05 Integrals and The Fundamental Theorem of Calculus

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **4 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

This is an advanced course for those students who have completed Precalculus. The course includes topics of a first semester college calculus program. Major areas of concentration are the theory of limits, differential calculus and its applications, and integral calculus and its applications.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

This unit establishes the relationship between differentiation and integration using the Fundamental Theorem of Calculus. Students begin by exploring the contextual meaning of areas of certain regions bounded by rate functions. Integration determines accumulation of change over an interval, just as differentiation determines instantaneous rate of change at a point. Students should understand that integration is a limiting case of a sum of products (areas) in the same way that differentiation is a limiting case of a quotient of differences (slopes). Future units will apply the idea of accumulation of change to a variety of realistic and geometric applications.

Essential questions:

- Interpret the meaning of The fundamental Theorem of Calculus.
- What types of problems benefit form the use of integrals?
- What is a definite integral and how does it relate to other areas of Calculus?
- How are indefinite integrals related to derivatives?

CONTENT AREA STANDARDS

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

F.TF

A. Extend the domain of trigonometric functions using the unit circle

B. Model periodic phenomena with trigonometric functions

C. Prove and apply trigonometric identities

MA.9-12.EU 3.1	Antidifferentiation is the inverse process of differentiation.
MA.9-12.EU 3.2	The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.
MA.9-12.EU 3.3	The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.
MA.9-12.EU 3.4	The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

NJSLS-CLKS

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a)

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

9.4.12.DC.7: Evaluate the influence of digital communities on the nature, content and responsibilities of careers, and other aspects of society (e.g., 6.1.12.CivicsPD.16.a).

9.4.12.DC.8: Explain how increased network connectivity and computing capabilities of everyday objects allow for innovative technological approaches to climate protection.

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
LA.RI.11-12.3	Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
LA.RI.11-12.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
LA.RI.11-12.7	Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.D.CS1	Advocate and practice safe, legal, and responsible use of information and technology.

TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.2.12.C.4	Explain and identify interdependent systems and their functions.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.
TECH.8.2.12.D.CS3	Assess the impact of products and systems.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- An antiderivative of a function f is a function g whose derivative is f.
- Differentiation rules provide the foundation for finding antiderivatives.
- A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
- The definite integral of a continuous function f over the interval $[a, b]$, denoted by ∫ (from a to b) $f(x)dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is ∫ (from a to b) $f(x)dx = lim$ [as maxΔx_i approaches 0] Σ [i = 1 to n] $f(x_i*)\Delta x_i$, where x_i* is a value in the ith subinterval, Δx_i is the width of the ith subinterval, n is the number of subintervals, and maxΔx_i is the width of the largest subinterval. Another form of the definition is ∫ (from a to b) $f(x)dx = lim$ [as n approaches ∞] Σ [i = 1 to n] $f(x_i*)\Delta x_i$,, where $\Delta x_i = (b - a)/n$ and x_i* is a value in the ith subinterval.
- The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
- Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.
- Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
- In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
- Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
- The definition of the definite integral may be extended to functions with removable or jump discontinuities

- The definite integral can be used to define new functions; for example, $f(x)$ = ∫ (from 0 to x) e [superscript $-t^2$] dt.
- If f is a continuous function on the interval $[a, b]$, then d/dx (∫ (from a to x) $f(t)dt$) = $f(x)$, where x is between a and b.
- Graphical, numerical, analytical, and verbal representations of a function provide information about the function g defined as $g(x)$ = ∫ (from a to x) $f(t)dt$.
- The function defined by $F(x)$ = ∫ (from a to x) $f(t)dt$ is an antiderivative of f.
- If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then ∫ (from a to b) $f(x)dx$ = $F(b) - F(a)$.
- The notation ∫ $f(x)dx = F(x) + C$ means that $F'(x) = f(x)$, and ∫ $f(x)dx$ is called an indefinite integral of the function f.
- Many functions do not have closed form antiderivatives.
- A function defined as an integral represents an accumulation of a rate of change.
- The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
- The limit of an approximating Riemann sum can be interpreted as a definite integral.
- For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
- Areas of certain regions in the plane can be calculated with definite integrals.
- Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
- The definite integral can be used to express information about accumulation and net change in many applied contexts.
- Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.

Procedural Knowledge

Students will be able to:

- Recognize antiderivatives of basic functions.
- Interpret the definite integral as the limit of a Riemann sum.
- Express the limit of a Riemann sum in integral notation.
- Approximate a definite integral.
- Calculate a definite integral using areas and properties of definite integrals.
- Analyze functions defined by an integral.
- Calculate antiderivatives.
- Evaluate definite integrals.
- Interpret the meaning of a definite integral within a problem.
- Apply definite integrals to problems involving the average value of a function.
- Apply definite integrals to problems involving motion.
- Apply definite integrals to problems involving area, volume.

EVIDENCE OF LEARNING

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Alternate Assessments

- Portfolios
- Verbal Assessment (instead of written)
- Multiple choice
- Modified Rubrics
- Performance Based Assessments

Formative Assessments

- Student feedback/questioning/observation
- Exit Ticket
- Error analysis
- Specific skill assessment/questions
- Survey/polling
- Reflection questions
- Scored/evaluated class work or homework
- Task completion

Summative Assessments

- Lesson Quizzes
- Unit Test

- Performance Tasks

RESOURCES (Instructional, Supplemental, Intervention Materials)

Core Instructional Materials

Textbook - Calculus AP Edition: Finney, et al. ISBM 0-13-201408-4

Supplemental Materials

Internet based resources such as:

Khan Academy

Albert.IO

DeltaMath

Teacher produced materials

https://www.geogebra.org/material/show/id/27457

http://www.ck12.org/book/CK-12-Texas-Instruments-Calculus-Student-Edition/section/5.1/

https://teacher.desmos.com/activitybuilder/custom/55c529e088a1570a43103e15#preview/6cc78a31-3451-4921-b2d2-56b34fbe949e

http://www.tmhs.tellurideschool.org/common/pages/DisplayFile.aspx?itemId=6733976

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby solve and analyze situations taken from business, physics, engineering, biology, statistics, geography, and numerous other fields. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.