

01 - Trigonometric Functions

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **2 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

The study of Precalculus comes between the study of Algebra2/Trig and Calculus. This course develops many new and rigorous techniques for the analysis and application of various types of functions and equations. The course begins with an expansion of the study of Trigonometry to included trigonometric functions, trigonometric equations, and analytic trigonometry through the use of trigonometric identities. Then an understanding of vectors is developed and applied to study the complex number system. Students are exposed to polar graphing and polar equations. A review of some fundamental functions and their properties along with the application of parametric equations lays a foundation for more advanced study in Calculus. The concept and properties of limits and established and applied to further analyze various functions. The derivative is defined and computational techniques and properties established. Several applications of derivatives such as: optimization, related rates, and graphical analysis are examined. Lastly, the calculation and application of an antiderivative is introduced.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

This unit will expand on the Trigonometric relationships and functions we developed in previous courses to examine the “Trigonometric Functions of Real Numbers”. In this unit we will use the unit circle approach to define trigonometric functions. We will graph the sine and cosine functions and find periods, amplitude, and phase shifts. Applications such as harmonic motions will be discussed. Combinations of sinusoidal functions will be discussed through a technique called the addition of ordinates. Lastly, we will discuss the graphs of the other trigonometric functions (tangent, cotangent, secant, and cosecant).

- Define trigonometric functions using the unit circle approach.
- Graph a sinusoidal function and determine its amplitude, period, and phase shift
- Graph tangent, cotangent, secant, and cosecant functions including those with shifts/transformations

Essential Questions:

- How are Trig functions of real numbers similar/different from Trig functions of acute angles?
- What is needed to accurately graph a trigonometric function?
- How are periodic or sinusoidal functions used to analyze harmonic motion?
- How are the graphs of other trig functions related to the graphs of Sine and Cosine?

CONTENT AREA STANDARDS

A.CED

A. Create equations that describe numbers or relationships

A.REI

A. Understand solving equations as a process of reasoning and explain the reasoning

B. Solve equations and inequalities in one variable

C. Solve systems of equations

D. Represent and solve equations and inequalities graphically

A.SSE

A. Interpret the structure of expressions

B. Write expressions in equivalent forms to solve problems

HS Functions

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

F.TF

A. Extend the domain of trigonometric functions using the unit circle

B. Model periodic phenomena with trigonometric functions

C. Prove and apply trigonometric identities

MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-TF.A	Extend the domain of trigonometric functions using the unit circle
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

NJSLS-CLKS

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a)

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

9.4.12.DC.7: Evaluate the influence of digital communities on the nature, content and responsibilities of careers, and other aspects of society (e.g., 6.1.12.CivicsPD.16.a).

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
LA.RI.11-12.3	Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
LA.RI.11-12.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
LA.RI.11-12.7	Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
LA.RI.11-12.10a	By the end of grade 11, read and comprehend literary nonfiction at grade level text-complexity or above with scaffolding as needed.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)

TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.C.CS2	Communicate information and ideas to multiple audiences using a variety of media and formats.
TECH.8.1.12.D.CS1	Advocate and practice safe, legal, and responsible use of information and technology.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.2.12.C.4	Explain and identify interdependent systems and their functions.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.
TECH.8.2.12.D.CS3	Assess the impact of products and systems.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- The definition of trigonometric functions using the unit circle approach is consistent with both of the previous definitions (right triangle trigonometry and trigonometric functions of nonacute angles in the Cartesian Plane).
- The x-coordinates and y-coordinates of points on the unit circle are related to the values of cosine and sine functions.
- One can visualize the periodic properties of trigonometric (circular) functions.
- The graphs of the sine and cosine functions are called sinusoidal graphs
- There is a cyclic nature to periodic functions
- Harmonic motion can be modeled and analyzed as a sinusoidal function
- Domain restrictions are related to vertical asymptotes
- Vertical asymptotes follow a specific pattern in periodic functions
- There exist relationships between the graphs of the cosine and secant functions and the sine and cosecant functions

Procedural Knowledge

Students will be able to:

- Draw the unit circle showing the special angles, and label the cosine and sine values
- Determine the domain and range of trigonometric functions
- Classify trigonometric functions as odd or even
- Graph the sine and cosine functions
- Determine the domain and range of the sine and cosine functions
- Determine the amplitude and period of sinusoidal functions
- Determine the phase shift of a sinusoidal function
- Solve harmonic motion problems
- Graph sums of functions
- Determine the domain and range of the tangent, cotangent, secant and cosecant functions
- Graph basic tangent, cotangent, secant and cosecant functions
- Determine the period of tangent, cotangent, secant and cosecant functions
- Graph translated tangent, cotangent, secant and cosecant functions

EVIDENCE OF LEARNING

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Alternate Assessment

- Portfolios
- Verbal Assessment (instead of written)
- Multiple choice
- Modified Rubrics
- Performance Based Assessments

Formative Assessments

- Student feedback/questioning/observation
- Exit Ticket
- Error analysis
- Specific skill assessment/questions
- Survey/polling
- Reflection questions
- Scored/evaluated class work or homework
- Task completion

Summative Assessments

- Lesson Quizzes
- Unit Test
- Performance Tasks

RESOURCES (Instructional, Supplemental, Intervention Materials)

Core Instructional Resources

Textbook: Precalculus with limits 2/E - Young ISBN 978-0-470-90412-1

Supplemental Resources

Internet based resources such as:

[Khan Academy](#)

[Albert.IO](#)

[DeltaMath](#)

Teacher produced materials

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby solve and analyze situations taken from business, physics, engineering, biology, statistics, geography, and numerous other fields. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.