05 Polynomial Functions and Equations

Content Area: **TEMPLATE**
Course(s):
Time Period: **Full Year**
Length: **4 -5 weeks**
Status: **Published**

General Overview, Course Description or Course Philosophy

This course is an extension of Algebra 1. Emphasis is upon the development of insights into the structure of algebra as a deductive process. The content includes function foundations, equations and inequalities, polynomial functions and equations, rational functions and equations, radical expressions and equations, exponential and logarithmic functions and equations, trigonometric functions and equations, introductory data analysis, and probability.

OBJECTIVES, ESSENTIAL QUESTIONS, ENDURING UNDERSTANDINGS

Objectives:

- Understand that polynomial functions are smooth and continuous.
- Demonstrate how to use polynomial functions to model complex scenarios and relationships between a domain and range.
- Analyzing the graph of a polynomial function provide information about the properties of the function.
- Solve real-world problems that involve polynomial functions.

Essential Questions:

- How do polynomial functions model real-world problems and their solutions?
- Why are complex numbers necessary?
- How are operations and properties of complex numbers related to those of real numbers?
- What does the degree of a polynomial tell you about its related polynomial function?
- For a polynomial function, how are factors, zeros and x0intercepts related?
- For a polynomial function, how are factors and roots related?

Enduring Understanding:

- The characteristics of polynomial functions and their representations are useful in solving real-world problems.
- The domain and range of polynomial functions can be extended to include the set of complex numbers.
- A polynomial function has distinguishing "behaviors". You can look at its algebraic form and know something about its graph and you can look at its graph and know something about its algebraic form.

- The zeros of a polynomial functions can help you understand the behaviors of its graph.
- Polynomials can be divided using steps that are similar to the long division steps that you use to divide whole numbers.
- The degree of a polynomial equation tell you how many roots the equation has.
- You can use a pattern of coefficients to write the expansion of $(a + b)^n$

CONTENT AREA STANDARDS

A.APR

A. Perform arithmetic operations on polynomials

B. Understand the relationship between zeros and factors of polynomials

C. Use polynomial identities to solve problems

D. Rewrite rational expressions

F.BF

A. Build a function that models a relationship between two quantities

B. Build new functions from existing functions

F.IF

A. Understand the concept of a function and use function notation

B. Interpret functions that arise in applications in terms of the context

C. Analyze functions using different representations

F.LE

A. Construct and compare linear and exponential models and solve problems

B. Interpret expressions for functions in terms of the situation they model

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.

MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.K-12.4	Model with mathematics.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.K-12.5	Use appropriate tools strategically.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.K-12.6	Attend to precision.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.A-APR.C.4	Prove polynomial identities and use them to describe numerical relationships.
MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

RELATED STANDARDS (Technology, 21st Century Life & Careers, ELA Companion Standards are Required)

LA.K-12.NJSLSA.R7	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
CS.K-12.2.a	Cultivate working relationships with individuals possessing diverse perspectives, skills, and personalities.
CS.K-12.3.a	Identify complex, interdisciplinary, real-world problems that can be solved

	computationally.
CS.K-12.3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.
CS.K-12.3.c	Evaluate whether it is appropriate and feasible to solve a problem computationally.
CS.K-12.4.a	Extract common features from a set of interrelated processes or complex phenomena.
WRK.K-12.P.1	Act as a responsible and contributing community members and employee.
TECH.K-12.P.5	Utilize critical thinking to make sense of problems and persevere in solving them.
TECH.K-12.P.6	Model integrity, ethical leadership and effective management.

STUDENT LEARNING TARGETS

Refer to the 'Declarative Knowledge' and 'Procedural Knowledge sections.

Declarative Knowledge

Students will understand that:

- Characteristics of a polynomial function and how ti applies to function behavior
- Long and synthetic division are used to determine the relationship between zeros and factors of a polynomial
- The Remainder Theorem determines whether $(x - a)$ is a divisor of a polynomial
- Key features of y-intercept, zeros, and nature of roots algebraically
- Descartes' Rule of Signs determines the maximum number of positive and negative real roots of a polynomial
- Complex roots always come in pairs
- The Fundamental Theorem of Algebra determines the number of roots of a polynomial both real and complex

Procedural Knowledge

Students will be able to:

- Determine key features of polynomials algebraically
- Perform polynomial operations
- Given a graph, explain how to determine if there exists complex roots
- Utilize technology to approximate real zeros
- Explain the relationship between zeros/potential zeros and factors of polynomials
- Factor polynomials algebraically over real and complex numbers
- Identify, evaluate and graph polynomial functions
- Extend polynomial identities to complex numbers

- Graph polynomial functions expressed symbolically
- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative)
- Interpret complicated expressions by viewing one or more of their parts as a single entity
- Interpret parts of an expression, such as terms, factors, and coefficients in terms of a context
- Prove polynomial Identities Analysis
- Recognize even and odd functions from their graphs and algebraic expressions for them
- Relate the domain of a function to its graph and to the quantitative relationship it describes
- Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms
- Apply the Remainder Theorem
- Calculate and interpret the average rate of change of a function
- Interpret key features of graphs and tables of a function that models a relationship between two quantities
- Sketch graphs showing key features given a verbal description of the relationship
- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication
- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line)
- Use polynomial identities to describe numerical relationships
- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context
- Use the structure of an expression to identify ways to rewrite it
- Use the zeros to construct a rough graph of the function defined by the polynomial
- Add, subtract, and multiply polynomials
- Identify zeros of polynomials when suitable factorizations are available
- Know the Remainder Theorem

EVIDENCE OF LEARNING

Refer to the 'Formative Assessments' and 'Summative Assessments' sections.

Benchmark Assessments

Benchmark Assessments conducted three times per year, using Pear Assessment (Standards Based

Assessments)

Alternative Assessments

- Portfolios
- Verbal Assessment (instead of written)
- Multiple choice
- Modified Rubrics
- Performance Based Assessments

Formative Assessments

- Class Discussion
- Teacher observation
- Exit/Entrance Tickets
- Classwork
- Homework

Summative Assessments

- Test
- Quizzes
- Projects

RESOURCES (Instructional, Supplemental, Intervention Materials)

Core Instructional Materials

- Sullivan Algebra and Trigonometry Textbook (chapter 5)

Supplemental Materials

- Khan Academy
- Polynomial Reference
- Deltamath
- Illustrative Mathematics Tasks by standard
- Illustrative Mathematics Curriculum
- Desmos
- Birthday Polynomial

INTERDISCIPLINARY CONNECTIONS

Interdisciplinary connections are frequently addressed through modeling and application problems whereby students solve and analyze situations such as curvature of rollercoasters, cost analysis, company evaluation, voltage, projectiles, and gas laws in chemistry. Examples can be found in topic specific textbook problems and digital resources.

ACCOMMODATIONS & MODIFICATIONS FOR SUBGROUPS

See link to Accommodations & Modifications document in course folder.