08 Exponential and Logarithmic Functions

Content Area: **TEMPLATE**
Course(s):
Time Period: **Full Year**
Length: **4 - 6 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

This course is an extension of Algebra 1. Emphasis is upon the development of insights into the structure of algebra as a deductive process. The content includes function foundations, equations and inequalities, polynomial functions and equations, rational functions and equations, radical expressions and equations, exponential and logarithmic functions and equations, trigonometric functions and equations, introductory data analysis, and probability.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Objectives:

- Apply the concept of exponential functions to be able to solve real world problems.
- Define logarithms in terms of exponential functions.
- Apply the laws of logarithms to simplify expressions.
- Solve and simplify expressions by converting from log form to exponential form.

Essential Questions:

- How do you model a quantity that changes regularly over time by the same percentage?
- How are exponents and logarithms and their functions related?
- How do exponential and logarithmic functions model real-world problems?

Enduring Understanding:

- You can represent repeated multiplication with a function in the form of $y = ab^x$ where b is a positive number other than 1.
- The exponential function $y = b^x$ is one-to-one, so its inverse $x = b^y$ is a function. To express "y as a function of x" for the inverse you write $y = log_b x$
- Logarithms and exponents have corresponding properties
- You can use logarithms to solve exponential equations; and conversely, you can use exponents to solve logarithmic properties

- The function $y = e^x$ and $y = ln\ x$ are inverse functions.

CONTENT AREA STANDARDS

MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
MA.F-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MA.F-BF.B.4b	Verify by composition that one function is the inverse of another.
MA.F-BF.B.4c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MA.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MA.F-LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.B.4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
MA.A-SSE.B.3c	Use the properties of exponents to transform expressions for exponential functions.

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

CS.K-12.4.a	Extract common features from a set of interrelated processes or complex phenomena.

CS.K-12.4.c	Create modules and develop points of interaction that can apply to multiple situations and reduce complexity.
CS.K-12.4.d	Model phenomena and processes and simulate systems to understand and evaluate potential outcomes.
LA.K-12.NJSLSA.R7	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
PFL.9.1.K12.P.4	Demonstrate creativity and innovation.
PFL.9.1.K12.P.5	Utilize critical thinking to make sense of problems and persevere in solving them.
PFL.9.1.K12.P.6	Model integrity, ethical leadership and effective management.
PFL.9.1.K12.P.8	Use technology to enhance productivity increase collaboration and communicate effectively.
PFL.9.1.K12.P.9	Work productively in teams while using cultural/global competence.

STUDENT LEARNING TARGETS

Refer to the 'Declarative Knowledge' and 'Procedural Knowledge sections.

Declarative Knowledge

Students will understand that:

- The inverse relationship between exponents and logarithms

Procedural Knowledge

Students will be able to:

- Read values of an inverse function from a graph or a table, given that the function has an inverse
- Verify by composition that one function is the inverse of another
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs
- Create equations sin two or more variables to represent relationships between quantities
- Derive the formula for the same of a finite geometric series
- For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology
- Graph exponential and logarithmic functions expressed symbolically

- Graph trigonometric functions expressed symbolically
- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative)
- Interpret complicated expressions by viewing one or more of their parts as a single entity
- Interpret the parameters in a linear or exponential function in terms of a context
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function
- Recognize even and odd functions from their graphs and algebraic expressions for them
- Use the properties of exponents to interpret expressions for exponential functions in various form
- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms
- Create equations and inequalities with one variable that describes numbers or relationships.
- Find inverse functions
- Graph equations on coordinate axes with labels and scales
- Interpret key features of graphs and tables of a function that models a relationship between two quantities
- Sketch graphs showing key features given a verbal description of the relationship
- Use created equations and inequalities in one variable to solve problems
- Use the properties of exponents to transform expressions for exponential functions
- Use the formula for the sum of a finite geometric series to solve problems (when the common ratio is not 1)

EVIDENCE OF LEARNING

Refer to the 'Formative Assessments' and 'Summative Assessments' sections.

Formative Assessments

- Class Discussion
- Teacher observation
- Exit/Entrance Tickets
- Classwork
- Homework

Summative Assessments

- Quizzes
- Test
- Projects

RESOURCES (Instructional, Supplemental, Intervention Materials)

- Sullivan Algebra and Trigonometry Textbook (Chapter 1, section 4)
- Khan Academy
- Radical Expressions Reference
- Deltamath
- Illustrative Mathematics Tasks by standard
- Illustrative Mathematics Curriculum
- Desmos
- Logarithims of Reddit Ranking
- Comparing Investments

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby students solve and analyze situations such as rent-to-buy, debt, sound, star brightness, and pH balance in chemistry. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.