04-Antiderivatives/Integrals: Integration Techniques, Fundamental Theorem of Calculus, Integration by Parts, Improper Integrals

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **20 Blocks**
Status: **Published**

General Overview, Course Description or Course Philosophy

Integration determines accumulation of change over an interval. The rules of differentiation are the basis of antidiffentiation so they are crucial prerequisite knowledge. The integral is the limiting case of the partioned area between the curve y = f(x) and the x-axis or x = g(y) and the y-axis as the number of partitions goes to infinity. Many integrals can be evaluated by applying the techniques of antidifferentiation. Not all functions have a closed form antideritive.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Enduring Understandings:

- Antidifferentiation is the inverse process of differentiation.
- Antiderivatives ask the question: what function has this derivative?
- The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.
- Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
- In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
- Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
- The definition of the definite integral may be extended to functions with removable or jump discontinuities.
- (BC) An improper integral is an integral that has one or both limits at infinity.
- (BC) Improper integrals can be determined using limits of definite integrals.
- The definite integral can be used to define new functions: If f(x) is a continuous function on the interval [a, b] defined by an integral of f(x) from 0 to x then the derivative is f(x)
- Many functions do not have closed form antiderivatives.
- Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, (BC) integration by parts
- The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.

Essential Questions:

- How do you recognize an elementary type of integrand?
- How do you determine if all of the requisite parts are present in an integrand?
- Explain the two parts of the Fundamental Theorem of Calculus.
- How do you apply integration by parts? What processes do you have to be able to do in order to use it?
- How do you use the techiques of integration, limits, and the Fundamental Theorem to evaluate improper integrals?

CONTENT AREA STANDARDS

MA.9-12.3	Integrals and the Fundamental Theorem of Calculus
MA.9-12.EK 3.1A1	An antiderivative of a function f is a function g whose derivative is f.
MA.9-12.EK 3.1A2	Differentiation rules provide the foundation for finding antiderivatives.
MA.9-12.EK 3.2A1	A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
MA.9-12.EK 3.2A2	The definite integral of a continuous function f over the interval $[a, b]$, denoted by ∫ (from a to b) $f(x)dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is ∫ (from a to b) $f(x)dx$ = lim [as maxΔx_i approaches 0] Σ [i = 1 to n] $f(x_i*)\Delta x_i$, where x_i* is a value in the ith subinterval, Δx_i is the width of the ith subinterval, n is the number of subintervals, and maxΔx_i is the width of the largest subinterval. Another form of the definition is ∫ (from a to b) $f(x)dx$ = lim [as n approaches ∞] Σ [i = 1 to n] $f(x_i*)\Delta x_i$,, where $\Delta x_i = (b - a)/n$ and x_i* is a value in the ith subinterval.
MA.9-12.EK 3.2A3	The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
MA.9-12.EK 3.2B1	Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.
MA.9-12.EK 3.2B2	Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
MA.9-12.EK 3.2C1	In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
MA.9-12.EK 3.2C2	Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.

MA.9-12.EK 3.2C3	The definition of the definite integral may be extended to functions with removable or jump discontinuities
MA.9-12.EK 3.2D1	(BC) An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration.
MA.9-12.EK 3.2D2	(BC) Improper integrals can be determined using limits of definite integrals.
MA.9-12.EK 3.3A1	The definite integral can be used to define new functions; for example, $f(x)$ = ∫ (from 0 to x) e [superscript $-t^2$] dt.
MA.9-12.EK 3.3A2	If f is a continuous function on the interval $[a, b]$, then d/dx (∫ (from a to x) $f(t)dt$) = $f(x)$, where x is between a and b.
MA.9-12.EK 3.3A3	Graphical, numerical, analytical, and verbal representations of a function provide information about the function g defined as $g(x)$ = ∫ (from a to x) $f(t)dt$.
MA.9-12.EK 3.3B1	The function defined by $F(x)$ = ∫ (from a to x) $f(t)dt$ is an antiderivative of f.
MA.9-12.EK 3.3B2	If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then ∫ (from a to b) $f(x)dx = F(b) - F(a)$.
MA.9-12.EK 3.3B3	The notation $\int f(x)dx = F(x) + C$ means that $F'(x) = f(x)$, and $\int f(x)dx$ is called an indefinite integral of the function f.
MA.9-12.EK 3.3B4	Many functions do not have closed form antiderivatives.
MA.9-12.EK 3.3B5	Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, (BC) integration by parts, and nonrepeating linear partial fractions.
MA.9-12.EU 3.1	Antidifferentiation is the inverse process of differentiation.
MA.9-12.EU 3.2	The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.
MA.9-12.EU 3.3	The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.
MA.9-12.LO 3.1A	Recognize antiderivatives of basic functions.
MA.9-12.LO 3.2B	Approximate a definite integral.
MA.9-12.LO 3.2C	Calculate a definite integral using areas and properties of definite integrals.
MA.9-12.LO 3.2D	(BC) Evaluate an improper integral or show that an improper integral diverges.
MA.9-12.LO 3.3A	Analyze functions defined by an integral.
MA.9-12.LO 3.2Aa	Interpret the definite integral as the limit of a Riemann sum.
MA.9-12.LO 3.2Ab	Express the limit of a Riemann sum in integral notation.
MA.9-12.LO 3.3Ba	Calculate antiderivatives.
MA.9-12.LO 3.3Bb	Evaluate definite integrals.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

TECH.8.1.12.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- An antiderivative of a function is a function whose derivative is the integrand.
- Differentiation rules provide the foundation for finding antiderivatives.
- A Riemann sum, which requires a partition of an interval is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
- The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
- Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.
- Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
- In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
- Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
- The definition of the definite integral may be extended to functions with removable or jump discontinuities.
- (BC) An improper integral is an integral that has one or both limits at infinity.
- (BC) Improper integrals can be determined using limits of definite integrals.
- The definite integral can be used to define new functions: If f(x) is a continuous function on the interval [a, b] defined by an integral of f(x) from 0 to x then the derivative is f(x)
- Many functions do not have closed form antiderivatives.
- Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, (BC) integration by parts

Procedural Knowledge

Students will be able to:

- Convert a sum into sigma notation
- Find the value of a sum in sigma notatio
- Approximate area under a curve using rectangles and left endpoints, right endpoints, or midpoints
- Approximate area under a curve using the Trapezoidal Rule
- Write the limit of a Riemann Sum as a definite integral
- Use areas to evaluate integrals
- Find antiderivative using power rules
- Find antiderivative of 1/x
- Find the antiderivative of trigonometric and exponential functions
- Find antiderivatives that require a trig identity
- Recognize the requisite parts for a closed form integral and complete the integration
- Evaluate definite integrals using the FTC
- Calculate derivatives of integrals using the FTC
- Solve antiderivatives by applying integration by parts
- Identify an integrand as a square root or inverse sine and complete the integration
- Identify an integrand as an ln or an inverse tangent and complete the integration
- Complete solutions for improper integrals

EVIDENCE OF LEARNING

Formative Assessments

- Exit Slips
- Marzano Scales
- Explain why antiderivatives have an infinite number of solutions
- Explain why a missing coefficient is not a problem when finding an antiderivative
- Explain why a missing inner derivative is a problem when finding an antiderivative; ie, why an integral can fail to have a closed form solution?
- Summarize (and any question as well as its answer you had) what was covered in class today
- Homework

Summative Assessments

Cumulative Tests

RESOURCES (Instructional, Supplemental, Intervention Materials)

- TI-84 Graphing calculator;
- Teacher designed worksheets
- Calculus Early Transcendentals, Anton, Bivens, and Davis
- Calculus, Farrand and Poxon
- https://tutorial.math.lamar.edu/
- solutions at https://www.slader.com/textbook/9780470647691-calculus-early-transcendentals-10th-edition/
- https://ia801309.us.archive.org/23/items/Calculus10thEditionH.Anton/Calculus%2010th%20edition%20H.%20Anton.pdf

INTERDISCIPLINARY CONNECTIONS

All examples are from the last link in the resources. Add 24 to the cited page number in order to go directly to the page.

- Area: page 321, Quick Check 1
- Area: page 347, Example 6
- Integrating Rates of Change: page 371, Physics
- Integrating Rates of Change: page 371, Population
- Integrating Rates of Change: page 371, Engineering
- Integrating Rates of Change: page 371, Economics

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.