03-Applications of Derivatives: Graphing, Optimization, Related Rates, PVA, L'Hopital's Rule, Intermediate Value Theorem, Extreme Value Theorem, Rolle's Theorem, Mean Value Theorem

Content Area:	**Math**
Course(s):	
Time Period:	**Full Year**
Length:	**24 Blocks**
Status:	**Published**

General Overview, Course Description or Course Philosophy

Using derivatives to describe the rate of change of one variable with respect to another variable allows student s to understand change in a variety of contexts.

Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyzing the gra ph of a function
(for example,determining whether a function is increasing or decreasing and finding concavity and extremeval ues),
and solving problems involving rectilinearmotion, and be familiar with a variety of real
world applications including related rates and optimization.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Enduring Understandings:

- A function's derivative, which is itself a function, can be used to understand the behavior of the function.
- The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.
- The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
- The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
- The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
- The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
- The derivative can be used to express information about rates of change in applied contexts.
- L'Hopital's Rule can be used to evaluate the indeterminate limit forms zero/zero or infinity/infinity
- The Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval.

Essential Questions:

- How are problems about position, velocity, and acceleration of a particle in motion over time structurally similar to problems about the volume of a rising balloon over an interval of heights, the population of London over the 14th century, or the metabolism of a dose of medicine over time?
- How is the first derivative of f(x) used to determine relative maximums and minimums of f(x)?
- How is the first derivative of f(x) used to determine intervals an which f(x) is increasing/decreasing?
- How is the second derivative of f(x) used to determine where f(x) is concave up/down?
- How are the first derivative of f(x) and the Extreme Value Theorem used to find the absolute maximum and minimum of f(x)?
- How do you solve an optimization problem?
- How do you solve a related rates problem?
- What are the relationships of position, velocity, and acceleration?
- When can L'Hopital's Rule be used to evaluate an indeterminate limit?

CONTENT AREA STANDARDS

MA.9-12.2	Derivatives
MA.9-12.EK 1.1C3	Limits of the indeterminate forms 0/0 and ∞/∞ may be evaluated using L'Hospital's Rule.
MA.9-12.EK 1.2B1	Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.
MA.9-12.EK 2.2A1	First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
MA.9-12.EK 2.2A2	Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
MA.9-12.EK 2.2A3	Key features of the graphs of f, f' and f'' are related to one another.
MA.9-12.EK 2.2B1	A continuous function may fail to be differentiable at a point in its domain.
MA.9-12.EK 2.2B2	If a function is differentiable at a point, then it is continuous at that point.
MA.9-12.EK 2.3A1	The unit for $f'(x)$ is the unit for f divided by the unit for x.
MA.9-12.EK 2.3A2	The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
MA.9-12.EK 2.3B1	The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
MA.9-12.EK 2.3B2	The tangent line is the graph of a locally linear approximation of the function near the point of tangency.

MA.9-12.EK 2.3C1	The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
MA.9-12.EK 2.3C2	The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
MA.9-12.EK 2.3C3	The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
MA.9-12.EK 2.3D1	The derivative can be used to express information about rates of change in applied contexts.
MA.9-12.EK 2.4A1	If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b), the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.
MA.9-12.EU 2.2	A function's derivative, which is itself a function, can be used to understand the behavior of the function.
MA.9-12.EU 2.3	The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.
MA.9-12.EU 2.4	The Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval.
MA.9-12.LO 2.2A	Use derivatives to analyze properties of a function.
MA.9-12.LO 2.2B	Recognize the connection between differentiability and continuity.
MA.9-12.LO 2.3B	Solve problems involving the slope of a tangent line.
MA.9-12.LO 2.3C	Solve problems involving related rates, optimization, rectilinear motion, (BC) and planar motion.
MA.9-12.LO 2.3D	Solve problems involving rates of change in applied contexts.
MA.9-12.LO 2.4A	Apply the Mean Value Theorem to describe the behavior of a function over an interval.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.
TECH.8.1.12.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
- Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
- Key features of the graphs of function, its first derivative, and its second derivative are related to one another.
- A continuous function may fail to be differentiable at a point in its domain.
- If a function is differentiable at a point, then it is continuous at that point.
- A continuous function may fail to be differentiable at a point in its domain.
- If a function is differentiable at a point, then it is continuous at that point.
- The unit for the derivative is the unit of the function divided by the unit of x.
- The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
- The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
- The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
- The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
- The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
- The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
- The derivative can be used to express information about rates of change in applied contexts.
- L'Hopital's Rule can be used to evaluate the indeterminate limit forms zero/zero or infinity/infinity.

Procedural Knowledge

Students will be able to:

- Graph with intercepts, limits, first derivative chart, second derivative chart, relative maximums/minimums polynomial, rational, exponential, logarithmic, and trigonometric functions
- Solve problems involving the slope of a tangent line
- Identify key words in a problem
- Draw a diagram when appropriate
- Solve optimization problems
- Find an expression for the quantity to be maximized/minimized
- Identify any restrictions
- Use differentiation techniques to determine critical points
- Answer questions within physical constraints
- Identify unchanging quantities

- Identify the mathematical relationship among the quantities
- Use implicit differentiation to solve the related rates problem
- Solve PVA problems
- Apply The Intermediate Value Theorem
- Apply The Extreme Value Theorem
- Apply Rolle's Theorem
- Apply The Mean Value Theorem
- Solve indeterminate forms by the application of L'Hopital's Rule

EVIDENCE OF LEARNING

Formative Assessments

- Marzano Scales
- Exit Slips
- Summarize (and any question as well as its answer you had) what was covered in class today.
- Given a function, explain the process of creating a graph.
- Given limits, intercepts, first and second derivative charts of a polynomial function, sketch a possible graph of the function.
- Given an optimization problem, develop a solution and give a detailed explanation of the process.
- How do you identify key features of a problem?
- Given a related rates problem, develop a solution and give a detailed explanation of the process.
- Homework

Summative Assessments

Topic tests

RESOURCES (Instructional, Supplemental, Intervention Materials)

- TI-84 Graphing calculator;
- Teacher designed worksheets
- Calculus Early Transcendentals, Anton, Bivens, and Davis

- Calculus, Farrand and Poxon
- solutions at https://www.slader.com/textbook/9780470647691-calculus-early-transcendentals-10th-edition/
- https://tutorial.math.lamar.edu/
- https://ia801309.us.archive.org/23/items/Calculus10thEditionH.Anton/Calculus%2010th%20edition%20H.%20Anton.pdf

INTERDISCIPLINARY CONNECTIONS

All examples are from the last link in the resources. Add 24 to the cited page number in order to go directly to the page.

- Enviromental Science: page 205, example 2
- Baseball: page 206, example 3
- Photography: page 206, example 4
- Engineering: page 207, example 5
- Farming: page 275, example 1
- Manufactoring: page 276, example 2
- Engineering: page 277, example 3
- Geometry: page 278, example 4

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.