08-Mathematical Practices

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **Ongoing**
Status: **Published**

General Overview, Course Description or Course Philosophy

Mathematical information may be organized or presented graphically, numerically, analytically, or verbally. Mathematicians must be able to communicate effectively in all of these contexts and transition seamlessly from one representation to another. These standards must be consistently applied in order to achieve competence and fluency.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Enduring Understandings:

- Adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive discourse contribute positively to mathematical success.
- It is important to determine expressions and values using mathematical procedures and rules.
- Ihey must be able to translate mathematical information from a single representation or across multiple representations.
- Justification of reasoning and solutions is an integral component of successful work.
- Must always use correct notation, language, and mathematical conventions to communicate results or solutions.

Essential Questions:

- How do you apply reasoning with definitions and theorems?
- How do you connect concepts?
- How do you implement algebraic/computational processes?
- How so you connect multiple representations?
- How do you build notational fluency?
- How do you communicate mathematics to others?

CONTENT AREA STANDARDS

MA.9-12.MPAC	Mathematical Practices for AP Calculus (MPACs)
MA.9-12.MPAC 6	Communicating
MA.9-12.MPAC 3	Implementing algebraic/computational processes
MA.9-12.MPAC 2	Connecting concepts
MA.9-12.MPAC 1	Reasoning with definitions and theorems
MA.9-12.MPAC 4	Connecting multiple representations
MA.9-12.MPAC 2.a	relate the concept of a limit to all aspects of calculus;
MA.9-12.MPAC 4.a	associate tables, graphs, and symbolic representations of functions;
MA.9-12.MPAC 5.a	know and use a variety of notations (e.g., $f'(x)$, y', dx/dy);
MA.9-12.MPAC 6.a	clearly present methods, reasoning, justifications, and conclusions;
MA.9-12.MPAC 3.a	select appropriate mathematical strategies;
MA.9-12.MPAC 1.a	use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results.
MA.9-12.MPAC 2.b	use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;
MA.9-12.MPAC 1.b	confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem;
MA.9-12.MPAC 6.b	use accurate and precise language and notation;
MA.9-12.MPAC 3.b	sequence algebraic/computational procedures logically;
MA.9-12.MPAC 5.b	connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);
MA.9-12.MPAC 4.b	develop concepts using graphical, symbolical, verbal, or numerical representations with and without technology;
MA.9-12.MPAC 4.c	identify how mathematical characteristics of functions are related in different representations;
MA.9-12.MPAC 5.c	connect notation to different representations (graphical, numerical, analytical, and verbal); and
MA.9-12.MPAC 1.c	apply definitions and theorems in the process of solving a problem;
MA.9-12.MPAC 6.c	explain the meaning of expressions, notation, and results in terms of a context (including units);
MA.9-12.MPAC 2.c	connect concepts to their visual representations with and without technology; and
MA.9-12.MPAC 3.c	complete algebraic/computational processes correctly;
MA.9-12.MPAC 3.d	apply technology strategically to solve problems;
MA.9-12.MPAC 5.d	assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.
MA.9-12.MPAC 1.d	interpret quantifiers in definitions and theorems (e.g., “for all,” “there exists”);
MA.9-12.MPAC 2.d	identify a common underlying structure in problems involving different contextual situations.

MA.9-12.MPAC 6.d	explain the connections among concepts;
MA.9-12.MPAC 4.d	extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
MA.9-12.MPAC 4.e	construct one representational form from another (e.g., a table from a graph or a graph from given information); and
MA.9-12.MPAC 3.e	attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
MA.9-12.MPAC 1.e	develop conjectures based on exploration with technology; and
MA.9-12.MPAC 6.e	critically interpret and accurately report information provided by technology; and
MA.9-12.MPAC 3.f	connect the results of algebraic/computational processes to the question asked.
MA.9-12.MPAC 6.f	analyze, evaluate, and compare the reasoning of others.
MA.9-12.MPAC 1.f	produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.
MA.9-12.MPAC 4.f	consider multiple representations (graphical, numerical, analytical, and verbal) of a function to select or construct a useful representation for solving a problem.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.
TECH.8.1.12.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- Applying problem solving steps and strategies based on the context can increase efficiency and accuracy while assisting with the development of conceptual understanding
- Applying mathematical tools based on the context can increase efficiency and accuracy while assisting with the development of conceptual understanding
- Mathematical modeling and varied representations of mathematical situations assists with the development of conceptual understanding

Procedural Knowledge

Students will be able to:

Apply reasoning with definitions and theorems by

- using definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;
- confirming that hypotheses have been satisfied in order to apply the conclusion of a theorem;
- applying definitions and theorems in the process of solving a problem;
- interpreting quantifiers in definitions and theorems (e.g., “for all,” “there exists”);
- developing conjectures based on exploration with technology; and
- producing examples and counterexamples to clarify understanding of definitions, investigate whether converses of theorems are true or false, or to test conjectures

Connect concepts by:

- relating the concept of a limit to all aspects of calculus;
- using the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;
- connecting concepts to their visual representations with and without technology; and
- identifying a common underlying structure in problems involving different contextual situations.

Implement algebraic/computational processes by:

- selecting appropriate mathematical strategies;
- sequencing algebraic/computational procedures logically;
- completing algebraic/computational processes correctly;
- applying technology strategically to solve problems;
- attending to precision graphically, numerically, analytically, and verbally and specify units of measure; and
- connecting the results of algebraic/computational processes to the question asked

Connect multiple representations by:

- associating tables, graphs, and symbolic representations of functions;
- developing concepts using graphical, symbolical, verbal, or numerical representations with and without technology;
- identifying how mathematical characteristics of functions are related in different representations;
- extracting and interpreting mathematical content from any presentation of a function (e.g., utilize information from a table of values);
- constructing one representational form from another (e.g., a table from a graph or graph from given information); and
- considering multiple representations (graphical, numerical, analytical, and verbal of a function to select or construct a useful representation for solving a problem.

Build notational fluency by:

- knowing and use a variety of notations
- connecting notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);
- connecting notation to different representations (graphical, numerical, analytical, and verbal); and assigning meaning to notation, accurately interpreting the notation in a given problem and across different content

Communicate by:

- clearly presenting methods, reasoning, justifications, and conclusions;
- using accurate and precise language and notation;
- explaining the meaning of expressions, notation, and results in terms of a contex (including units);
- explaining the connections among concepts;
- critically interpreting and accurately reporting information provided by technology; and
- analyzing, evaluating, and comparing the reasoning of others.

EVIDENCE OF LEARNING

Formative Assessments

- Teacher observations, student-student discussions, conferences
- Explain the process used to solve the problem
- Identify the theorems and defintions used in order to solve the problem

Summative Assessments

N/A

RESOURCES (Instructional, Supplemental, Intervention Materials)

- TI-84 Graphing calculator;
- Teacher designed worksheets
- Calculus Early Transcendentals, Anton, Bivens, and Davis
- Calculus, Farrand and Poxon

- https://tutorial.math.lamar.edu/
- solutions at https://www.slader.com/textbook/9780470647691-calculus-early-transcendentals-10th-edition/
- https://ia801309.us.archive.org/23/items/Calculus10thEditionH.Anton/Calculus%2010th%20edition%20H.%20Anton.pdf

INTERDISCIPLINARY CONNECTIONS

Mathematics is used in many other disciplines in order to develop hypotheses and to substantiatiate conclusions.

These disciplines include:

- Astronomy
- Biology
- Economics
- Engineering
- Medicine
- Physics

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.