05-Infinite Sequences and Series: Tests for Convergence

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **12 Blocks**
Status: **Published**

General Overview, Course Description or Course Philosophy

In this unit, student will discover that a sum of infinitely many terms may converge to a finite value. Intuition can be developed by exploring graphs, tables, and symbolic expressions for series that converge and diverge. The tests will include the Divergence Test, the Integral Test, the Limit Comparison Test, the Root Test, the Ratio Test, and the Comparison Test. Absolute or Conditional Convergence will be determined for Altenating Series.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Enduring Understandings:

- It is possible for the sum of an infinite number of terms to be bounded.
- The nth partial sum is defined as the sum of the first n terms of a sequence.
- An infinite series of numbers converges to a real number (or has sum), if and only if the limit of its sequence of partial sums exists and equals .
- A series may be absolutely convergent, conditionally convergent, or divergent. If a series converges absolutely, then it converges.
- In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the nth term test, the comparison test, the limit comparison test, the integral test, the Root Test, the ratio test, and the alternating series test.

Essential Questions:

- Why is the condition the limit of the nth term goes to zero as n goes to infinity a necessary condition for convergence but not a sufficient condition?
- How do you identify the comon series of numbers include geometric series, the harmonic series, and p-series?
- How do you identify the features of the nth-term in order to select an appropriate test for convergence?
- Why is the sufficient condition for the convergence of an alternating series so much weaker than that of a series of all positive terms?
- What is thee difference between absolute and conditional convergce?

CONTENT AREA STANDARDS

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

F.TF

A. Extend the domain of trigonometric functions using the unit circle

B. Model periodic phenomena with trigonometric functions

C. Prove and apply trigonometric identities

MA.9-12.4	Series (BC)
MA.9-12.EK 4.1A1	The nth partial sum is defined as the sum of the first n terms of a sequence.
MA.9-12.EK 4.1A2	An infinite series of numbers converges to a real number S (or has sum S), if and only if the limit of its sequence of partial sums exists and equals S.
MA.9-12.EK 4.1A3	Common series of numbers include geometric series, the harmonic series, and p-series.
MA.9-12.EK 4.1A4	A series may be absolutely convergent, conditionally convergent, or divergent.
MA.9-12.EK 4.1A5	If a series converges absolutely, then it converges.
MA.9-12.EK 4.1A6	In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the nth term test,

	the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.
MA.9-12.EK 4.1B1	FORMULA: If a is a real number and r is a real number such that IrI < 1, then the geometric series Σ [n = 0 to ∞] $ar^n = a/(1 - r)$.
MA.9-12.EK 4.1B2	If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series.
MA.9-12.EK 4.1B3	If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value.
MA.9-12.EU 4.1	The sum of an infinite number of real numbers may converge.
MA.9-12.LO 4.1A	Determine whether a series converges or diverges.
MA.9-12.LO 4.1B	Determine or estimate the sum of a series.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.
TECH.8.1.12.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- The nth partial sum is defined as the sum of the first n terms of a sequence.
- An infinite series of numbers converges to a real number (or has sum), if and only if the limit of its sequence of partial sums exists and equals S.
- Common series of numbers include geometric series, the harmonic series, and p-series.
- A series may be absolutely convergent, conditionally convergent, or divergent.
- If a series converges absolutely, then it converges.
- In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the nth term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.

- Find the sum of a convergent geometric series.

Procedural Knowledge

Students will be able to:

- Compute the limit of a sequence
- Find the sum of an infinite geometric series
- Apply the Divergence Test for infinite series
- Apply the Integral Test for infinite series
- Apply the Limit Comparison Test for infinite series
- Apply the Root Test for infinite series
- Apply the Ratio Test for infinite series
- Apply the Comparison Test for infinite series

EVIDENCE OF LEARNING

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Formative Assessments

- Exit Slips
- Marzano Scales
- Explain why a sequence with multiple cluster points does not have a limit.
- How do you determine which test for convergence is appropriate?
- Summarize (and any question as well as its answer you had) what was covered in class today.
- Homework

Summative Assessments

Cumulative Tests

RESOURCES (Instructional, Supplemental, Intervention Materials)

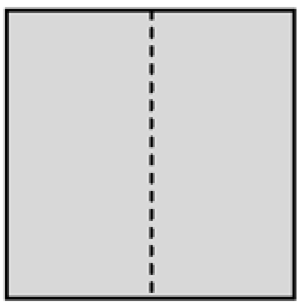
- Core Instructional Materials
    - Calculus Early Transcendentals, Anton, Bivens, and Davis
    - Calculus, Farrand and Poxon
    - Solutions at
        - https://www.slader.com/textbook/9780470647691-calculus-early-transcendentals-10th-edition/
        - https://ia801309.us.archive.org/23/items/Calculus10thEditionH.Anton/Calculus%2010th%20edition%20H.%20Anton.pdf

  Supplemental Materials

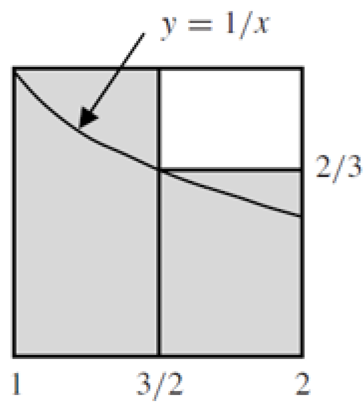
    - TI-84 Graphing calculator
    - Teacher designed worksheets
    - https://tutorial.math.lamar.edu/classes/calci/calci.aspx
    - https://www.khanacademy.org/math/old-ap-calculus-ab/ab-limits-continuity
- AProofWithoutWords

Proof Without Words: The Alternating Harmonic Series Sums to ln 2

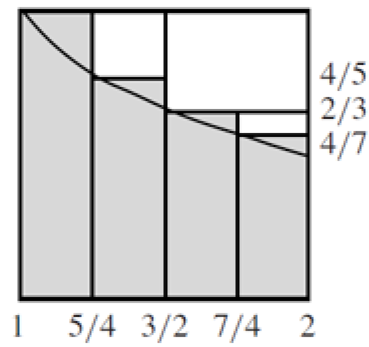
CLAIM. $\sum_{n=0}^{\infty}(-1)^n \frac{1}{n+1} = \ln 2.$



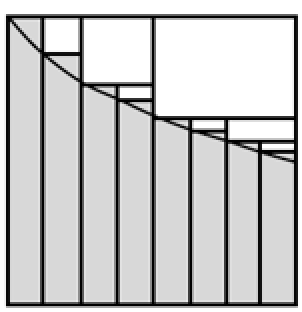
$1 \cdots$



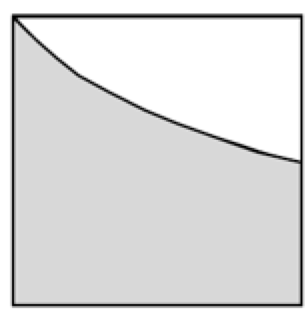
$-\frac{1}{2} + \frac{1}{3} - \cdots$



$-\frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \cdots$



$-\frac{1}{8} + \frac{1}{9} - \frac{1}{10} + \frac{1}{11} - \frac{1}{12} + \frac{1}{13} - \frac{1}{14} + \frac{1}{15} - \cdots$



$= \int_1^2 \frac{1}{x}\,dx = \ln 2$

—Matt Hudelson
Washington State University
Pullman WA 99164

Summary We demonstrate graphically the result that the alternating harmonic series sums to the natural logarithm of two. This is accomplished through a sequence of strategic replacements of rectangles with others of lesser area. In the limit, we obtain the region beneath the curve $y = 1/x$ and above the x-axis between the values of one and two.

INTERDISCIPLINARY CONNECTIONS

All examples are from the last link in the resources. Add 24 to the cited page number in order to go directly to the page.

- Beverton-Holt Model for Population Change from One Gereration to Another: page 613, Problems 29 and 30
- Playtine - Bouncing Ball: page 622, Problem 27
- Volume of an Infinite Staircase: page 622, Problem 28
- Algebra: sum of alternating harmonic series: page 640, example 2 and A Proof Without Words in the Resources

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.