03 - Vectors and the Complex Plane

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **3 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

The study of Precalculus comes between the study of Algebra2/Trig and Calculus. This course develops many new and rigorous techniques for the analysis and application of various types of functions and equations. The course begins with an expansion of the study of Trigonometry to included trigonometric functions, trigonometric equations, and analytic trigonometry through the use of trigonometric identities. Then an understanding of vectors is developed and applied to study the complex number system. Students are exposed to polar graphing and polar equations. A review of some fundamental functions and their properties along with the application of parametric equations lays a foundation for more advanced study in Calculus. The concept and properties of limits and established and applied to further analyze various functions. The derivative is defined and computational techniques and properties established. Several applications of derivatives such as: optimization, related rates, and graphical analysis are examined. Lastly, the calculation and application of an antiderivative is introduced.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

In this unit vectors are defined and combined with the Law of Sines and the Law of Cosines to find the resulting velocity and force vectors. The dot product is defined and used in physical problems like calculating work. Trigonometric functions are then used to define complex numbers in polar form. This form of complex numbers is used to perform operations and calculations. Complex numbers are examined in polar and rectangular form.

Objectives:

- define and determine the direction and magnitude of a vector
- find the dot product of two vectors
- express complex numbers in polar form
- use De Moivre's Theorem to find a complex number raised to a power
- convert between rectangular and polar coordinates

Essential Questions:

- What is a vector and what are it's components?
- How do vector operations relate to operations with other mathematical elements?
- How are complex number represented in the complex plane?
- What are advantages vs. disadvantages of polar and rectangular forms of complex numbers (including the real numbers)?

CONTENT AREA STANDARDS

A.CED

A. Create equations that describe numbers or relationships

A.REI

A. Understand solving equations as a process of reasoning and explain the reasoning

B. Solve equations and inequalities in one variable

C. Solve systems of equations

D. Represent and solve equations and inequalities graphically

A.SSE

A. Interpret the structure of expressions

B. Write expressions in equivalent forms to solve problems

HS Functions

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

F.TF

A. Extend the domain of trigonometric functions using the unit circle

B. Model periodic phenomena with trigonometric functions

C. Prove and apply trigonometric identities

MA.N-CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
MA.N-CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
MA.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
MA.N-CN.B.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
MA.N-CN.B.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
MA.N-CN.B.6	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
MA.N-VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lvert\lvert\boldsymbol{v}\rvert\rvert$, $\boldsymbol{v}$).
MA.N-VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.N-VM.B.4	Add and subtract vectors.
MA.N-VM.B.5	Multiply a vector by a scalar.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

NJSLS-CLKS

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a)

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

9.4.12.DC.7: Evaluate the influence of digital communities on the nature, content and responsibilities of careers, and other aspects of society (e.g., 6.1.12.CivicsPD.16.a).

9.4.12.DC.8: Explain how increased network connectivity and computing capabilities of everyday objects allow for innovative technological approaches to climate protection.

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
LA.RI.11-12.3	Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
LA.RI.11-12.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
LA.RI.11-12.7	Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
LA.RI.11-12.10a	By the end of grade 11, read and comprehend literary nonfiction at grade level text-complexity or above with scaffolding as needed.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.6	Attend to precision.

MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.C.CS2	Communicate information and ideas to multiple audiences using a variety of media and formats.
TECH.8.1.12.D.CS1	Advocate and practice safe, legal, and responsible use of information and technology.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.2.12.C.4	Explain and identify interdependent systems and their functions.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.
TECH.8.2.12.D.CS3	Assess the impact of products and systems.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- Vectors and scalars are different and follow different rules
- There are geometric and algebraic representations of vectors
- The dot product is specific to vector multiplication
- A complex number can be represented in both rectangular and polar forms
- The horizontal axis of the complex planes relates to the real number component while the vertical axis relates to the imaginary component of a complex number
- Identities exists that allow for the multiplication and division of complex numbers in polar form
- De Moivre's Theorem provides a method for raising a complex number to a power in polar form
- Coordinates can be translated between polar and rectangular forms

Procedural Knowledge

Students will be able to:

- Represent vectors geometrically and algebraically
- Find the magnitude and direction of a vector
- Operate with vectors
- Calculate unit vectors
- Express vectors in terms of their horizontal and vertical components
- Find the dot product of two vectors
- Use the dot product to determine the angel between two vectors
- Graph/plot points on the complex plane
- Convert between polar and rectangular forms of complex numbers
- Operate with complex numbers in both polar and rectangular forms
- Find nth roots of complex numbers

EVIDENCE OF LEARNING

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Alternate Assessments

- Portfolios
- Verbal Assessment (instead of written)
- Multiple choice
- Modified Rubrics
- Performance Based Assessments

Formative Assessments

- Student feedback/questioning/observation

- Exit Ticket
- Error analysis
- Specific skill assessment/questions
- Survey/polling
- Reflection questions
- Scored/evaluated class work or homework
- Task completion

Summative Assessments

- Lesson Quizzes
- Unit Test
- Performance Tasks

RESOURCES (Instructional, Supplemental, Intervention Materials)

Core Instructional Resources

Textbook: Precalculus with limits 2/E - Young ISBN 978-0-470-90412-1

Supplemental Resources

Internet based resources such as:

Khan Academy

Albert.IO

DeltaMath

Teacher produced materials

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby solve and analyze situations taken from business, physics, engineering, biology, statistics, geography, and numerous other fields. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.