04 Applications of Derivatives

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **4-5 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

This is an advanced course for those students who have completed Precalculus. The course includes topics of a first semester college calculus program. Major areas of concentration are the theory of limits, differential calculus and its applications, and integral calculus and its applications.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Having developed proficiency in various techniques of differentiation, students are now ready to apply their skills to a variety of modeling and analytic situations. First, students evaluate and solve functions generated by a derivative to identify critical values and extrema of a function. Students then apply these skills and techniques to solve problems related to motion (position, velocity, acceleration), graphical features of functions, optimization, and related rates.a

Essential Questions:

- What features of a function are expressed through its derivative?
- How are derivatives used to solve problems related to real world phenomena?
- What types of problems would not be solvable if not for the derivative?

CONTENT AREA STANDARDS

N.CN

A. Perform arithmetic operations with complex numbers

B. Represent complex numbers and their operations on the complex plane

C. Use complex numbers in polynomial identities and equations

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

F.TF

A. Extend the domain of trigonometric functions using the unit circle

B. Model periodic phenomena with trigonometric functions

C. Prove and apply trigonometric identities

MA.9-12.2	Derivatives
MA.9-12.EU 2.2	A function's derivative, which is itself a function, can be used to understand the behavior of the function.
MA.9-12.EU 2.3	The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.
MA.9-12.EU 2.4	The Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval.
MA.9-12.MPAC 1.c	apply definitions and theorems in the process of solving a problem;
MA.9-12.MPAC 1.f	produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

NJSLS-CLKS

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a)

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice

(e.g., 1.1.12acc.C1b, 2.2.12.PF.3).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

9.4.12.DC.7: Evaluate the influence of digital communities on the nature, content and responsibilities of careers, and other aspects of society (e.g., 6.1.12.CivicsPD.16.a).

9.4.12.DC.8: Explain how increased network connectivity and computing capabilities of everyday objects allow for innovative technological approaches to climate protection.

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
LA.RI.11-12.3	Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
LA.RI.11-12.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
LA.RI.11-12.7	Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.

MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.D.CS1	Advocate and practice safe, legal, and responsible use of information and technology.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.2.12.C.4	Explain and identify interdependent systems and their functions.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.
TECH.8.2.12.D.CS3	Assess the impact of products and systems.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
- Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
- Key features of the graphs of f, f' and f'' are related to one another.
- A continuous function may fail to be differentiable at a point in its domain.
- If a function is differentiable at a point, then it is continuous at that point.
- The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
- The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
- The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
- The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
- The derivative can be used to express information about rates of change in applied contexts.
- Solutions to differential equations are functions or families of functions.
- If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b), the

Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.

Procedural Knowledge

Students will be able to:

- Use derivatives to analyze properties of a function.
- Recognize the connection between differentiability and continuity.
- Interpret the meaning of a derivative within a problem.
- Solve problems involving the slope of a tangent line.
- Solve problems involving related rates, optimization, rectilinear motion.
- Solve problems involving rates of change in applied contexts.
- Verify solutions to differential equations.
- Estimate solutions to differential equations.
- Apply the Mean Value Theorem to describe the behavior of a function over an interval.

EVIDENCE OF LEARNING

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Alternate Assessments

- Portfolios
- Verbal Assessment (instead of written)

- Multiple choice
- Modified Rubrics
- Performance Based Assessments

Formative Assessments

- Student feedback/questioning/observation
- Exit Ticket
- Error analysis
- Specific skill assessment/questions
- Survey/polling
- Reflection questions
- Scored/evaluated class work or homework
- Task completion

Summative Assessments

- Lesson Quizzes
- Unit Test
- Performance Tasks

RESOURCES (Instructional, Supplemental, Intervention Materials)

Core Instructional Materials

Textbook - Calculus AP Edition: Finney, et al. ISBM 0-13-201408-4 (chapter 4)

Supplemental Materials

Internet based resources such as:

Khan Academy

Albert.IO

DeltaMath

Teacher produced materials

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby solve and analyze situations taken from business, physics, engineering, biology, statistics, geography, and numerous other fields. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.