02-Derivatives: Differentiation Techniques, Implicit Differentiation, Logarithmic Differentiation

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **20 Blocks**
Status: **Published**

General Overview, Course Description or Course Philosophy

A derivative is the instantaneous rate of change. The derivative is defined as the limit of the average rate of change of a function over an interval as the length of the interval goes to zero. Analytical rules for computing basic derivatives are presented. They are then extended to composite functions. The importance of identifying the type of function before applying a derivative rule is paramount. Implicit differentiation is a technique that is employed when it is relation is not convient or possible to express it as y = f(x). Implicit Differentiation is used to find the derivative of an inverse function. Logarithmic differention together with implicit differentiation is used to find a the derivative of a function with a variable in both the base and the the power.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Enduring Understandings:

The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.

- Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
- The chain rule provides a way to differentiate composite functions.
- The chain rule is the basis for implicit differentiation.
- The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
- The derivative of y = (f(x))^g(x) is found by using the Properties of Logarithms and Implicit Differentiation.
- Differentiating the first derivative produces the second derivative provided that it exists and so on.

Essential Questions:

- Explain the definition of the derivative.
- Compare the average rate of change to the instantaneous rate of change.

- How do you determine the appropriate derivative rule?
- Explain what features must be identified in a composite function and the steps needed to find the derivative using the chain rule.
- Why are the Product and Quotinet Rules crucial in the process of implicit diffentiation?
- What is the definition of an inverse function and how is it used to find the derivative using implicit differentiation?
- Explain how the properties of logarithms and implicit differentionn are used to find the derivative of y = (f(x))^g(x).

CONTENT AREA STANDARDS

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

F.TF

A. Extend the domain of trigonometric functions using the unit circle

B. Model periodic phenomena with trigonometric functions

C. Prove and apply trigonometric identities

MA.9-12.2	Derivatives
MA.9-12.EK 2.1A1	The difference quotients $[f(a+h)-f(a)] / h$ and $[f(x)-f(a)] / (x-a)$ express the average rate of change of a function over an interval.
MA.9-12.EK 2.1A2	The instantaneous rate of change of a function at a point can be expressed by lim [as h approaches 0] $[f(a+h)-f(a)] / h$ or lim [as x approaches a] $[f(x)-f(a)] / (x-a)$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
MA.9-12.EK 2.1A3	The derivative of f is the function whose value at x is lim [as h approaches 0] $[f(x+h)-f(x)] / h$ provided this limit exists.
MA.9-12.EK 2.1A4	For $y = f(x)$, notations for the derivative include dy/dx, $f'(x)$, and y'.
MA.9-12.EK 2.1A5	The derivative can be represented graphically, numerically, analytically, and verbally.
MA.9-12.EK 2.1B1	The derivative at a point can be estimated from information given in tables or graphs.
MA.9-12.EK 2.1C1	Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
MA.9-12.EK 2.1C2	Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
MA.9-12.EK 2.1C3	Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
MA.9-12.EK 2.1C4	The chain rule provides a way to differentiate composite functions.
MA.9-12.EK 2.1C5	The chain rule is the basis for implicit differentiation.
MA.9-12.EK 2.1C6	The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
MA.9-12.EK 2.1D1	Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f.
MA.9-12.EK 2.1D2	Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include d^2y/dx^2, $f''(x)$ and y''. Higher order derivatives can be denoted d^ny/dx^n or $f^{(n)}(x)$.
MA.9-12.EU 2.1	The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.
MA.9-12.LO 2.1A	Identify the derivative of a function as the limit of a difference quotient.
MA.9-12.LO 2.1B	Estimate derivatives.
MA.9-12.LO 2.1C	Calculate derivatives.
MA.9-12.LO 2.1D	Determine higher order derivatives.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

9.1.12.PB.6: Describe and calculate interest and fees that are applied to various forms of spending, debt and saving.

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.

CRP.K-12.CRP11	Use technology to enhance productivity.
TECH.8.1.12.E.CS3	Evaluate and select information sources and digital tools based on the appropriateness for specific tasks.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- The difference quotients and express the average rate of change of a function over an interval.
- The instantaneous rate of change of a function at a point can be expressed by or provided that the limit exists.
- A derivative of the function whose value at is provided by the limit of the difference quotient provided the limit exists.
- There are various notations for the derivative of a function.
- The derivative can be represented graphically, numerically, analytically, and verbally.
- The derivative at a point can be estimated from information given in tables or graphs.
- Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
- The chain rule provides a way to differentiate composite functions.
- The chain rule is the basis for implicit differentiation.
- The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
- Differentiating the first derivative produces the second derivative provided that it exists and so on.
- Higher order derivatives are represented with a variety of notations.

Procedural Knowledge

Students will be able to:

- Calculate the average value of a function
- Calculate derivatives
- Use definition to find derivative of f(x)=c, f(x)=mx+b, f(x)=ax2+bx+c, f(x)=ax3, f(x)=(ax+b)^(1/2), f(x)=c/(ax+b)
- Execute power rule to find derivatives
- Find derivatives of trigonometric, exponential, and logarithmic functions

- Find derivative of composite functions using chain, product and quotient rules
- Identify derivatives using implicit differentiation
- Find derivatives using logarithmic differentiation
- Find derivatives of inverse trig functions
- Determine higher order derivatives
- Estimate derivatives

EVIDENCE OF LEARNING

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Alternate Assessments

- Portfolios
- Verbal Assessment (instead of written)
- Multiple choice
- Modified Rubrics
- Performance Based Assessments

Formative Assessments

- Marzano Scales
- Exit slips
- Explain the relationship of the average rate of change and the instantaneous rate of change using a graph.

- Explain the relationship of the sign of the derivative to the graph of the function Compare the chain rule to the child's nested egg toy.
- Explain why in implicit differentiation you must include dy/dx but you do not have to include dx/dx.
- Summarize (and any question as well as its answer you had) what was covered in class today.
- Homework

Summative Assessments

Cumulative tests

RESOURCES (Instructional, Supplemental, Intervention Materials)

- Core Instructional Materials
    - Calculus Early Transcendentals, Anton, Bivens, and Davis
    - Calculus, Farrand and Poxon
    - Solutions at
        - https://www.slader.com/textbook/9780470647691-calculus-early-transcendentals-10th-edition/
        - https://ia801309.us.archive.org/23/items/Calculus10thEditionH.Anton/Calculus%2010th%20edition%20H.%20Anton.pdf

  Supplemental Materials

    - TI-84 Graphing calculator
    - Teacher designed worksheets
    - https://tutorial.math.lamar.edu/classes/calci/calci.aspx
    - https://www.khanacademy.org/math/old-ap-calculus-ab/ab-limits-continuity

INTERDISCIPLINARY CONNECTIONS

Average rate of change and instaneous rate of change (the derivative)

- Velocity can be viewed as rate of change—the rate of change of position with respect to
- time.
- Rates of change occur in other applications as well. For example:
- A microbiologist might be interested in the rate at which the number of bacteria in a colony changes with time.
- An engineer might be interested in the rate at which the length of a metal rod changes with temperature.

- An economist might be interested in the rate at which production cost changes with the quantity of a product that is manufactured.
- A medical researcher might be interested in the rate at which the radius of an artery changes with the concentration of alcohol in the bloodstream

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.