AP Calculus BC

Syllabus

Course Overview

The AP Calculus AB course at our school is taught over a full high school academic year. The primary textbook is *Calculus, Numerical and Algebraic*, by Finney, Demana, Waits, & Kennedy. However, many other resources are integrated throughout the course, as stated below. The primary goals of the course are to offer a sound and thorough understanding of Calculus concepts, in addition to adequately preparing students for success on the AP exam.

Throughout the course emphasis is placed on the "Rule of Four." There are a variety of ways to approach and solve problems. The four branches of the problem solving tree of mathematics are:

- Numerical analysis (where data points are known, but not an equation)
- Graphical analysis (where a graph is known, but again, not an equation)
- Analytic/algebraic analysis (traditional equation and variable manipulation)
- Verbal/written methods of representing problems (classic story problems as well as written justification of one's thinking in solving a problem)

Course Planner

Primary Textbook

Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic*. Reading, Mass.: Addison-Wesley, 2007.

The chapter numbers follow the textbook.

Chapter 1: Prerequisites for Calculus (6 days)

- Elementary functions:
- Linear, power, exponential/logarithmic, trigonometric / inverse trigonometric
- Parametric equations
- Getting familiar with the graphing calculator

Chapter 2: Limits and Continuity (7 days)

- Limits:
- Limit at a point, limit at infinity, infinite limits
- Properties of limits
    - Sandwich (or "Squeeze") Theorem

        A Graphical Exploration is used to investigate the Sandwich Theorem. Students graph y1 = x^2, y2 = -x^2, y3 = sin (1/x) in radian mode on graphing calculators. The limit as x approaches 0 of each function is explored in an attempt to "see" the limit as x approaches 0 of x^2 * sin (1/x). This helps tie the graphical implications of the Sandwich Theorem to the analytical applications of it.

- Continuity
- Tangent line to a curve
- Slope of a curve at a point

Chapter 3: Derivatives (15 days)

- Definition of f'
- Derivative at a point
- Relating the graphs of f and f'
- When does $f'(a)$ fail to exist?

  **An exploration is conducted with the calculator. Students graph y1 = absolute value of (x) + 1 and y2 = sqrt (x^2 + 0.0001) + 0.99. They investigate the graphs near x = 0 by zooming in repeatedly. The students discuss the local linearity of each graph and whether each function appears to be differentiable at x = 0
- Rules for differentiation:
  - Sum, product, quotient
- Chain rule
- Implicit differentiation
- Derivatives of trigonometric, inverse trigonometric, exponential, and logarithmic functions

Chapter 4: Applications of Derivatives (17 days)

- Mean value theorem
- Using the derivative to find:
  - Critical point(s) and extreme values
  - When the function is increasing or decreasing
  - Point(s) of inflection
  - When the function is concave up or concave down

  Students will all be given 3 graphs of f, f', and f'' for different "classic" functions. Each student will have their own function and will have to explain written and verbally to the class the connection between f, f', and f''. For example: when x<3, *f is decreasing* because $f' < 0$, and *f is concave down* because $f'' < 0$, etc.

- Optimization problems
- Using the tangent line to approximate function values
- Newton's method
- Differentials and change
- Related rates

Chapter 5: The Definite Integral (14 days)

- RAM (rectangle approximation method)
- Riemann sums
- Finding a formula for an antiderivative
- Using a definite integral to find area, volume, average value of a function
- Fundamental theorem of calculus

  Examples of problems similar to AP free response questions are used here. Emphasis is placed on written explanation of units and why the derivative or integral was used in the context of the real-world application. Students are also given functions where analytical integration is not possible and they must use the graphing calculator.

• Approximating the definite integral:
  • Trapezoidal rule, Simpson's rule, error analysis

Chapter 6: Differential Equations and Mathematical Modeling (15 days)

• Slope fields
• Antiderivatives and the indefinite integral
• Techniques of integration:
  • Substitution, integration by parts, trigonometric substitution, partial fractions
• Separable differential equations
• Euler's method
• Exponential growth and decay
• Logistic growth

Chapter 7: Applications of Definite Integrals (12 days)

• Using the definite integral to discuss:
• Net change—motion on a line, consumption over time
• Area, volume, length of a curve, surface area of a solid of revolution
• Work, fluid force

Chapter 8: L'Hôpital's Rule, Improper Integrals, Partial Fractions (13 days)

• Indeterminate forms and L'Hôpital's rule
• Relative rates of growth
• Improper integrals (partial fractions and trig substitutions—done with Chapter 6)

Chapter 10: Parametric, Vector, and Polar Functions (14 days)

• Parametric functions:
• Derivative at a point
• $\frac{d^2y}{dx^2}$
• Length of a curve, surface area of a solid of revolution • Vectors:
• Angle between vectors
• Scalar product
• Using vectors to describe motion in the plane
• Polar coordinates and pole graphs:
• Slope, horizontal and vertical tangent lines • Area, length of a curve

Chapter 9: Infinite Series (18 days)

- Geometric series
- Power series:
  • Term-by-term differentiation and integration to find power series of new functions
- Taylor's series / Maclaurin series
- Lagrange form of the remainder
- Tests for convergence/divergence:
  - *nth term test*

- Direct comparison
- Ratio test
- Integral test
- Limit comparison test
- Alternating series test (Leibniz's theorem)

- Radius and interval of convergence

Review/Test Preparation

2 weeks

- Multiple-choice practice (Items from past exams—1997, 1998, 2003, and 2008
  Test taking strategies are emphasized
  Individual and group practice are both used

- Free-response practice (Released items from the AP Central website) Rubrics are reviewed so students see the need for complete answers
  Individually written responses are crafted. Attention to full explanations is emphasized

Course Assessments

Students are assessed using a variety of methods including *group activities, oral presentations, quizzes and exams.* Students are expected to explain all solutions both verbally and in written form during the course of the year. Proper mathematical notation is also stressed throughout the course and is consistently assessed. All major exams are "mini" AP examinations, in which all problems consist of multiple choice and open-ended AP examination questions that have been released by *College Board* from previous years. (Students are given these examinations approximately once every 3 weeks.) Therefore, the actual AP Calculus Examination that the students take in the spring has been modeled all year in this course. Each test has a calculator section and a non-calculator section, like the AP test.

Technology and Computer Software

Calculators

All students use a TI-83 or TI-84 graphing calculator in the classroom. The teacher uses a TI-84+ model in instruction. We use the calculator as a tool to illustrate ideas and make discoveries about functions in Calculus.
The following technology objectives are required of the students in this course:

- The ability to graph functions in arbitrary viewing windows
- The ability to find the zeros of functions
- The ability to find numerical derivatives of functions
- The ability to calculate the value of a definite integral

.
Students are also required to make connections between the graphs of functions and their analysis, and conclusions about the behavior of functions when using a graphing calculator.

Teaching Strategies

We try to learn by discovery whenever possible in class. Students have a much better grasp of the concepts of the calculus if they understand where it comes from and why it matters. Sometimes the discovery will be individually as a warm up, other times it is in small groups of four and sometimes it will be as an entire class.

Functions represented analytically make up one segment of our study of calculus. There are so many situations that use functions described verbally, graphically, or numerically, and we work with those along the way. Major emphasis is placed on an understanding of the relationship between the various representations of functions and how the calculus completes our previous understanding of them.

Teacher Resources

Smart board Presentations:

The teacher uses Smart board presentations almost daily to enhance instruction and offer visuals of concepts being studied.

The use of multiple websites are used throughout the course, offering a more pictorial approach to many topics.

In addition, online simulation software is being used to bring the smart board "to life", allowing students to study many concepts in real time. For example, students have the ability to physically drag a tangent along a curve, studying how the derivative changes, in addition to making connections between f, f', and f'' graphically. When studying volumes, students have the ability to view color animations of volumes being created by revolving a region about an axis, allowing for 3-dimensional viewing of solids. (This is especially effective when teaching the shell method!)

Additional Text Resources:

Ron Larson, Bruce Edwards.
AP edition Calculus, 9th edition. Pearson: Brooks/Cole, 2010.

Jon Rogawski, Ray Cannon.
Calculus – Graphical, Numerical, Algebraic, 3rd edition. Freeman, 2012.

Howard Anton, Irl Bivens, & Stephen Davis.
Calculus – Early Transcendentals, Single Variable, 10th edition. John Wiley & Sons, 2012.

Finney, Ross L., Franklin D. Damana, Bert K. Waits, and Daniel Kennedy.
Calculus – Graphical, Numerical, Algebraic, 3rd edition. Pearson: Prentice Hall, 2007.

Best, George W., J. Richard Lux.
Preparing for the (AB) AP Calculus Examination. Venture Publishing, 1998.

Collegeboard
AP Calculus: Multiple-Choice Question Collection, 1969-1998.

Finney, Ross L., Franklin D. Damana, Bert K. Waits, and Daniel Kennedy.

Calculus – Graphical, Numerical, Algebraic, 3rd edition. Pearson: Prentice Hall, 2007.

Lederman, David, Lin McMullin.
Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (AB) Examination. D & S Marketing, Inc., 2003.

The College Board
Released Exams: AP Calculus AB and Calculus BC, 1996, 1997, 1998, 2003, 2008.