01-Limits and Continuity

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **12 Blocks**
Status: **Published**

General Overview, Course Description or Course Philosophy

The concept of a limit is essential for the understanding of definitions, formulas, and theorems in calculus.

Students must understand the concept and to be able to apply techniques for finding the limit of a function as it approaches a point or at infinity.

The concepts of continuity at a point and over an interval will be explored.

The different types of discontinuity will be identified.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Enduring Understandings:

- The concept of a limit can be used to understand the behavior of functions.
- A limit is evaluated close to a domain value not at the domain value.
- Limits can be evaluated using analytical, graphical, and/or tabular representations.
- Continuity is a key property of functions that is defined using limits.

Essential Questions:

- Why is it important to represent limits analytically using correct notation?
- Compare evaluating limits in analytical representation to those in graphical representation to those in tablular representation.
- How do you determine the correct technique needed to evaluate a limit?
- Explain the Squeeze Theorem.
- How do you determine continuity of a fuction at a point or on an interval?
- Explain the Intermediate Value Theorem: (Intermediate-Value Theorem: If f is continuous on a closed interval [a, b] and k is any number between f (a) and f (b), inclusive, then there is at least one number x in the interval [a, b] such that f (x) = k.)
- Explain all components of the definition of continuity.
- What are the types of discontinuity?
- Explain the Intermediate Value Theorem: (Intermediate-Value Theorem: If f is continuous on a closed interval [a, b] and k is any number between f (a) and f (b), inclusive, then there is at least one number x in the interval [a, b] such that f (x) = k.) Why is continuity of the function a nessasay condition of the theorem?

CONTENT AREA STANDARDS

MA.9-12.1	Limits
MA.9-12.EK 1.1A1	Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is lim [as x approaches c] $f(x) = R$.
MA.9-12.EK 1.1A2	The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
MA.9-12.EK 1.1A3	A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
MA.9-12.EK 1.1B1	Numerical and graphical information can be used to estimate limits.
MA.9-12.EK 1.1C1	Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
MA.9-12.EK 1.1C2	The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
MA.9-12.EK 1.1C3	Limits of the indeterminate forms 0/0 and ∞/∞ may be evaluated using L'Hospital's Rule.
MA.9-12.EK 1.1D1	Asymptotic and unbounded behavior of functions can be explained and described using limits.
MA.9-12.EK 1.1D2	Relative magnitudes of functions and their rates of change can be compared using limits.
MA.9-12.EK 1.2A1	A function is continuous at $x = c$ provided that $f(c)$ exists, lim [as x approaches c] $f(x)$ exists, and lim [as x approaches c] $f(x) = f(c)$.
MA.9-12.EK 1.2A2	Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
MA.9-12.EK 1.2A3	Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
MA.9-12.EK 1.2B1	Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.
MA.9-12.EU 1.1	The concept of a limit can be used to understand the behavior of functions.
MA.9-12.EU 1.2	Continuity is a key property of functions that is defined using limits.
MA.9-12.LO 1.1B	Estimate limits of functions.
MA.9-12.LO 1.1C	Determine limits of functions.
MA.9-12.LO 1.1D	Deduce and interpret behavior of functions using limits.
MA.9-12.LO 1.2A	Analyze functions for intervals of continuity or points of discontinuity.
MA.9-12.LO 1.2B	Determine the applicability of important calculus theorems using continuity.
MA.9-12.LO 1.1Aa	Express limits symbolically using correct notation.
MA.9-12.LO 1.1Ab	Interpret limits expressed symbolically.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.
TECH.8.1.12.E.CS3	Evaluate and select information sources and digital tools based on the appropriateness for specific tasks.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will know that:

- Given a function f(s), the limit of as x approaches a is a real number R if can be made arbitrarily close to R by taking sufficiently close to a (but not equal to).
- The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
- A limit might not exist for some functions at particular values of a. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right. Numerical and graphical information can be used to estimate limits.
- Numerical and graphical information can be used to estimate limits. Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
- The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
- Asymptotic and unbounded behavior of functions can be explained and described using limits.
- Relative magnitudes of functions and their rates of change can be compared using limits.
- The Intermediate Value Theorem guarantees that a function will attain a given value on a closed interval. (Intermediate-Value Theorem) If f is continuous on a closed interval [a, b] and k is any number between f (a) and f (b), inclusive, then there is at least one number x in the interval [a, b] such that f (x) = k.

Procedural Knowledge

Students will be able to:

- Determine limits by direct evaluation
- Use algebraic techniques for indeterminate forms
- Find limits using a graphing calculator
- Find limits using tabular information
- Create graphs of piecewise functions
- Find limits of piecewise functions
- Determine continuity
- Identify types of discontinuity.
- Apply the Intermediate Value Theorem.

EVIDENCE OF LEARNING

Formative Assessments

- Marzano Scales
- Explain what a limit means and use a graph to illustrate.
- Explain why a function may not be defined at a point, but the limit may exist at that point.
- Explain and give examples of why 0/0, infinity/infinity, infinity-infinity are indeterminate forms.
- Summarize (and any question as well as its answer you had) what was covered in class today.
- Homework

Summative Assessments

Cumuative tests

RESOURCES (Instructional, Supplemental, Intervention Materials)

- TI-84 Graphing calculator
- Teacher designed worksheets

- Calculus Early Transcendentals, Anton, Bivens, and Davis
- Calculus, Farrand and Poxon
- https://tutorial.math.lamar.edu/classes/calci/calci.aspx
- https://www.khanacademy.org/math/old-ap-calculus-ab/ab-limits-continuity
- solutions at https://www.slader.com/textbook/9780470647691-calculus-early-transcendentals-10th-edition/
- https://ia801309.us.archive.org/23/items/Calculus10thEditionH.Anton/Calculus%2010th%20edition%20H.%20Anton.pdf

INTERDISCIPLINARY CONNECTIONS

All examples are from the last link in the resources. Add 24 to the cited page number in order to go directly to the page.

- Physics: Sky diver: page 99 problem 65
- Physics: Ohm's Law: page 109 problem 75
- Physics: Continuity in applications: page 111
- Algebra: Approximating Roots: page 116 example 5
- Economics: Compounded Continuously Interest: page 129 problem 21

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.