- o [Light it Up](#)
- o [Correlation and the Regression Line](#)
- o [Bathtub water levels](#)
- https://www.illustrativemathematics.org/
  - o [Alg 1.2 Linear Equations, Inequalities and systems - Lessons 1-11, 18-20](#)
  - o [Alg 1.3 Two-Variable Statistics](#)

INTERDISCIPLINARY CONNECTIONS

Data collection/analysis

Statistics

Financial/Economic/Business/Entrepenerial Literacy

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.