# 07-Parametric and Polar Functions

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **12 Blocks**
Status: **Published**

## General Overview, Course Description or Course Philosophy

This unit is an introductios to Parametric and Polar functions that are alternatives to the Rectangular (Cartesian) representation of curves.

Graphs of functions must pass the vertical line test, a limitation that excludes curves with self-intersections or even such basic curves as circles. In this section we will study an alternative method for describing curves algebraically that is not subject to the severe restriction of the vertical line test. We will then derive formulas required to find slopes, tangent lines, and arc lengths of these parametric curves.

Sometimes a moving point has a special affinity for some fixed point, such as a planet moving in an orbit under the central attraction of the Sun. In such cases, the path of the particle is best described by its angular direction and its distance from the fixed point. In this section we will discuss the Polar coordinate system that is based on this idea.

## OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Enduring Understandings:

- A parametric function is a set of parametric equations (x = x(t) and y = y(t)) that describe a curve on the x-y plane.  That curve is not necessarily a function in rectangular form of y = f(x).
- A polar function (r = f(theta)) is defined by a distance r from a fixed point (pole) and an angular rotation theta from the polar axis in a counter clockwise direction.  Many complex curves can be described very simply in polar form but not in rectangular form.
- Frequently, it will be useful to superimpose a rectangular xy-coordinate system on top of a polar coordinate system, making the positive x-axis coincide with the polar axis. If this is done, then every point P will have both rectangular coordinates (x, y) and polar coordinates
- A point has a unique description in rectangular form.  That same point has an infinite number of descriptors in polar form.
- The techniques and applications of differentiation and integration can be extended from rectangular functions to parametric and polar functions.

Essential Quesitons:

- How do you create a parametric graph?
- How do you find dy/dx of a function in parametric form?
- How do you write the equation of a tangent line at a given point of a function in parametric form?
- How do you use derivatives to explain the features of the graph of a function in parametric form?

- Hoq do you find the length of a parmetric function?
- How do you create a polar graph?
- How do you find dy/dx of a function in polar form?
- How do you write the equation of a tangent line at a given point of a function in polar form?
- How do you use derivatives to explain the features of the graph of a function in polar form?
- Hoq do you find the length of a polar function?

## CONTENT AREA STANDARDS

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|---|---|
| MA.9-12.EK 2.1C7 | (BC) Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates. |
| MA.9-12.EK 2.2A4 | (BC) For a curve given by a polar equation $r = f(\theta)$, derivatives of $r$, $x$, and $y$ with respect to θ and first and second derivatives of $y$ with respect to $x$ can provide information about the curve. |
| MA.9-12.EK 3.4D1 | Areas of certain regions in the plane can be calculated with definite integrals. (BC) Areas bounded by polar curves can be calculated with definite integrals. |
| MA.9-12.EK 3.4D3 | (BC) The length of a planar curve defined by a function or by a parametrically defined curve can be calculated using a definite integral. |
| MA.9-12.MPAC 6.d | explain the connections among concepts; |
| MA.9-12.MPAC 3.f | connect the results of algebraic/computational processes to the question asked. |
| MA.F-IF.C | Analyze functions using different representations |

## RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

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|---|---|
| CRP.K-12.CRP2 | Apply appropriate academic and technical skills. |
| CRP.K-12.CRP4 | Communicate clearly and effectively and with reason. |
| CRP.K-12.CRP6 | Demonstrate creativity and innovation. |
| CRP.K-12.CRP8 | Utilize critical thinking to make sense of problems and persevere in solving them. |
| CRP.K-12.CRP12 | Work productively in teams while using cultural global competence. |
| TECH.8.1.12.E | Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information. |
| TECH.8.1.12.F | Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources. |

## STUDENT LEARNING TARGETS

## Declarative Knowledge

Students will understand that:

- A curve can be described by either a Cartesian or a Parametric representation.
- Parametric derivatives are an extension of derivatives of functions in rectangular form.
- Length of a parametric function is an extension of length of a function in rectangular form.
- A curve can be describe by eithe a Cartesian or a Polar representation.
- Some families of polar functions are identified by name.
- Polar derivatives are extension of derivatives in rectangular form.
- Polar areas are an extension of areas enclosed by functions in rectangular form.
- Length of a function in polar form is an extension of length of a function in rectangular form.

## Procedural Knowledge

Students will be able to:

- Convert from parametric to rectangular form
- Create a parametric function graph from a chart
- Use a graphing calculator to graph a parametric function
- Compute parametric derivatives
- Use derivatives to explain the features of the parametric graph
- Use a graphing calculator to compute the length of a parametric function
- Convert between polar and rectangular forms.
- Plot points on a polar graph.
- Create a polar function graph from a chart.
- Use a graphing calculator to graph a polar function
- Compute polar derivatives
- Use a graphing calculator to compute polar areas
- Use a graphing calculator to compute length of a polar function

## EVIDENCE OF LEARNING

## Formative Assessments

- Marzano Scales
- Explain the process of converting from parametric to rectangular form.
- Explain the process of converting from rectangular form to polar form.
- Explain the process of converting from polar form to rectangular form.
- Identify the common types of polar functions.
- Why do we use symmetries when computing polar areas.
- Summarize (and any question as well as its answer you had) what was covered in class today.
- Homework

## Summative Assessments

Test

## RESOURCES (Instructional, Supplemental, Intervention Materials)

- TI-84 Graphing calculator;
- Teacher designed worksheets
- Calculus Early Transcendentals, Anton, Bivens, and Davis
- Calculus, Farrand and Poxon
- https://tutorial.math.lamar.edu/
- https://mathworld.wolfram.com/Cardioid.html
- solutions at https://www.slader.com/textbook/9780470647691-calculus-early-transcendentals-10th-edition/
- https://ia801309.us.archive.org/23/items/Calculus10thEditionH.Anton/Calculus%2010th%20edition%20H.%20Anton.pdf

## INTERDISCIPLINARY CONNECTIONS

All examples are from the last link in the resources. Add 24 to the cited page number in order to go directly to the page.

- Aerodynamics Paper Airplane Flight: page 696, Example 5
- Physics Lissajous Curve: page 703, Problem 55
- Engineering Amusment Park Rides (Scrambler/Calypso): page 704, Problem 73
- Involute of a Circle Walking a Dog: page 704, Problem 74
- Science: page 713, Spirals in Nature
- Science: page 717, Problem 56 Butterfly Curves
- Astronomy: p 719, Problem 75
- Appllications of Conic Sections: page 743, Optics

- Applications of Conic Sections:  page 743, Architecture
- Applications of Conic Sections:  page 743, Navigation
- Astronomy:  page 759, Kepler's Laws
- Astronomy Haley's Comet:  page 760, Example 4
- Astronomy Lunar Orbit:  page 761, Example 5
- Astonomy Orbits Pluto:  page 762, Problem 23
- Astronomy Orbits Mercury:  page 762, Problem 24
- Astronomy Orbits Hale Bott Comet:  page 762, Problem 25
- Astronomy Orbits Mars:  page 763, Problem 26
- Astronomy Orbits Vanguard I around Earth:  page 763, Problem 27
- Astonomy Orbits Space Ship Gallileo around Jupiter:  page 763, Problem 28

## ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.