Calculus Unit 2: Derivatives and Applications of Derivatives

Content Area: **Math**
Course(s):
Time Period: **MP2**
Length: **45**
Status: **Published**

Evidence (Assessments)

Formative/Summative:

- Written section assessments
- Review Games
- Practice exercises and assignments
- White board demonstrations
- Desmos Activities
- Written Topic Assessments
- Technology Assessments
- Benchmark 1 Assessment
- AP Benchmark 1

NJSLS

HSF.LE.A.1

ID.A.4

F.IF.7

F.IF.8

F.IF.9

ID.A.1

F.IF.4

Rationale, Transfer Goals, and Enduring Understandings

Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. Students build the derivative using the concept of limits and use the derivative primarily to compute the instantaneous rate of change of a function. Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyzing the graph of a function (for example, determining whether a function is increasing or decreasing and finding concavity and extreme values), and solving problems involving rectilinear motion. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. In addition, students should be able to solve separable differential equations, understand and be able to apply the Mean Value Theorem, and be familiar with a variety of real-world applications including related rates, optimization, and growth and decay models. (College Board)

Essential Questions

- How can a state determine the rate of change in high school graduates at a particular level of public investment in education (in graduates per dollar) based on a model for the number of graduates as a function of the state's education budget?

  Why do mathematical properties and rules for simplifying and evaluating limits apply to differentiation?

  If you knew that the rate of change in high school graduates at a particular level of public investment in education (in graduates per dollar) was a positive number, what might that tell you about the number of graduates at that level of investment?

Content

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    - The difference quotients express the average rate of change of a function over an interval.
    - The instantaneous rate of change of a function at x=a can be expressed by h0f(a+h)-f(a)h or xaf(x)-f(a)x-a, provided the limit exists. There are equivalent forms of the definition of the derivative and are denoted f'(a).

  The derivative of f is the function whose value at x is h0f(x+h)-f(x)h, provided this limit exists

- For y=f(x), notations for the derivative include dy/dx, f'(x), and y'.
- The derivative can be represented graphically, numerically, analytically, and verbally.
- The derivative of a function at a point is the slope of the line tangent to a graph of the function at that point.
- The derivative at a point can be estimated from information given in tables or graphs.
- Technology can be used to calculate or estimate the value of a derivative of a function at a point.
- If a function is differentiable at a point then it is continuous at that point. In particular, if a point is not in the domain of f, then it is not in the domain of f'.
- A continuous function may fail to be differentiable at a point in its domain.
- Direct application of the definition of the derivative and specific rules can be used to calculate the derivative four functions of the form
- f(x)=x'.
- Sums, differences, and constant multiples of functions can be differentiated using derivative rules.
- The power rule combined with sum, difference, and constant multiple properties can be used to find the derivatives for polynomial functions.
- Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions.
- The chain rule provides a way to differentiate composite functions.
- The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
- The derivative can be used to express information about rates of change in applied contexts.
- The derivative can be used to solve optimization problems; that is, finding a minimum or maximum value of a function on a given interval.

Skills

- Determine average rates of change using difference quotients.
- Represent the derivative of a function as the limit of a difference quotient.
- Represent the derivative of a function as the limit of a difference quotient.
- Determine the equation of a line tangent to a curve at a given point.
- Estimate derivatives.
- Explain the relationship between differentiability and continuity.
- Calculate derivatives of familiar functions.
- Interpret a limit as a definition of a derivative.
- Calculate derivatives of products and quotients of differentiable functions.
- Calculate derivatives of compositions of differentiable functions.
- Calculate derivatives of implicitly defined functions.
- Calculate derivatives of inverse and inverse trigonometric functions.
- Determine higher order derivatives of a function.
- Interpret the meaning of a derivative in context.
- Calculate rates of change in applied contexts.
- Approximate a value on a curve using the equation of a tangent line.
- Determine limits of functions that result in indeterminate forms.
- Justify conclusions about functions by applying the Mean Value Theorem over an interval.
- Calculate minimum and maximum values in applied contexts or analysis of functions.

Activities/Strategies

Math practice individually, whole group, and small group.

Peer group leadership

Student presentations of concepts and demonstration of skills

Students given access to online textbook

Partners or group work (groups formed heterogeneously according to ability)

Open Source activities below from Illustrative Math, Desmos, Geogebra:

- Sketchy Derivatives
- Average Value of a Function
- Derivative Match
- Functions and their derivatives
- Mean Value Theorem
- Related Rates Activities
- Optimization Problems
- Sketch the Derivative

Spiraling for Mastery

Content or Skill for this Unit	Spiral Focus from Previous Unit
• Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations. • Provide reasons or rationales for solutions and conclusions. • Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems. • Apply appropriate mathematical rules or procedures, with and without technology.	• Find Limits using graphs • Find one-sided limits using graphs • determine if a limit exists • Find limits using limit laws • Find limits of polynomials and rational functions • Find limits at vertical asymptotes using graphs

• Provide reasons or rationales for solutions and conclusions.	• Determine end behavior using graphs

Career Awareness, Exploration, Preparation, and Training

9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth

21st Century Skills

CRP2. Apply appropriate academic and technical skills.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

Interdisciplinary Connections

NJSLS ELA

NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

NJSLA Science

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength,

and speed of waves traveling in various media.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the

change in energy of the other component(s) and energy flows in and out of the system are known.