

07 - Introduction to Derivatives

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **4 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

The study of Precalculus comes between the study of Algebra2/Trig and Calculus. This course develops many new and rigorous techniques for the analysis and application of various types of functions and equations. The course begins with an expansion of the study of Trigonometry to included trigonometric functions, trigonometric equations, and analytic trigonometry through the use of trigonometric identities. Then an understanding of vectors is developed and applied to study the complex number system. Students are exposed to polar graphing and polar equations. A review of some fundamental functions and their properties along with the application of parametric equations lays a foundation for more advanced study in Calculus. The concept and properties of limits and established and applied to further analyze various functions. The derivative is defined and computational techniques and properties established. Several applications of derivatives such as: optimization, related rates, and graphical analysis are examined. Lastly, the calculation and application of an antiderivative is introduced.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

- Understand and Apply the definition of a derivative
- Describe how the derivative is related to the instantaneous rate of change and slope of a tangent line.
- Recognize and appropriately use notation related to derivatives.
- Understand and Apply various rules for differentiation such as: the power rule, sum and difference rule, product rule, quotient rule, constant multiple rule, and chain rule to determine the derivative of various functions.

Essential Questions:

- What is a derivate?
- How can one use the definition of a derivative to calculate the derivative of a function?
- How and when are the different rules for differentiation used to compute derivatives?
- What information about a fuction can be determined from its derivative?

CONTENT AREA STANDARDS

A.CED

A. Create equations that describe numbers or relationships

A.REI

- A. Understand solving equations as a process of reasoning and explain the reasoning**
- B. Solve equations and inequalities in one variable**
- C. Solve systems of equations**
- D. Represent and solve equations and inequalities graphically**

A.SSE

- A. Interpret the structure of expressions**
- B. Write expressions in equivalent forms to solve problems**

HS Functions

F.BF

- A. Build a function that models a relationship between two quantities**
- B. Build new functions from existing functions**

F.IF

- A. Understand the concept of a function and use function notation**
- B. Interpret functions that arise in applications in terms of the context**
- C. Analyze functions using different representations**

F.LE

- A. Construct and compare linear and exponential models and solve problems**
- B. Interpret expressions for functions in terms of the situation they model**

F.TF

- A. Extend the domain of trigonometric functions using the unit circle**
- B. Model periodic phenomena with trigonometric functions**
- C. Prove and apply trigonometric identities**

MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.9-12.EU 2.1	The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.
MA.9-12.EU 2.2	A function's derivative, which is itself a function, can be used to understand the behavior of the function.
MA.9-12.EU 2.3	The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.
MA.9-12.EU 2.4	The Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

NJSLS-CLKS

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a)

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

9.4.12.DC.7: Evaluate the influence of digital communities on the nature, content and responsibilities of careers, and other aspects of society (e.g., 6.1.12.CivicsPD.16.a).

9.4.12.DC.8: Explain how increased network connectivity and computing capabilities of everyday objects allow for innovative technological approaches to climate protection.

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an

intended audience (e.g., S-ID.B.6b, HS-LS2-4).

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
LA.RI.11-12.3	Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
LA.RI.11-12.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
LA.RI.11-12.7	Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.D.CS1	Advocate and practice safe, legal, and responsible use of information and technology.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.

TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.2.12.C.4	Explain and identify interdependent systems and their functions.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.
TECH.8.2.12.D.CS3	Assess the impact of products and systems.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- The difference quotients $[f(a + h) - f(a)] / h$ and $[f(x) - f(a)] / (x - a)$ express the average rate of change of a function over an interval.
- The instantaneous rate of change of a function at a point can be expressed by \lim [as h approaches 0] $[f(a + h) - f(a)] / h$ or \lim [as x approaches a] $[f(x) - f(a)] / (x - a)$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
- The derivative of f is the function whose value at x is \lim [as h approaches 0] $[f(x + h) - f(x)] / h$ provided this limit exists.
- For $y = f(x)$, notations for the derivative include dy/dx , $f'(x)$, and y' .
- The derivative can be represented graphically, numerically, analytically, and verbally. 👍👎💬
- The derivative at a point can be estimated from information given in tables or graphs.
- Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
- The chain rule provides a way to differentiate composite functions.
- The chain rule is the basis for implicit differentiation.
- The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
- Differentiating f' produces the second derivative f'' , provided the derivative of f' exists; repeating this process produces higher order derivatives of f .
- Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include d^2y/dx^2 , $f''(x)$ and y'' . Higher order derivatives can be denoted $d^n y/dx^n$ or $f^{(n)}(x)$.
- First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
- Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
- A continuous function may fail to be differentiable at a point in its domain.

- If a function is differentiable at a point, then it is continuous at that point.

Procedural Knowledge

Students will be able to:

- Identify the derivative of a function as the limit of a difference quotient.
- Estimate derivatives.
- Calculate derivatives.
- Determine higher order derivatives.
- Use derivatives to analyze properties of a function.
- Recognize the connection between differentiability and continuity.
- Interpret the meaning of a derivative within a problem.
- Solve problems involving the slope of a tangent line.

EVIDENCE OF LEARNING

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Alternative Assessments

- Portfolios
- Verbal Assessment (instead of written)
- Multiple choice
- Modified Rubrics
- Performance Based Assessments

Formative Assessments

- Student feedback/questioning/observation
- Exit Ticket
- Error analysis
- Specific skill assessment/questions
- Survey/polling
- Reflection questions
- Scored/evaluated class work or homework
- Task completion

Summative Assessments

- Lesson Quizzes
- Unit Test
- Performance Tasks

RESOURCES (Instructional, Supplemental, Intervention Materials)

Core Instructional Resources

Textbook - Calculus AP Edition: Finney, et al. ISBN 0-13-201408-4

Supplemental Resources

Internet based resources such as:

[Khan Academy](#)

[Albert.IO](#)

[DeltaMath](#)

Teacher produced materials

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby solve and analyze situations taken from business, physics, engineering, biology, statistics, geography, and numerous other fields. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.