

05 - Conics and Parametric Equations

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **4 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

The study of Precalculus comes between the study of Algebra2/Trig and Calculus. This course develops many new and rigorous techniques for the analysis and application of various types of functions and equations. The course begins with an expansion of the study of Trigonometry to included trigonometric functions, trigonometric equations, and analytic trigonometry through the use of trigonometric identities. Then an understanding of vectors is developed and applied to study the complex number system. Students are exposed to polar graphing and polar equations. A review of some fundamental functions and their properties along with the application of parametric equations lays a foundation for more advanced study in Calculus. The concept and properties of limits and established and applied to further analyze various functions. The derivative is defined and computational techniques and properties established. Several applications of derivatives such as: optimization, related rates, and graphical analysis are examined. Lastly, the calculation and application of an antiderivative is introduced.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

In this unit we define various conic sections and analyze their equations and graphs. Then we examine parametric equations, which give orientation along a plane curve. Students will develop an understanding of how and why a second degree equation in two variables corresponds to a conic sections (parabola, ellipse, or hyperbola). Students will develop techniques for graphing these conics. Students will recognize and describe parametric equations and use them to model projectile motion.

Essential Questions:

How do the graphs/equations of conics relate to other common equations/graphs?

What features of a conic can be analyzed from its graph/equation?

How are parametric equations related to non-parametric equations?

What types of situations are well suited to be modeled by a parametric equation?

CONTENT AREA STANDARDS

A.CED

A. Create equations that describe numbers or relationships

A.REI

A. Understand solving equations as a process of reasoning and explain the reasoning

B. Solve equations and inequalities in one variable

C. Solve systems of equations

D. Represent and solve equations and inequalities graphically

A.SSE

A. Interpret the structure of expressions

B. Write expressions in equivalent forms to solve problems

HS Functions

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

F.TF

A. Extend the domain of trigonometric functions using the unit circle

B. Model periodic phenomena with trigonometric functions

C. Prove and apply trigonometric identities

MA.F-BF.B	Build new functions from existing functions
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-LE.A	Construct and compare linear and exponential models and solve problems
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
MA.A-CED.A	Create equations that describe numbers or relationships
MA.G-GPE.A.2	Derive the equation of a parabola given a focus and directrix.
MA.G-GPE.A.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

NJSLS-CLKS

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a)

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

9.4.12.DC.7: Evaluate the influence of digital communities on the nature, content and responsibilities of careers, and other aspects of society (e.g., 6.1.12.CivicsPD.16.a).

9.4.12.DC.8: Explain how increased network connectivity and computing capabilities of everyday objects allow for innovative technological approaches to climate protection.

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an

intended audience (e.g., S-ID.B.6b, HS-LS2-4).

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6).

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
LA.RI.11-12.3	Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
LA.RI.11-12.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
LA.RI.11-12.7	Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
LA.RI.11-12.10a	By the end of grade 11, read and comprehend literary nonfiction at grade level text-complexity or above with scaffolding as needed.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.C.CS2	Communicate information and ideas to multiple audiences using a variety of media and formats.
TECH.8.1.12.D.CS1	Advocate and practice safe, legal, and responsible use of information and technology.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.

TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.2.12.C.4	Explain and identify interdependent systems and their functions.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.
TECH.8.2.12.D.CS3	Assess the impact of products and systems.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- Each conic is the intersection of a plane and a cone
- The equations of the three conics are related to the same general form of a second degree equation in two variables
- Conics have foci, a directrix, and axis of symmetry
- A circle is a particular ellipse
- The result of increasing the value of a the parameter reveal the orientation of a curve or the direction of motion along it
- Time is often used as a parameter in parametric equations

Procedural Knowledge

Students will be able to:

- Define and classify conics
- Graph Conics
- Identify key features of a conic from its equation and/or graph
- Write the equation of various conics
- Solve application problems involving conics
- Graph parametric equations
- Generate an equation in rectangular form that corresponds to a graph defined parametrically
- Generate a parametric equation for the graph that is defined by an equation in rectangular form

EVIDENCE OF LEARNING

Alternate Assessments

- Portfolios
- Verbal Assessment (instead of written)
- Multiple choice
- Modified Rubrics
- Performance Based Assessments

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Formative Assessments

- Student feedback/questioning/observation
- Exit Ticket
- Error analysis
- Specific skill assessment/questions
- Survey/polling
- Reflection questions
- Scored/evaluated class work or homework
- Task completion

Summative Assessments

- Lesson Quizzes
- Unit Test

- Performance Tasks

RESOURCES (Instructional, Supplemental, Intervention Materials)

Core Instructional Resources

Textbook: Precalculus with limits 2/E - Young ISBN 978-0-470-90412-1 (chapter 9)

Supplemental Resources

Internet based resources such as:

[Khan Academy](#)

[Albert.IO](#)

[DeltaMath](#)

Teacher produced materials

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby solve and analyze situations taken from business, physics, engineering, biology, statistics, geography, and numerous other fields. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.