02 Limits and Continuity

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **4 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

This is an advanced course for those students who have completed Precalculus. The course includes topics of a first semester college calculus program. Major areas of concentration are the theory of limits, differential calculus and its applications, and integral calculus and its applications.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Objectives:

- Determine average and instantaneous rates of change
- Understand and apply the definition of a limit
- Calculate one-sided and two-sided limits
- Relate properties of limits to characteristics of functions

Essential Questions:

- What are limits and how are they calculated?
- How are properties from Algebra and Arithmatic applied to limits?
- How are limits and the continuity of a function related?

Enduring Understandings:

- Calculus is concerned with change and motion and deals with quantities that approach other quantities
- The idea of a limit underlies the various branches of calculus

CONTENT AREA STANDARDS

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

F.TF

A. Extend the domain of trigonometric functions using the unit circle

B. Model periodic phenomena with trigonometric functions

C. Prove and apply trigonometric identities

MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.9-12.EK 1.1A1	Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is lim [as x approaches c] $f(x) = R$.
MA.9-12.EK 1.1A2	The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
MA.9-12.EK 1.1A3	A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
MA.9-12.EK 1.1B1	Numerical and graphical information can be used to estimate limits.
MA.9-12.EK 1.1C1	Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
MA.9-12.EK 1.1C2	The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
MA.9-12.EK 1.1C3	Limits of the indeterminate forms 0/0 and ∞/∞ may be evaluated using L'Hospital's Rule
MA.9-12.EK 1.1D1	Asymptotic and unbounded behavior of functions can be explained and described using

	limits.
MA.9-12.EK 1.2A1	A function is continuous at $x = c$ provided that $f(c)$ exists, lim [as x approaches c] $f(x)$ exists, and lim [as x approaches c] $f(x) = f(c)$.
MA.9-12.EK 1.2A2	Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
MA.9-12.EK 1.2A3	Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
MA.9-12.EK 1.2B1	Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.
MA.9-12.EU 1.1	The concept of a limit can be used to understand the behavior of functions.
MA.9-12.EU 1.2	Continuity is a key property of functions that is defined using limits.
MA.9-12.LO 1.1B	Estimate limits of functions.
MA.9-12.LO 1.1C	Determine limits of functions.
MA.9-12.LO 1.1D	Deduce and interpret behavior of functions using limits.
MA.9-12.LO 1.2A	Analyze functions for intervals of continuity or points of discontinuity.
MA.9-12.LO 1.2B	Determine the applicability of important calculus theorems using continuity.
MA.9-12.LO 1.1Aa	Express limits symbolically using correct notation.
MA.9-12.LO 1.1Ab	Interpret limits expressed symbolically.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

NJSLS-CLKS

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a)

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

9.4.12.DC.7: Evaluate the influence of digital communities on the nature, content and responsibilities of careers, and other aspects of society (e.g., 6.1.12.CivicsPD.16.a).

9.4.12.DC.8: Explain how increased network connectivity and computing capabilities of everyday objects allow for innovative technological approaches to climate protection.

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an

intended audience (e.g., S-ID.B.6b, HS-LS2-4).

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
LA.RI.11-12.3	Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
LA.RI.11-12.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
LA.RI.11-12.7	Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.2.12.C.4	Explain and identify interdependent systems and their functions.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.

STUDENT LEARNING TARGETS

Declarative Knowledge

Students will understand that:

- A limit is a specifically defined mathematical term.
- The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
- A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
- Asymptotic and unbounded behavior of functions can be explained and described using limits.
- A function is continuous at $x = c$ provided that $f(c)$ exists, lim [as x approaches c] $f(x)$ exists, and lim [as x approaches c] $f(x) = f(c)$.
- The concept of a limit can be used to understand the behavior of functions.

Procedural Knowledge

Students will be able to:

- Evaluate limits graphically
- Evaluate limits numerically (using table on calculator)
- Evaluate limits algebraically by direct substitution
- Evaluate limits of the form 0/0 using algebraic techniques (factoring/canceling. rationalizing, simplifying, sinx/x)
- Evaluate limits at infinity
- Evaluate limits of piecewise functions
- Determine vertical and horizontal asymptotes using limits

EVIDENCE OF LEARNING

Alternate Assessments

- Portfolios
- Verbal Assessment (instead of written)
- Multiple choice

- Modified Rubrics
- Performance Based Assessments

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based Assessments)

Formative Assessments

- Student feedback/questioning/observation
- Exit Ticket
- Error analysis
- Specific skill assessment/questions
- Survey/polling
- Reflection questions
- Scored/evaluated class work or homework
- Task completion

Summative Assessments

- Lesson Quizzes
- Unit Test
- Performance Tasks

RESOURCES (Instructional, Supplemental, Intervention Materials)

Core Instructional Materials

Textbook - Calculus AP Edition: Finney, et al. ISBM 0-13-201408-4 (chapter 2)

Supplemental Materials

Internet based resources such as:

Khan Academy

Albert.IO

DeltaMath

https://www.khanacademy.org/math/precalculus/limit-topic-precalc/limits-precalc/v/introduction-to-limits-hd

http://www.mathwarehouse.com/calculus/continuity/what-are-types-of-discontinuities.php

Teacher produced materials

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby solve and analyze situations taken from business, physics, engineering, biology, statistics, geography, and numerous other fields. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.