Linear and Quadratic Functions

Content Area: **Math**
Course(s):
Time Period: **MP2**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
Unit 2 focuses on extending students' understanding of linear functions and polynomials. This unit focuses on extending students' previous understanding of functions to include quadratic functions: graphing them, using them to model real-world situations, and comparing them to linear and exponential functions. Students also expand their knowledge of quadratic functions. Students learn to solve quadratic equations using tables, graphs, and factoring. Students also solve quadratic equations using square roots, completing the square, and the quadratic formula. Students learn different methods, such as graphing, elimination, and substitution, for solving linear-quadratic systems.	In this unit, students will build capacity related to applying algebraic concepts to real-world situations and using equations to model real-world situations. Applications of functions and linear relationships are skills that are built upon in future coursework.

NJSLS

MATH.9-12.F.BF.A.1	Write a function that describes a relationship between two quantities.
MATH.9-12.S.ID.B.6.a	Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.S.ID.B.6.b	Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.

MATH.9-12.S.ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
MATH.9-12.A.CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
MATH.9-12.A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MATH.9-12.A.CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MATH.9-12.F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MATH.9-12.F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MATH.9-12.F.IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.F.IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MATH.9-12.A.REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
MATH.9-12.F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MATH.9-12.F.LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure

Unit Focus

Enduring Understandings	Essential Questions
• When a linear equation is written in slope-intercept form, y=mx+b, m is the slope, and the line intersects the y-axis at (0, b), so the y-intercept is b. • The point-slope form of a linear equation is used to write the equation of a line using the slope and any point on the line. • The standard form of a linear equation is helpful for identifying the x- and y-intercepts. These are used to graph the line and to aid in understanding the constraints within a real-world context. • Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly. • The equations of lines can be used to help identify whether the lines are parallel or perpendicular. • Parallel lines have the same slope but different x- or y-intercepts; perpendicular lines have slopes that are opposite reciprocals. • The intersection of the graphs of a pair of linear equations is used to estimate the solution to the system. If two lines intersect at a point, there is exactly one solution. If two lines are the same, there are infinitely many solutions. If two lines are parallel, there is no solution. • Substitution is one method for solving systems of equations. The process involves solving one equation for a variable and substituting the solution into the system's other equation. This results in an equation in one variable. Solve for the variable and substitute its value into one of the original equations in the system to find the value of the second variable. • Elimination is an alternate method for solving systems of equations when it is not easy to use	• What information does the slope-intercept form of a linear equation reveal about a line? • What information does the point-slope form of a linear equation reveal about a line? • What information does the standard form of a linear equation reveal about a line? • How can the equations of lines help identify whether the lines are parallel, perpendicular, or neither? • How can you use a graph to illustrate the solution to a system of linear equations? • How do you use substitution to solve a system of linear equations? • Why does the elimination method work when solving a system of equations? • How does the graph of a linear inequality in two variables help you identify the solutions of the inequality? • How is the graph of a system of linear inequalities related to the solutions of the system of inequalities? • What is the quadratic parent function and how can you recognize the key features of its graph? • How can the vertex form of a quadratic function help you sketch the graph of a function? • How is the standard form of a quadratic function different from the vertex form? • What kinds of real-world situations can be modeled by quadratic functions? • How can you determine whether a linear, exponential, or quadratic function best models data?

substitution. Multiply one or both equations by a constant to get like coefficients that are opposite to use elimination. • The graph of a linear inequality in two variables shows that solutions of the inequality as a half-plane above or below the boundary line. The boundary line is included in the solution when the inequality symbol is ≤ or ≥ excluded when the inequality symbol is < or >. • Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly. • Systems of linear inequalities can be solved by graphing. The solution of a system of inequalities is the intersection of the corresponding half-planes, excluding the boundary lines in the case of a strict inequality. • Linear, quadratic, and exponential functions are differentiated by their average rates of change over different intervals. A linear function models a relationship between x and y in which the differences between successive y-values are constant. A quadratic function models a relationship in which the second differences, or the difference between the first differences, are constant. An exponential function models a relationship where the ratios of consecutive y-values are constant. Critical Knowledge and Skills	

Instructional Focus

Learning Targets

- Write linear equations in two variables using slope-intercept form to represent the relationship between two quantities.
- Interpret the slope and the intercept of a linear model.
- Write and graph linear equations in point-slope form.
- Analyze different forms of a line to interpret the slope and y-intercept of a linear model in the context of data
- Write and graph linear equations in standard form.
- Use linear equations in standard form to interpret both the x- and y-intercepts in the context of given data.
- Use mathematical modeling to represent a problem situation and to propose a solution.
- Test and verify the appropriateness of their math models.
- Explain why the results from their mathematical models might not align exactly with the problem situation.
- Create equations to represent lines that are parallel or perpendicular to a given line.
- Graph lines to show an understanding of the relationship between the slopes of parallel and perpendicular lines.
- Solve real-world problems with parallel or perpendicular lines.
- Graph systems of linear equations in two variables to find an approximate solution.
- Write a system of linear equations in two variables to represent real-world problems.
- Use the substitution method to solve systems of equations.
- Represent situations as a system of equations and interpret solutions as viable/nonviable options for the
  situation.
- Solve systems of linear equations and prove that the sum of one equation and a multiple of the other produces a system with the same solutions as the original system.
- Represent constraints with a system of equations in a modeling context.
- Graph solutions to linear inequalities in two variables.
- Represent constraints with inequalities and interpret solutions as viable or nonviable options in a modeling context.
- Use mathematical modeling to represent a problem situation and to propose a solution.
- Test and verify the appropriateness of their math models.
- Explain why the results from their mathematical models might not align exactly with the problem situation.
- Graph a solution set of a system of linear inequalities in two variables.
- Interpret solutions of linear inequalities in a modeling context.

Prerequisite Skills

- Graphing two lines. Finding a solution of two lines
- solving a system algebraically and finding a solution of 2 lines
- solving a system algebraically and finding a solution of 2 lines
- comparing numbers
- graphing and reading a graph
- solving equations and finding a solution

- graphing inequalities
- types of lines for solution sets ex: no solution, parallel lines
- graphing inequalities and finding solutions
- different types of solutions of systems of equations
- evaluating an algebra equation and interpreting its meaning
- function notation
- defining variables, function notation
- domain and range
- finding a domain of the function from a given graph
- domain and range, graphing, finding slope
- finding an x- and y-intercept from a graph and writing it as an ordered pair, finding the slope from a graph and finding the domain of a function
- standard form of an equation, slope-intercept form of an equation, graphing an equation, creating a "t" chart of points for an equation
- slope, x-intercept, slope intercept form, negative reciprocals
- finding the slope of a line graphed, finding the slope of a table of values, absolute value
- Identify key features of the graph of a quadratic function using graphs, tables, and equations.
- Explain the effect of the value of a on the quadratic parent function.
- Identify key features of the graph of quadratic functions written in vertex form.
- Graph quadratic functions in vertex form.
- Graph quadratic functions in standard form and show intercepts, maxima, and minima.
- Determine how the values of a, b, and c affect the graph of $f(x)=ax^2+bx+c$.
- Identify key features of parabolas.
- Compare properties of quadratic functions presented in different forms (algebraically, in a table, graphically).
- Use quadratic functions fitted to data to model real-world situations.
- Use the vertical motion model to write an equation.
- Compare a model to a data set by analyzing and evaluating residuals.
- Use mathematical modeling to represent a problem situation and to propose a solution.
- Test and verify the appropriateness of their math models.
- Explain why the results from their mathematical models might not align exactly with the problem situation.
- Determine which model - linear, exponential, or quadratic - best fits a set of data.
- Use fitted function to solve problems in the context of data.
- Use a graph to identify the x-intercepts as solutions of a quadratic equation.
- Use a graphing calculator to make a table of values to approximate or solve a quadratic equation.
- Use the Zero-Product Property and factoring to find the solutions of a quadratic equation.

- Apply factoring to solve real-world problems.
- Use the zeros of a quadratic equation to sketch a graph.
- Write the factored form of a quadratic function from a graph.
- Use properties of exponents to rewrite radical expressions.
- Multiply radical expressions.
- Write a radical expression to model or represent a real-world problem.
- Solve quadratic equations by finding square roots.
- Determine reasonable solutions for real-world problems.
- Solve a quadratic trinomial by completing the square to transform a quadratic equation into a perfect square trinomial.
- Use completing the square to write a quadratic equation in vertex form.
- Derive the quadratic formula by completing the square.
- Solve quadratic equations in one variable by using the quadratic formula.
- Use the discriminant to determine the number and type of solutions to a quadratic equation.
- Use mathematical modeling to represent a problem situation.□ Test and verify the appropriateness of their math models.
- Explain why the results might not exactly match the problem situation.
- Describe a linear-quadratic system of equations.
- Solve a linear-quadratic system of equations by graphing, elimination, or substitution.

Common Misconceptions

- In solving systems graphically, students may mistake a system with infinite solutions for a system with no solutions.
- When solving by substitution, students may substitute into the same equation they used to isolate the variable.
- Students may attempt to solve a system of linear equations using a less efficient method.
- When students are graphing a system of inequalities, they might shade the wrong side of the line.
- When students are graphing a system of inequalities, they might confuse whether the boundary line should be solid or dashed.
- When students are graphing a system of inequalities, they might not know what to do if a point falls ON one of the lines in the system.
- When students are graphing a system of inequalities whose graph forms a set of parallel lines, they might assume that the system has no solutions.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity

• Writing Euqations in Different Forms • Using Slope • Systems of Linear Equations • Systems of Linear Inequalities • Quadratic Functions • Modeling with Quadratic Functions • Solving Quadratic Equations • Solving Linear-Quadratic Systems of Equations	• Operations on Real Numbers (Grade 8) • Solving Linear Equations (Grade 8) • Literal Equations (Algebra I) • Solving Inequalities (Grade 7) • Slope Intercept Form (Grade 8) • Standard Form (Algebra I) • Analyzing Graphs of Functions (Grade 8) • Using Functions to Model Relationships (Grade 8/Algebra I) • Solving Linear Equations (Grade 8/Algebra I) • Graphing Quadratic Functions (Algebra I)	• IXL • Math Diagnostic and Intervention System Actibities

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests • Unit Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
• Savvas EnVision Algebra I	• IXL • Delta Math • Desmos • Khan Academy

Career Readiness, Life Literacies, and Key Skills

TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.
TECH.9.4.12.IML.1	Compare search browsers and recognize features that allow for filtering of information.
TECH.9.4.12.IML.7	Develop an argument to support a claim regarding a current workplace or societal/ethical issue such as climate change (e.g., NJSLSA.W1, 7.1.AL.PRSNT.4).

Interdisciplinary Connections

ELA.RI.AA.9–10.7	Describe and evaluate the argument and specific claims in an informational text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and reasoning.
ELA.SL.PE.9–10.1.C	Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.
ELA.SL.ES.9–10.3	Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, identifying any false reasoning or distorted evidence.
9-12.HS-PS2-1	Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
9-12.HS-PS2-4	Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.