FST Unit 1 - Advanced Functions and Modeling

Content Area: **Math**
Course(s):
Time Period: **Full Year**
Length: **60**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
In this unit, students go beyond Algebra 1 by learning about different types of functions, such as polynomial, rational, exponential, and logarithmic. They also expand their understanding of numbers to include complex numbers. Students practice transforming parent functions, identifying key graph features, working with rational exponents, and using multiple methods to solve equations, including graphing with technology. The unit concludes by exploring how exponential and logarithmic functions are related, preparing students for modeling tasks later in the course.	Function analysis is central to advanced high school math. It helps students connect separate algebraic skills into a single approach for understanding change. This unit encourages students to think about math more deeply and introduces the main types of functions used in higher-level math, science, economics, and data modeling. Teaching this unit gives students important algebraic and graphical skills in three main areas: Students build a strong understanding by studying how patterns change, how rules vary, and how functions can be transformed. They learn to adjust parameters, examine important features such as domain, maximum and minimum values, end behavior, and symmetry, and work with rational exponents. This helps them compare different mathematical models with confidence. Students expand their skills by working with complex numbers (a + bi) and higher-degree equations. They practice completing the square, dividing polynomials, using the Factor and Remainder Theorems, and analyzing asymptotes. These tools help them break down complicated expressions and solve challenging problems, including non-linear systems and rational models. Exponential and logarithmic functions help students model real-world situations like compound interest, population growth, radioactive decay, and sound intensity. By understanding that logarithms are the inverse of exponentials and using technology to analyze intersections, students learn to solve challenging equations and quantify non-linear processes in modern STEM fields. Ultimately, this unit prepares students for Precalculus and Calculus. It gives them the skills to model real-world situations, work with abstract functions, and think carefully about how math works.

NJSLS

N.RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
F.BF.A.1.a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
N.RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
A.APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$.
A.APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
F.BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
A.APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
N.CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
N.CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
F.BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
N.CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
A.REI.B.4.a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
A.REI.B.4.b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
A.REI.C.6	Solve systems of linear equations algebraically (include using the elimination method) and graphically, focusing on pairs of linear equations in two variables.
F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions

	and absolute value functions.
A.REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
F.IF.C.7.d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
A.REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
A.REI.D.12	Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F.LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
F.LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice

1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure
8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• You can look at any function in different ways, such as with equations, graphs, tables, or words. Being able to move easily between these forms helps you understand the function better. • The complex number system builds on real numbers, making it possible for every quadratic and, more generally, every	• How does changing the equation of a function affect its graph? Also, how can you use a graph to figure out what the equation might be? • Why do we use complex numbers, and what do they stand for? • How can you predict the end behavior and zeros of a polynomial or rational function just

polynomial equation to have a solution. • A function's graph shows important features like intercepts, maximum and minimum points, symmetry, asymptotes, and how the function behaves at the ends. You can often predict these from the function's equation. • Exponential and logarithmic functions are opposites, or inverses, of each other. This connection is useful for solving equations, not just for making graphs.	by looking at its equation? • How are exponential functions and their inverses related, and how does this connection help you solve equations?

Instructional Focus

Learning Targets

Lesson 1 — Foundations of Linear and Non-Linear Functions

- I can find an explicit formula or recursive rule for a pattern described in words.
- I can find and interpret the average rate of change of a function from a table, graph, or equation.
- I can write a sequence both recursively and with an explicit formula, and switch between the two.
- I can solve a system of linear equations by graphing or by solving it algebraically.
- I can graph the solution to a system of linear inequalities.

Lesson 2 — Parent Functions and Transformations

- I can name and describe the basic shape of each parent function family.
- I can predict how shifting, stretching, or reflecting a function changes its graph.
- I can find the value of a transformation parameter by comparing a graph to its parent function.
- I can graph a square root function, including any domain restrictions.
- I can graph a cube root function.
- I can explain what a rational exponent means and connect it to radical notation.
- I can rewrite an expression with radicals or rational exponents using the rules of exponents.
- I can explain why the sum or product of a rational and an irrational number is irrational.
- I can identify a function's intercepts, extrema, symmetry, and end behavior from its graph.
- I can compare two functions given in different forms (table, graph, equation, words).

Lesson 3 — Quadratics and Complex Numbers

- I can explain what the number i is and identify a complex number written as a + bi.
- I can add, subtract, and multiply complex numbers.
- I can complete the square to derive the quadratic formula.
- I can solve any quadratic equation by factoring, square roots, completing the square, or the quadratic formula.
- I can write the complex solutions to a quadratic equation in the form a ± bi.

- I can solve a system made of one linear and one quadratic equation, algebraically and graphically.

Lesson 4 — Higher-Degree Polynomials

- I can divide polynomials using long division.
- I can divide polynomials using synthetic division.
- I can use the Remainder Theorem to find the remainder of a polynomial division without dividing.
- I can use the Factor Theorem to test whether a given value is a zero of a polynomial.
- I can find the real zeros of a polynomial and describe how each zero's multiplicity affects the graph.
- I can determine a polynomial's end behavior from its leading term.
- I can sketch a polynomial's graph using its zeros and end behavior.
- I can build a polynomial function to model a real situation.

Lesson 5 — Rational Functions and Asymptotes

- I can simplify a rational expression and rewrite it as a quotient plus a remainder over the divisor.
- I can find the asymptotes and end behavior of a rational function and use them to graph it.
- I can solve a rational equation and check for extraneous solutions.
- I can solve a radical equation and check for extraneous solutions.

Lesson 6 — Modeling with Exponential Functions

- I can build an exponential growth or decay model from a real situation.
- I can explain what the parameters in an exponential model mean in context.
- I can graph an exponential function and identify its intercept and end behavior.
- I can use an exponential model to solve a compound interest or growth/decay problem.

Lesson 7 — Inverses and Logarithmic Functions

- I can find the inverse of a function and solve $f(x) = c$.
- I can explain how a logarithm is the inverse of an exponential function.
- I can use logarithms to solve an exponential equation.
- I can graph a logarithmic function and an exponential function together as inverses.
- I can solve an equation graphically by finding where two functions intersect, using technology.

Prerequisite Skills

- Solving linear equations and inequalities in one variable.
- Basic function notation and evaluating functions for given inputs.
- Factoring quadratic expressions and solving quadratics with real solutions.
- Properties of integer exponents.
- Graphing linear, quadratic, and exponential parent functions.
- Solving systems of two linear equations by graphing or substitution.

Common Misconceptions

- Students may treat an equation as the only “real” representation of a function and struggle to extract the same information from a graph or table.
- Students often believe an even-degree polynomial with no real zeros “has no solutions” rather than recognizing complex solutions exist.

- Students frequently apply horizontal transformations in the wrong direction (treating f(x + k) as a right shift instead of a left shift).
- Students often assume all zeros of a polynomial cause the graph to cross the x-axis, missing that even multiplicity causes the graph to touch and turn.
- Students commonly view logarithms and exponents as unrelated procedures rather than as inverse operations.
- Students frequently forget to check for extraneous solutions after solving rational or radical equations, accepting values that make a denominator zero or a radicand negative.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
Function Transformations	Reapplied in Unit 2 to graph transformed sine/cosine functions (amplitude, period, midline, phase shift)	Warm-up in Unit 2, Lesson 10, Day 1: revisit the four transformation types on a non-trig parent function before introducing trig transformations
Key Features of graphs: Intercepts, extrema, symmetry, end behavior	Reapplied throughout Units 2 and 3 when interpreting graphs of periodic functions and statistical distributions	Warm-Up Prompts in Units 2–3 asking students to identify the same key features on a new graph type
Solving equations graphically using technology	Reused in Unit 2 to solve trigonometric equations graphically and in Unit 3 to assess linear/exponential model fit	Include one “solve/assess graphically” problem on each Unit 2 and Unit 3 quiz
Exponential growth/decay modeling and parameter interpretation	Directly connects to Unit 3, Lesson 14 (Linear Regression and Residuals), where students choose among linear/quadratic/exponential models for data	Buffer-day review (end of Trimester 1) revisiting exponential model-building before Unit 3 begins
Complex numbers and solving quadratics by all methods.	Underlies the Fundamental-Theorem-of-Algebra framing revisited briefly in Lesson 4 (Higher-Degree Polynomials) when discussing total zero count	Quick-check problem in Lesson 4 asking students to state how many total zeros (real + complex) a given polynomial has

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests and Quizzes • Unit 1 Benchmark (Link-It) - Covers Unit 1, Lessons 1 - 5 • Unit 2 Benchmark (Link-it) - Covers Unit 1, Lesson 6 - 7 and Unit 2, Lessons 8 - 9

Resources

Key Resources	Supplemental Resources
Saavas Algebra 2 Math Medic	iXL Delta Math Desmos Khan Academy Skew the Script Teacher-made worksheets FST Pacing Guide

Career Readiness, Life Literacies, and Key Skills

9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
9.4.12.CT.3	Enlist input from a variety of stakeholders (e.g., community members, experts in the field) to design a service learning activity that addresses a local or global issue (e.g., environmental justice).
9.1.12.CDM.8	Compare and compute interest and compound interest and develop an amortization table using business tools.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

Interdisciplinary Connections

HS-PS1-8	Develop models to illustrate the changes in the composition of the nucleus of the atom and the energy released during the processes of fission, fusion, and radioactive decay.
HS-LS2-1	Use mathematical and/or computational representations to support explanations of factors that affect carrying capacity of ecosystems at different scales.
8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.