06-Applications of Integrals: Areas, Volumes, Average Value of a Function, Arc Length

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **16 Blocks**
Status: **Published**

General Overview, Course Description or Course Philosophy

Students will learn how to find the average value of a function, model particle motion and net change, and determine areas, volumes, and arc length. When finding areas or volumes, they must be able to sketch the curve(s) and to find the points of intersection, if necessary, to determine the limits of integration. Many times the TI-84 graphing calculator will be used to accomplish this and to complete the numerical solution.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Enduring Understandings:

- The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.
- Given the velocity of a particle and an initial condition, displacement can be determined as well as total distanced travelled.
- The area between curves is computed by integrating the difference of the "larger" and the "smaller" functions with appropriate limits of integration.
- Not all solids of revolution have solid interiors; some have holes or channels that create interior surfaces.
- The **average value** of a function on an interval is the height of a rectangle that has the same width as the interval and has the same area as the function on that interval. It is computed by multiplying the integral of f(x) from a to b by 1/(b-a).
- Finding cross sectional volumes is accomplished by the accumulation of the areas of the cross sections.
- Finding the length of a curve is an extension of the Pythagorean Theorem.

Essential Questions:

- Explain displacement and total distance as the accumulation of velocity.
- What do you have to determine to find the area between curves? Why is a sketch helpful?
- What do you have to determine to find a volume of revolution? Why is a sketch helpful?
- Why would you want to compute the average value of a function?

CONTENT AREA STANDARDS

MA.9-12.EK 3.4A1	A function defined as an integral represents an accumulation of a rate of change.
MA.9-12.EK 3.4A2	The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
MA.9-12.EK 3.4A3	The limit of an approximating Riemann sum can be interpreted as a definite integral.
MA.9-12.EK 3.4B1	The average value of a function f over an interval $[a, b]$ is $1/(b-a) \int$ (from a to b) $f(x)dx$.
MA.9-12.EK 3.4C1	For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
MA.9-12.EK 3.4C2	(BC) The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve given by parametric or vector-valued functions.
MA.9-12.EK 3.4D1	Areas of certain regions in the plane can be calculated with definite integrals. (BC) Areas bounded by polar curves can be calculated with definite integrals.
MA.9-12.EK 3.4D2	Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
MA.9-12.EK 3.4E1	The definite integral can be used to express information about accumulation and net change in many applied contexts.
MA.9-12.EU 3.4	The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.
MA.9-12.LO 3.4A	Interpret the meaning of a definite integral within a problem.
MA.9-12.LO 3.4B	Apply definite integrals to problems involving the average value of a function.
MA.9-12.LO 3.4C	Apply definite integrals to problems involving motion.
MA.9-12.LO 3.4D	Apply definite integrals to problems involving area, volume, (BC) and length of a curve.
MA.9-12.LO 3.4E	Use the definite integral to solve problems in various contexts.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.
TECH.8.1.12.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills

to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- A function defined as an integral represents an accumulation of a rate of change.
- The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
- The limit of an approximating Riemann sum can be interpreted as a definite integral
- The average value of a function f(x) on an interval from a to b can be computed by (1/(b – a))(integral from a to b of f(x))
- For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time
- Areas of certain regions in the plane can be calculated with definite integrals.
- Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
- The definite integral can be used to express information about accumulation and net change in many applied contexts.

Procedural Knowledge

Students will be able to:

- Determine the integral to compute the area
- Find the average value of a function
- Find arc length of a function in rectangular form
- Determine the integral to compute the volume of revolution using washers.
- Determine the integral to compute the volume of a solid with known cross section
- Solve PVA problems
- Use a graphing calculator to complete the computation

EVIDENCE OF LEARNING

Formative Assessments

- Marzano Scales
- Exit Slips
- Explain upper and lower curves
- Explain left and right curves
- Why is a sketch useful in determining the integral?
- Why why is it important to find points of intersection?
- Compare the difference in the set up of the integrals when rotating around a horizontal line to rotating around a vertical line
- Summarize (and any question as well as its answer you had) what was covered in class today.
- Homework

Summative Assessments

- Construction of a cross section model.
- Topic Tests

RESOURCES (Instructional, Supplemental, Intervention Materials)

- TI-84 Graphing calculator;
- Teacher designed worksheets
- Calculus Early Transcendentals, Anton, Bivens, and Davis
- Calculus, Farrand and Poxon
- https://tutorial.math.lamar.edu/
- solutions at https://www.slader.com/textbook/9780470647691-calculus-early-transcendentals-10th-edition/
- https://ia801309.us.archive.org/23/items/Calculus10thEditionH.Anton/Calculus%2010th%20edition%20H.%20Anton.pdf

INTERDISCIPLINARY CONNECTIONS

All examples are from the last link in the resources. Add 24 to the cited page number in order to go directly to the page.

- Racing: page 415, Example 3
- Racing: page 420, Problem 46
- Population: page 420, Problen 47
- Medicine: page 420, Problem 48

- Geometry: page 423, Example 1
- Geometry: page 425, Exanple 2
- Geometry: page 426, Example 3
- Geometry: page 427, Example 5
- Engineering: page 430, Problem 50
- Basketball: page 442, Problem 22
- Engineering: page 442, Problem 23
- Golf: page 443, Problem 25

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.