

02 - Analytic Trig

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **4 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

The study of Precalculus comes between the study of Algebra2/Trig and Calculus. This course develops many new and rigorous techniques for the analysis and application of various types of functions and equations. The course begins with an expansion of the study of Trigonometry to included trigonometric functions, trigonometric equations, and analytic trigonometry through the use of trigonometric identities. Then an understanding of vectors is developed and applied to study the complex number system. Students are exposed to polar graphing and polar equations. A review of some fundamental functions and their properties along with the application of parametric equations lays a foundation for more advanced study in Calculus. The concept and properties of limits and established and applied to further analyze various functions. The derivative is defined and computational techniques and properties established. Several applications of derivatives such as: optimization, related rates, and graphical analysis are examined. Lastly, the calculation and application of an antiderivative is introduced.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

In this unit we will verify trigonometric identities. Specific identities that we will discuss are sum and difference, double-angle and half-angle, and product-to-sum. Inverse trigonometric function will be defined. Trigonometric identities and inverse trig functions will be used to solve trigonometric equations.

Learning Objectives for the Unit:

- Verify trigonometric identities.
- Use the sum and difference identities to simplify trigonometric expressions.
- Use the double-angle and half-angle identities to simplify trigonometric expressions.
- Use the product-to-sum and sum-to-product identities to simplify trigonometric expressions.
- Evaluate the inverse trigonometric functions to for specific values.
- Solve trigonometric equations.

Essential Questions:

- What are trigonometric identities and how are they used?
- How are the various identities developed based on previous identities?
- What dictates the process used to solve a particular trigonometric equation?
- Compare and contrast trig functions and their inverse functions.

CONTENT AREA STANDARDS

A.CED

A. Create equations that describe numbers or relationships

A.REI

A. Understand solving equations as a process of reasoning and explain the reasoning

B. Solve equations and inequalities in one variable

C. Solve systems of equations

D. Represent and solve equations and inequalities graphically

A.SSE

A. Interpret the structure of expressions

B. Write expressions in equivalent forms to solve problems

HS Functions

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

F.TF

A. Extend the domain of trigonometric functions using the unit circle

B. Model periodic phenomena with trigonometric functions

C. Prove and apply trigonometric identities

MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.F-TF.C	Prove and apply trigonometric identities
MA.F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

NJSLS-CLKS

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a)
- 9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
- 9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
- 9.4.12.DC.7: Evaluate the influence of digital communities on the nature, content and responsibilities of careers, and other aspects of society (e.g., 6.1.12.CivicsPD.16.a).
- 9.4.12.DC.8: Explain how increased network connectivity and computing capabilities of everyday objects allow for innovative technological approaches to climate protection.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for

accomplishing a specified task (e.g., W.11-12.6.).

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
LA.RI.11-12.3	Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
LA.RI.11-12.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
LA.RI.11-12.7	Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
LA.RI.11-12.10a	By the end of grade 11, read and comprehend literary nonfiction at grade level text-complexity or above with scaffolding as needed.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.C.CS2	Communicate information and ideas to multiple audiences using a variety of media and formats.
TECH.8.1.12.D.CS1	Advocate and practice safe, legal, and responsible use of information and technology.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.

TECH.8.2.12.C.4	Explain and identify interdependent systems and their functions.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.
TECH.8.2.12.D.CS3	Assess the impact of products and systems.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- there is more than one way to verify an identity
- identities must hold for all values in the domain of the functions that are related in the identities
- a trigonometric function of a sum is not the sum of the trigonometric functions
- the double-angle identities are derived from the sum identities
- the half-angle identities are derived from the double-angle identities
- the sum and difference identities are used to derive the product-to-sum identities
- the product-to-sum identities are used to derive the sum-to-product identities
- different notations for inverse trigonometric functions fit specific contexts
- domain restrictions on trigonometric functions are needed for inverse trigonometric functions to exist
- properties of inverse functions are extended to develop inverse trigonometric identities
- solving trigonometric equations is similar to solving algebraic equations
- the goal in solving a trigonometric equation is to find the value(s) for the independent variable that make(s) the equation a true statement

Procedural Knowledge

Students will be able to:

- Apply fundamental identities
- Simplify trigonometric expressions using identities
- Verify trigonometric identities
- Find exact value of trigonometric functions of certain rational multiples of π by using the sum and difference identities
- Develop new identities from the sum and difference identities
- Use the double-angle identities to find exact values of certain trigonometric functions
- Use the double-angle identities to help in verifying identities
- Use the half-angle identities to find exact values of certain trigonometric functions
- Use the half-angle identities to help in verifying identities
- Express products of trigonometric functions as sums of trigonometric functions
- Express sums of trigonometric functions as products of trigonometric functions
- Develop inverse trigonometric functions
- Find values of inverse trigonometric functions

- Graph inverse trigonometric functions
- Solve trigonometric equations by inspection
- Solve trigonometric equations using algebraic techniques
- Solve trigonometric equations using inverse functions
- Solve trigonometric equations (involving more than one trigonometric function) using trigonometric identities

EVIDENCE OF LEARNING

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Alternate Assessments

- Portfolios
- Verbal Assessment (instead of written)
- Multiple choice
- Modified Rubrics
- Performance Based Assessments

Formative Assessments

- Student feedback/questioning/observation
- Exit Ticket
- Error analysis
- Specific skill assessment/questions
- Survey/polling
- Reflection questions

- Scored/evaluated class work or homework
- Task completion

Summative Assessments

- Lesson Quizzes
- Unit Test
- Performance Tasks

RESOURCES (Instructional, Supplemental, Intervention Materials)

Core Instructional Resources

Textbook: Precalculus with limits 2/E - Young ISBN 978-0-470-90412-1

Supplemental Resources

Internet based resources such as:

[Khan Academy](#)

[Albert.IO](#)

[DeltaMath](#)

Teacher produced materials

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby solve and analyze situations taken from business, physics, engineering, biology, statistics, geography, and numerous other fields. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.

