AP Statistics Unit 3 - Inferences for Categorical and Quantitative Data

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Unit Overview

Unit Summary	Unit Rationale
This unit encompasses the major forms of statistical inference used to draw conclusions about populations from sample data, with a focus on proportions, means, categorical data, and regression slopes. Students develop a comprehensive understanding of how to construct and interpret confidence intervals, conduct significance tests, and make informed decisions based on data. Students begin by exploring statistical questions suggested by sample variability and learn to identify appropriate inference procedures for estimating or testing population proportions. They will construct and interpret confidence intervals for single proportions and for the difference between two proportions, and analyze the relationship between margin of error, confidence level, and sample size. In significance testing, students identify hypotheses, verify conditions, calculate test statistics and p-values, and use the results to support or reject claims. A deep understanding of Type I and Type II errors and the factors affecting these probabilities is emphasized throughout. In Part 7, students shift focus to inference for population means using t-distributions. They learn to construct and interpret confidence intervals and perform significance tests for a single mean, the difference in means for paired data, and the difference between two independent means. Emphasis is placed on verifying conditions, calculating margins of error, and understanding how sample size and variability affect inference. Students build fluency in interpreting p-values and supporting claims with statistical evidence.	The ability to make data-based decisions is a critical skill in today's world, and statistical inference provides the framework for drawing valid conclusions from data. This unit, covering inference for proportions, means, categorical data, and regression slopes, is essential for developing students' analytical thinking and statistical literacy. It builds on foundational concepts of sampling variability and probability to empower students to estimate population parameters and evaluate claims using real-world data. Throughout this unit, students engage with all aspects of statistical inference—formulating hypotheses, verifying assumptions, calculating confidence intervals and test statistics, and interpreting results in context. These skills are not only central to the AP Statistics curriculum but are also foundational to fields such as science, healthcare, economics, education, and social sciences. Students learn to distinguish between correlation and causation, assess the strength of evidence, and understand the limitations of their conclusions. By applying inference methods to categorical and quantitative data, students gain a well-rounded understanding of how statistical tools can answer meaningful questions. Emphasis on interpreting p-values, margin of error, and the consequences of Type I and Type II errors helps students critically evaluate research findings and data-based claims. Ultimately, this unit prepares students to be informed consumers of data and thoughtful contributors to data-driven discussions in their academic and professional lives.

Part 8 introduces chi-square tests as tools for evaluating relationships in categorical data. Students perform goodness-of-fit tests to assess whether an observed distribution matches an expected one, and they conduct tests for homogeneity and independence in two-way tables. The process involves calculating expected counts, verifying assumptions, computing test statistics, and interpreting p-values. Students justify conclusions about categorical variables based on the strength of statistical evidence. The unit concludes with inference for linear regression slopes. Students analyze scatterplots to form questions about relationships between quantitative variables. They construct and interpret confidence intervals for the slope of a least-squares regression line and conduct hypothesis tests to evaluate the presence of a linear relationship. Mastery of conditions for inference, understanding the impact of sample size, and interpreting slope results in context are key goals. By the end of this unit, students will be proficient in using statistical reasoning and methods to estimate population parameters and test claims using a variety of models. They will be able to support data-based conclusions in context and recognize the scope and limitations of statistical inference.	

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MA.9-12.6.1.VAR-1.H.1	Variation in shapes of data distributions may be random or not.
MA.9-12.6.2.UNC-4.A.1	The appropriate confidence interval procedure for a one-sample proportion for one categorical variable is a one sample z-interval for a proportion.
MA.9-12.6.2.UNC-4.B.1	In order to make assumptions necessary for inference on population proportions, means, and slopes, we must check for independence in data collection methods and for selection of the appropriate sampling distribution.
MA.9-12.6.2.UNC-4.B.2.a.i	Data should be collected using a random sample or a randomized experiment.
MA.9-12.6.2.UNC-4.B.2.a.ii	When sampling without replacement, check that $n \leq 10\%N$, where N is the size of the population.
MA.9-12.6.2.UNC-4.B.2.b.i	For categorical variables, check that both the number of successes, $n\hat{p}$, and the number of failures, $n(1-\hat{p})$ are at least 10 so that the sample size is large enough to support an

	assumption of normality.
MA.9-12.6.2.UNC-4.C.1	Based on sample data, the standard error of a statistic is an estimate for the standard deviation for the statistic. The standard error of $\hat{p}$ is $SE_{\hat{p}} = \sqrt{(\hat{p}(1-\hat{p}))/n}$.
MA.9-12.6.2.UNC-4.C.2	A margin of error gives how much a value of a sample statistic is likely to vary from the value of the corresponding population parameter.
MA.9-12.6.2.UNC-4.C.3	For categorical variables, the margin of error is the critical value (z^*) times the standard error (SE) of the relevant statistic, which equals $z^*\sqrt{(\hat{p}(1-\hat{p}))/n}$ for a one sample proportion.
MA.9-12.6.2.UNC-4.C.4	The formula for margin of error can be rearranged to solve for n, the minimum sample size needed to achieve a given margin of error. For this purpose, use a guess for $\hat{p}$ or use $\hat{p}$ = 0.5 in order to find an upper bound for the sample size that will result in a given margin of error.
MA.9-12.6.2.UNC-4.D.1	In general, an interval estimate can be constructed as point estimate ± (margin of error). For a one-sample proportion, the interval estimate is $\hat{p} \pm z^*\sqrt{(\hat{p}(1-\hat{p}))/n}$.
MA.9-12.6.2.UNC-4.D.2	Critical values represent the boundaries encompassing the middle C% of the standard normal distribution, where C% is an approximate confidence level for a proportion.
MA.9-12.6.2.UNC-4.E.1	Confidence intervals for population proportions can be used to calculate interval estimates with specified units.
MA.9-12.6.3.UNC-4.F.1	A confidence interval for a population proportion either contains the population proportion or it does not, because each interval is based on random sample data, which varies from sample to sample.
MA.9-12.6.3.UNC-4.F.2	We are C% confident that the confidence interval for a population proportion captures the population proportion.
MA.9-12.6.3.UNC-4.F.3	In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the population proportion.
MA.9-12.6.3.UNC-4.F.4	Interpreting a confidence interval for a one-sample proportion should include a reference to the sample taken and details about the population it represents.
MA.9-12.6.3.UNC-4.G.1	A confidence interval for a population proportion provides an interval of values that may provide sufficient evidence to support a particular claim in context.
MA.9-12.6.3.UNC-4.H.1	When all other things remain the same, the width of the confidence interval for a population proportion tends to decrease as the sample size increases. For a population proportion, the width of the interval is proportional to $1/\sqrt{n}$.
MA.9-12.6.3.UNC-4.H.2	For a given sample, the width of the confidence interval for a population proportion increases as the confidence level increases.
MA.9-12.6.3.UNC-4.H.3	The width of a confidence interval for a population proportion is exactly twice the margin of error.
MA.9-12.6.4.VAR-6.D.1	The null hypothesis is the situation that is assumed to be correct unless evidence suggests otherwise, and the alternative hypothesis is the situation for which evidence is being collected.
MA.9-12.6.4.VAR-6.D.2	For hypotheses about parameters, the null hypothesis contains an equality reference (=, ≥, or ≤), while the alternative hypothesis contains a strict inequality (<, >, or ≠). The type of inequality in the alternative hypothesis is based on the question of interest. Alternative hypotheses with < or > are called one-sided, and alternative hypotheses with ≠ are called two-sided. Although the null hypothesis for a one-sided test may include an inequality symbol, it is still tested at the boundary of equality.
MA.9-12.6.4.VAR-6.D.3	The null hypothesis for a population proportion is: $H_0 : p = p_0$, where p_0 is the null hypothesized value for the population proportion.
MA.9-12.6.4.VAR-6.D.4	A one-sided alternative hypothesis for a proportion is either $H_a : p < p_0$ or $H_a : p > p_0$. A two-sided alternate hypothesis is $H_a : p_1 \neq p_2$.

MA.9-12.6.4.VAR-6.D.5	For a one-sample z-test for a population proportion, the null hypothesis specifies a value for the population proportion, usually one indicating no difference or effect.
MA.9-12.6.4.VAR-6.E.1	For a single categorical variable, the appropriate testing method for a population proportion is a one-sample z-test for a population proportion.
MA.9-12.6.4.VAR-6.F.1.a.i	Data should be collected using a random sample or a randomized experiment.
MA.9-12.6.4.VAR-6.F.1.a.ii	When sampling without replacement, check that $n \leq 10\%N$.
MA.9-12.6.4.VAR-6.F.1.b.i	Assuming that H_0 is true ($p = p_0$), verify that both the number of successes, np_0 and the number of failures, $n(1 - p_0)$ are at least 10 so that that the sample size is large enough to support an assumption of normality.
MA.9-12.6.5.VAR-6.G.1	The distribution of the test statistic assuming the null hypothesis is true (null distribution) can be either a randomization distribution or when a probability model is assumed to be true, a theoretical distribution (z).
MA.9-12.6.5.VAR-6.G.2	When using a z-test, the standardized test statistic can be written: test statistic = (sample statistic null value of the parameter)/(standard deviation of the statistic). This is called a z-statistic for proportions.
MA.9-12.6.5.VAR-6.G.3	The test statistic for a population proportion is: $z = (\hat{p} - p_0)/\sqrt{(p_0(1 - p_0))/n}$.
MA.9-12.6.5.VAR-6.G.4	A p-value is the probability of obtaining a test statistic as extreme or more extreme than the observed test statistic when the null hypothesis and probability model are assumed to be true. The significance level may be given or determined by the researcher.
MA.9-12.6.5.DAT-3.A.1.a	The proportion at or above the observed value of the test statistic, if the alternative is >.
MA.9-12.6.5.DAT-3.A.1.b	The proportion at or below the observed value of the test statistic, if the alternative is <.
MA.9-12.6.5.DAT-3.A.1.c	The proportion less than or equal to the negative of the absolute value of the test statistic plus the proportion greater than or equal to the absolute value of the test statistic, if the alternative is ≠.
MA.9-12.6.5.DAT-3.A.2	An interpretation of the p-value of a significance test for a one-sample proportion should recognize that the p-value is computed by assuming that the probability model and null hypothesis are true, i.e., by assuming that the true population proportion is equal to the particular value stated in the null hypothesis.
MA.9-12.6.6.DAT-3.B.1	The significance level,α, is the predetermined probability of rejecting the null hypothesis given that it is true.
MA.9-12.6.6.DAT-3.B.2	A formal decision explicitly compares the p-value to the significance level, α. If the p-value ≤ α, reject the null hypothesis. If the p-value > α, fail to reject the null hypothesis.
MA.9-12.6.6.DAT-3.B.3	Rejecting the null hypothesis means there is sufficient statistical evidence to support the alternative hypothesis. Failing to reject the null means there is insufficient statistical evidence to support the alternative hypothesis.
MA.9-12.6.6.DAT-3.B.4	The conclusion about the alternative hypothesis must be stated in context.
MA.9-12.6.6.DAT-3.B.5	A significance test can lead to rejecting or not rejecting the null hypothesis, but can never lead to concluding or proving that the null hypothesis is true. Lack of statistical evidence for the alternative hypothesis is not the same as evidence for the null hypothesis.
MA.9-12.6.6.DAT-3.B.6	Small p-values indicate that the observed value of the test statistic would be unusual if the null hypothesis and probability model were true, and so provide evidence for the alternative. The lower the p-value, the more convincing the statistical evidence for the alternative hypothesis.
MA.9-12.6.6.DAT-3.B.7	p-values that are not small indicate that the observed value of the test statistic would not be unusual if the null hypothesis and probability model were true, so do not provide convincing statistical evidence for the alternative hypothesis nor do they provide evidence that the null hypothesis is true.
MA.9-12.6.6.DAT-3.B.8	A formal decision explicitly compares the p-value to the significance α. If the p-value ≤ α, then reject the null hypothesis, $H_0 : p = p_0$. If the p-value > α, then fail to reject the null

	hypothesis.
MA.9-12.6.6.DAT-3.B.9	The results of a significance test for a population proportion can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
MA.9-12.6.7.UNC-5.A.1	A Type I error occurs when the null hypothesis is true and is rejected (false positive).
MA.9-12.6.7.UNC-5.A.2	A Type II error occurs when the null hypothesis is false and is not rejected (false negative).
MA.9-12.6.7.UNC-5.B.1	The significance level,α, is the probability of making a Type I error, if the null hypothesis is true.
MA.9-12.6.7.UNC-5.B.2	The power of a test is the probability that a test will correctly reject a false null hypothesis.
MA.9-12.6.7.UNC-5.B.3	The probability of making a Type II error = $1 - power$.
MA.9-12.6.7.UNC-5.C.1	The probability of a Type II error decreases when any of the following occurs, provided the others do not change:
MA.9-12.6.7.UNC-5.C.1.i	Sample size(s) increases.
MA.9-12.6.7.UNC-5.C.1.ii	Significance level (α) of a test increases.
MA.9-12.6.7.UNC-5.C.1.iii	Standard error decreases.
MA.9-12.6.7.UNC-5.C.1.iv	True parameter value is farther from the null.
MA.9-12.6.7.UNC-5.D.1	Whether a Type I or a Type II error is more consequential depends upon the situation.
MA.9-12.6.7.UNC-5.D.2	Since the significance level,α, is the probability of a Type I error, the consequences of a Type I error influence decisions about a significance level.
MA.9-12.6.8.UNC-4.I.1	The appropriate confidence interval procedure for a two-sample comparison of proportions for one categorical variable is a two-sample z-interval for a difference between population proportions.
MA.9-12.6.8.UNC-4.J.1.a.i	Data should be collected using two independent, random samples or a randomized experiment.
MA.9-12.6.8.UNC-4.J.1.a.ii	When sampling without replacement, check that $n_1 \leq 10\% N_1$ and $n_2 \leq 10\% N_2$.
MA.9-12.6.8.UNC-4.J.1.b.i	For categorical variables, check that $n_1 \hat{p}_1$, $n_1(1 - \hat{p}_1)$, $n_2 \hat{p}_2$, and $n_2(1 - \hat{p}_2)$ are all greater than or equal to some predetermined value, typically either 5 or 10.
MA.9-12.6.8.UNC-4.K.1	For a comparison of proportions, the interval estimate is $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{[(\hat{p}_1(1 - \hat{p}_1))/n_1 + (\hat{p}_2(1 - \hat{p}_2))/n_2)]}$.
MA.9-12.6.8.UNC-4.L.1	Confidence intervals for a difference in proportions can be used to calculate interval estimates with specified units.
MA.9-12.6.9.UNC-4.M.1	In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the difference in population proportions.
MA.9-12.6.9.UNC-4.M.2	Interpreting a confidence interval for difference between population proportions should include a reference to the sample taken and details about the population it represents.
MA.9-12.6.9.UNC-4.N.1	A confidence interval for difference in population proportions provides an interval of values that may provide sufficient evidence to support a particular claim in context.
MA.9-12.6.10.VAR-6.H.1	For a two-sample test for a difference of two proportions, the null hypothesis specifies a value of 0 for the difference in population proportions, indicating no difference or effect.
MA.9-12.6.10.VAR-6.H.2	The null hypothesis for a difference in proportions is: $H_0 : p_1 = p_2$, or $H_0 : p_1 - p_2 = 0$.
MA.9-12.6.10.VAR-6.H.3	A one-sided alternative hypothesis for a difference in proportions is $H_a : p_1 < p_2$, or, $H_a : p_1 > p_2$. A two-sided alternative hypothesis for a difference of proportions is $H_a : p_1 \neq p_2$.
MA.9-12.6.10.VAR-6.I.1	For a single categorical variable, the appropriate testing method for the difference of two population proportions is a two-sample z-test for a difference between two population proportions.

MA.9-12.6.10.VAR-6.J.1.a.i	Data should be collected using two independent, random samples or a randomized experiment.
MA.9-12.6.10.VAR-6.J.1.a.ii	When sampling without replacement, check that $n_1 \leq 10\%N_1$ and $n_2 \leq 10\%N_2$.
MA.9-12.6.10.VAR-6.J.1.b.i	For the combined sample, define the combined (or pooled) proportion, pˆ[subscript c] = (n_1pˆ$_1$ + n_2pˆ$_2$)/($n_1 + n_2$). Assuming that H_0 is true ($p_1 - p_2 = 0$ or $p_1 = p_2$), check that n_1pˆ[subscript c], n_1(1 − pˆ[subscript c]), n_2pˆ[subscript c], and n_2(1 − pˆ[subscript c]) are all greater than or equal to some predetermined value, typically either 5 or 10.
MA.9-12.6.11.VAR-6.K.1	The test statistic for a difference in proportions is: z = [(pˆ$_1$ − pˆ$_2$) − 0]/[(√pˆ_c(1 − pˆ_c))√(1/n_1 + 1/n_2)], where pˆ_c = (n_1pˆ$_1$ + n_2pˆ$_2$)/($n_1 + n_2$).
MA.9-12.6.11.DAT-3.C.1	An interpretation of the p-value of a significance test for a difference of two population proportions should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population proportions are equal to each other.
MA.9-12.6.11.DAT-3.D.1	A formal decision explicitly compares the p-value to the significance α. If the p-value $\leq \alpha$, then reject the null hypothesis, $H_0 : p_1 = p_2$, or $H_0 : p_1 - p_2 = 0$. If the p-value $> \alpha$, then fail to reject the null hypothesis.
MA.9-12.6.11.DAT-3.D.2	The results of a significance test for a difference of two population proportions can serve as the statistical reasoning to support the answer to a research question about the two populations that were sampled.
MA.9-12.7.1.VAR-1.I.1	Random variation may result in errors in statistical inference.
MA.9-12.7.2.VAR-7.A.1	When s is used instead of σ to calculate a test statistic, the corresponding distribution, known as the t-distribution, varies from the normal distribution in shape, in that more of the area is allocated to the tails of the density curve than in a normal distribution.
MA.9-12.7.2.VAR-7.A.2	As the degrees of freedom increase, the area in the tails of a t-distribution decreases.
MA.9-12.7.2.UNC-4.O.1	Because σ is typically not known for distributions of quantitative variables, the appropriate confidence interval procedure for estimating the population mean of one quantitative variable for one sample is a one-sample t-interval for a mean.
MA.9-12.7.2.UNC-4.O.2	For one quantitative variable, X, that is normally distributed, the distribution of $t = (\bar{x} - \mu)/(s/\sqrt{n})$ is a t-distribution with $n - 1$ degrees of freedom.
MA.9-12.7.2.UNC-4.O.3	Matched pairs can be thought of as one sample of pairs. Once differences between pairs of values are found, inference for confidence intervals proceeds as for a population mean.
MA.9-12.7.2.UNC-4.P.1.a.i	Data should be collected using a random sample or a randomized experiment.
MA.9-12.7.2.UNC-4.P.1.a.ii	When sampling without replacement, check that $n \leq 10\%N$, where N is the size of the population.
MA.9-12.7.2.UNC-4.P.1.b.i	If the observed distribution is skewed, n should be greater than 30.
MA.9-12.7.2.UNC-4.P.1.b.ii	If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.
MA.9-12.7.2.UNC-4.Q.1	The critical value t^* with $n - 1$ degrees of freedom can be found using a table or computer-generated output.
MA.9-12.7.2.UNC-4.Q.2	The standard error for a sample mean is given by $SE = s/\sqrt{n}$, where s is the sample standard deviation.
MA.9-12.7.2.UNC-4.Q.3	For a one-sample t-interval for a mean, the margin of error is the critical value (t^*) times the standard error (SE), which equals $t^*(s/\sqrt{n})$.
MA.9-12.7.2.UNC-4.R.1	The point estimate for a population mean is the sample mean, $\bar{x}$.
MA.9-12.7.2.UNC-4.R.2	For the population mean for one sample with unknown population standard deviation, the confidence interval is $\bar{x} \pm t^* s/\sqrt{n}$.
MA.9-12.7.3.UNC-4.S.1	A confidence interval for a population mean either contains the population mean or it does not, because each interval is based on data from a random sample, which varies from

	sample to sample.
MA.9-12.7.3.UNC-4.S.2	We are C% confident that the confidence interval for a population mean captures the population mean.
MA.9-12.7.3.UNC-4.S.3	An interpretation of a confidence interval for a population mean includes a reference to the sample taken and details about the population it represents.
MA.9-12.7.3.UNC-4.T.1	A confidence interval for a population mean provides an interval of values that may provide sufficient evidence to support a particular claim in context.
MA.9-12.7.3.UNC-4.U.1	When all other things remain the same, the width of a confidence interval for a population mean tends to decrease as the sample size increases.
MA.9-12.7.3.UNC-4.U.2	For a single mean, the width of the interval is proportional to $1/\sqrt{n}$.
MA.9-12.7.3.UNC-4.U.3	For a given sample, the width of the confidence interval for a population mean increases as the confidence level increases.
MA.9-12.7.4.VAR-7.B.1	The appropriate test for a population mean with unknown σ is a one-sample t-test for a population mean.
MA.9-12.7.4.VAR-7.B.2	Matched pairs can be thought of as one sample of pairs. Once differences between pairs of values are found, inference for significance testing proceeds as for a population mean.
MA.9-12.7.4.VAR-7.C.1	The null hypothesis for a one-sample t-test for a population mean is $H_0 : \mu = \mu_0$, where μ_0 is the hypothesized value. Depending upon the situation, the alternative hypothesis is H_a : $\mu < \mu_0$ or $H_a : \mu > \mu_0$, or $H_a : \mu \neq \mu_0$.
MA.9-12.7.4.VAR-7.C.2	When finding the mean difference, μ[subscript d], between values in a matched pair, it is important to define the order of subtraction.
MA.9-12.7.4.VAR-7.D.1.a.i	Data should be collected using a random sample or a randomized experiment.
MA.9-12.7.4.VAR-7.D.1.a.ii	When sampling without replacement, check that $n \leq 10\%N$.
MA.9-12.7.4.VAR-7.D.1.b.i	If the observed distribution is skewed, n should be greater than 30.
MA.9-12.7.4.VAR-7.D.1.b.ii	If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.
MA.9-12.7.5.VAR-7.E.1	For a single quantitative variable when random sampling with replacement from a population that can be modeled with a normal distribution with mean μ and standard deviation σ, the sampling distribution of $t = (\bar{x} - \mu)/(s/\sqrt{n})$ has a t-distribution with $n - 1$ degrees of freedom.
MA.9-12.7.5.DAT-3.E.1	An interpretation of the p-value of a significance test for a population mean should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population mean is equal to the particular value stated in the null hypothesis.
MA.9-12.7.5.DAT-3.F.1	A formal decision explicitly compares the p-value to the significance α. If the p-value ≤ α, then reject the null hypothesis, $H_0 : \mu = \mu_0$. If the p-value > α, then fail to reject the null hypothesis.
MA.9-12.7.5.DAT-3.F.2	The results of a significance test for a population mean can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
MA.9-12.7.6.UNC-4.V.1	Consider a simple random sample from population 1 of size n_1, mean μ_1, and standard deviation σ_1 and a second simple random sample from population 2 of size n_2, mean μ_2, and standard deviation σ_2. If the distributions of populations 1 and 2 are normal or if both n_1 and n_2 are greater than 30, then the sampling distribution of the difference of means, $\bar{x}_1 - \bar{x}_2$ is also normal. The mean for the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is $\mu_1 - \mu_2$. The standard deviation of $\bar{x}_1 - \bar{x}_2$ + is $\sqrt{[(\sigma_1)^2/n_1 + (\sigma_2)^2/n_2]}$.
MA.9-12.7.6.UNC-4.V.2	The appropriate confidence interval procedure for one quantitative variable for two independent samples is a two-sample t-interval for a difference between population means.

MA.9-12.7.6.UNC-4.W.1.a.i	Data should be collected using two independent, random samples or a randomized experiment.
MA.9-12.7.6.UNC-4.W.1.a.ii	When sampling without replacement, check that $n_1 \leq 10\% N_1$ and $n_2 \leq 10\% N_2$.
MA.9-12.7.6.UNC-4.W.1.b.i	If the observed distributions are skewed, both n_1 and n_2 should be greater than 30.
MA.9-12.7.6.UNC-4.X.1	For the difference of two sample means, the margin of error is the critical value (t^*) times the standard error (SE) of the difference of two means.
MA.9-12.7.6.UNC-4.X.2	The standard error for the difference in two sample means with sample standard deviations, s_1 and s_2, is $\sqrt{[(s_1)^2/n_1) + (s_2)^2/n_2]}$.
MA.9-12.7.6.UNC-4.Y.1	The point estimate for the difference of two population means is the difference in sample means, $\bar{x}_1 - \bar{x}_2$.
MA.9-12.7.6.UNC-4.Y.2	For a difference of two population means where the population standard deviations are not known, the confidence interval is $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{(s_1^2/n_1 + s_2^2/n_2)}$ where $\pm t^*$ are the critical values for the central C% of a t-distribution with appropriate degrees of freedom that can be found using technology.
MA.9-12.7.7.UNC-4.Z.1	In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the difference of population means.
MA.9-12.7.7.UNC-4.Z.2	An interpretation for a confidence interval for the difference of two population means should include a reference to the samples taken and details about the populations they represent.
MA.9-12.7.7.UNC-4.AA.1	A confidence interval for a difference of population means provides an interval of values that may provide sufficient evidence to support a particular claim in context.
MA.9-12.7.7.UNC-4.AB.1	When all other things remain the same, the width of the confidence interval for the difference of two means tends to decrease as the sample sizes increase.
MA.9-12.7.8.VAR-7.F.1	For a quantitative variable, the appropriate test for a difference of two population means is a two-sample t-test for a difference of two population means.
MA.9-12.7.8.VAR-7.G.1	The null hypothesis for a two-sample t-test for a difference of two population means, μ_1 and μ_2, is: $H_0 : \mu_1 - \mu_2 = 0$, or $H_0 : \mu_1 = \mu_2$. The alternative hypothesis is $H_a : \mu_1 - \mu_2 < 0$, or $H_a : \mu_1 - \mu_2 > 0$, or $H_a : \mu_1 - \mu_2 \neq 0$, or $H_a : \mu_1 > \mu_2$, or $H_a : \mu_1 < \mu_2$, or $H_a : \mu_1 \neq \mu_2$.
MA.9-12.7.8.VAR-7.H.1.a.i	Data should be collected using simple random samples or a randomized experiment.
MA.9-12.7.8.VAR-7.H.1.a.ii	When sampling without replacement, check that $n_1 \leq 10\% N_1$ and $n_2 \leq 10\% N_2$.
MA.9-12.7.8.VAR-7.H.1.b.i	If the observed distribution is skewed, both n_1 and n_2 should be greater than 30.
MA.9-12.7.8.VAR-7.H.1.b.ii	If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers. This should be checked for BOTH samples.
MA.9-12.7.9.VAR-7.I.1	For a single quantitative variable, data collected using independent random samples or a randomized experiment from two populations, each of which can be modeled with a normal distribution, the sampling distribution of $t = [(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)]/\sqrt{(s_1^2/n_1 + s_2^2/n_2)}$ is an approximate t-distribution with degrees of freedom that can be found using technology. The degrees of freedom fall between the smaller of $n_1 - 1$ and $n_2 - 1$ and $n_1 + n_2 - 2$.
MA.9-12.7.9.DAT-3.G.1	An interpretation of the p-value of a significance test for a two-sample difference of population means should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population means are equal to each other.
MA.9-12.7.9.DAT-3.H.1	A formal decision explicitly compares the p-value to the significance α. If the p-value $\leq \alpha$, then reject the null hypothesis, $H_0 : \mu_1 - \mu_2 = 0$, or $H_0 : \mu_1 = \mu_2$. If the p-value $> \alpha$, then fail to reject the null hypothesis.
MA.9-12.7.9.DAT-3.H.2	The results of a significance test for a two-sample test for a difference between two population means can serve as the statistical reasoning to support the answer to a research question about the populations that were sampled.

MA.9-12.7.10	Skills Focus: Selecting, Implementing, and Communicating Inference Procedures
MA.9-12.8.1.VAR-1.J.1	Variation between what we find and what we expect to find may be random or not.
MA.9-12.8.2.VAR-8.A.1	Expected counts of categorical data are counts consistent with the null hypothesis. In general, an expected count is a sample size times a probability.
MA.9-12.8.2.VAR-8.A.2	The chi-square statistic measures the distance between observed and expected counts relative to expected counts.
MA.9-12.8.2.VAR-8.A.3	Chi-square distributions have positive values and are skewed right. Within a family of density curves, the skew becomes less pronounced with increasing degrees of freedom.
MA.9-12.8.2.VAR-8.B.1	For a chi-square goodness-of-fit test, the null hypothesis specifies null proportions for each category, and the alternative hypothesis is that at least one of these proportions is not as specified in the null hypothesis.
MA.9-12.8.2.VAR-8.C.1	When considering a distribution of proportions for one categorical variable, the appropriate test is the chi-square test for goodness of fit.
MA.9-12.8.2.VAR-8.D.1	Expected counts for a chi-square goodness-of-fit test are (sample size) (null proportion).
MA.9-12.8.2.VAR-8.E.1.a.i	Data should be collected using a random sample or randomized experiment.
MA.9-12.8.2.VAR-8.E.1.a.ii	When sampling without replacement, check that $n \leq 10\%N$.
MA.9-12.8.2.VAR-8.E.1.b.i	A conservative check for large counts is that all expected counts should be greater than 5.
MA.9-12.8.3.VAR-8.F.1	The test statistic for the chi-square test for goodness of fit is $\chi^2 = \Sigma$(Observed count − Expected count)^2/ (Expected count), with degrees of freedom = number of categories − 1.
MA.9-12.8.3.VAR-8.F.2	The distribution of the test statistic assuming the null hypothesis is true (null distribution) can be either a randomization distribution or, when a probability model is assumed to be true, a theoretical distribution (chi-square).
MA.9-12.8.3.VAR-8.G.1	The p-value for a chi-square test for goodness of fit for a number of degrees of freedom is found using the appropriate table or computer generated output.
MA.9-12.8.3.DAT-3.I.1	An interpretation of the p-value for the chi-square test for goodness of fit is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.
MA.9-12.8.3.DAT-3.J.1	A decision to either reject or fail to reject the null hypothesis is based on comparison of the p-value to the significance level, α.
MA.9-12.8.3.DAT-3.J.2	The results of a chi-square test for goodness of fit can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
MA.9-12.8.4.VAR-8.H.1	The expected count in a particular cell of a two-way table of categorical data can be calculated using the formula: expected count = [(row total)(column total)]/table total.
MA.9-12.8.5.VAR-8.I.1	The appropriate hypotheses for a chi-square test for homogeneity are: H_0: There is no difference in distributions of a categorical variable across populations or treatments. H_a: There is a difference in distributions of a categorical variable across populations or treatments.
MA.9-12.8.5.VAR-8.I.2	The appropriate hypotheses for a chi-square test for independence are: H_0: There is no association between two categorical variables in a given population or the two categorical variables are independent. H_a:Two categorical variables in a population are associated or dependent.
MA.9-12.8.5.VAR-8.J.1	When comparing distributions to determine whether proportions in each category for categorical data collected from different populations are the same, the appropriate test is the chi-square test for homogeneity.
MA.9-12.8.5.VAR-8.J.2	To determine whether row and column variables in a two-way table of categorical data might be associated in the population from which the data were sampled, the appropriate test is the chi-square test for independence.
MA.9-12.8.5.VAR-8.K.1.a.i	For a test for independence: Data should be collected using a simple random sample.

MA.9-12.8.5.VAR-8.K.1.a.ii	For a test for homogeneity: Data should be collected using a stratified random sample or randomized experiment.
MA.9-12.8.5.VAR-8.K.1.a.iii	When sampling without replacement, check that $n \leq 10\%N$.
MA.9-12.8.5.VAR-8.K.1.b.i	A conservative check for large counts is that all expected counts should be greater than 5.
MA.9-12.8.6.VAR-8.L.1	The appropriate test statistic for a chi-square test for homogeneity or independence is the chi-square statistic: $X^2 = \Sigma(\text{Observed count} - \text{Expected count})^2/\text{Expected count}$, with degrees of freedom equal to: (number of rows − 1)(number of columns − 1).
MA.9-12.8.6.VAR-8.M.1	The p-value for a chi-square test for independence or homogeneity for a number of degrees of freedom is found using the appropriate table or technology.
MA.9-12.8.6.VAR-8.M.2	For a test of independence or homogeneity for a two-way table, the p-value is the proportion of values in a chi-square distribution with appropriate degrees of freedom that are equal to or larger than the test statistic.
MA.9-12.8.6.DAT-3.K.1	An interpretation of the p-value for the chi-square test for homogeneity or independence is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.
MA.9-12.8.6.DAT-3.L.1	A decision to either reject or fail to reject the null hypothesis for a chi-square test for homogeneity or independence is based on comparison of the p-value to the significance level, α.
MA.9-12.8.6.DAT-3.L.2	The results of a chi-square test for homogeneity or independence can serve as the statistical reasoning to support the answer to a research question about the population that was sampled (independence) or the populations that were sampled (homogeneity).
MA.9-12.8.7	Skills Focus: Selecting an Appropriate Inference Procedure for Categorical Data
MA.9-12.9.1.VAR-1.K.1	Variation in points' positions relative to a theoretical line may be random or non-random.
MA.9-12.9.2.UNC-4.AC.1	Consider a response variable, y, that is linearly related to an explanatory variable, x. For a simple random sample of n observations, the sample regression line, $\hat{y} = a + bx$ is an estimate of the population regression $\mu_y = \alpha + \beta x$. For a particular observation, (x_i , y_i), the residual from the sample regression line, $y_i - \hat{y}_i = y_i - (a + bx_i)$, is an estimate of $y_i - (\alpha + \beta x_i)$, the deviation of the response variable from the population regression line. For all points (x, y) in the population, the standard deviation of all of the deviations of the response variable from the population regression line, σ, can be estimated by the standard deviation of the residuals from the sample regression line, $s = \sqrt{[\Sigma(y_i - \hat{y}_i)^2/(n - 2)]}$
MA.9-12.9.2.UNC-4.AC.2	For a simple random sample of n observations, let b represent the slope of a sample regression line. Then the mean of the sampling distribution for b equals the population slope: $\mu_b = \beta$. The standard deviation of the sampling distribution for b is $\sigma_b = \sigma/(\sigma_x\sqrt{n})$, where $\sigma_x = \sqrt{[(\Sigma (x_i - \bar{x})^2)/n]}$.
MA.9-12.9.2.UNC-4.AC.3	The appropriate confidence interval for the slope of a regression model is a t-interval for the slope.
MA.9-12.9.2.UNC-4.AD.1.a	The true relationship between x and y is linear. Analysis of residuals may be used to verify linearity.
MA.9-12.9.2.UNC-4.AD.1.b	The standard deviation for y, σ[subscript y], does not vary with x. Analysis of residuals may be used to check for approximately equal standard deviations for all x.
MA.9-12.9.2.UNC-4.AD.1.c.i	Data should be collected using a random sample or a randomized experiment.
MA.9-12.9.2.UNC-4.AD.1.c.ii	When sampling without replacement, check that $n \leq 10\%N$.
MA.9-12.9.2.UNC-4.AD.1.d.i	If the observed distribution is skewed, n should be greater than 30.
MA.9-12.9.2.UNC-4.AE.1	For the slope of a regression line, the margin of error is the critical value (t^*) times the standard error (SE) of the slope.
MA.9-12.9.2.UNC-4.AE.2	The standard error for the slope of a regression line with sample standard deviation, s, is $SE = s/[s_x\sqrt{(n - 1)}]$, where s is the estimate of σ and s_x is the sample standard deviation of the x values.

MA.9-12.9.2.UNC-4.AF.1	The point estimate for the slope of a regression model is the slope of the line of best fit, b.
MA.9-12.9.2.UNC-4.AF.2	For the slope of a regression model, the interval estimate is $b \pm t^*$ (SE[subscript b]).
MA.9-12.9.3.UNC-4.AG.1	In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the slope of the regression model, i.e., the true slope of the population regression model.
MA.9-12.9.3.UNC-4.AG.2	An interpretation for a confidence interval for the slope of a regression line should include a reference to the sample taken and details about the population it represents.
MA.9-12.9.3.UNC-4.AH.1	A confidence interval for the slope of a regression model provides an interval of values that may provide sufficient evidence to support a particular claim in context.
MA.9-12.9.3.UNC-4.AI.1	When all other things remain the same, the width of the confidence interval for the slope of a regression model tends to decrease as the sample size increases.
MA.9-12.9.4.VAR-7.J.1	The appropriate test for the slope of a regression model is a t-test for a slope.
MA.9-12.9.4.VAR-7.K.1	The null hypothesis for a t-test for a slope is: $H_0 : \beta = \beta_0$, where β_0 is the hypothesized value from the null hypothesis. The alternative hypothesis is $H_0 : \beta < \beta_0$ or $H_0 : \beta > \beta_0$, or $H_0 : \beta \neq \beta_0$.
MA.9-12.9.4.VAR-7.L.1.a	The true relationship between x and y is linear. Analysis of residuals may be used to verify linearity.
MA.9-12.9.4.VAR-7.L.1.b	The standard deviation for y, σ[subscript y], does not vary with x. Analysis of residuals may be used to check for approximately equal standard deviations for all x.
MA.9-12.9.4.VAR-7.L.1.c.i	Data should be collected using a random sample or a randomized experiment.
MA.9-12.9.4.VAR-7.L.1.c.ii	When sampling without replacement, check that $n \leq 10\%N$.
MA.9-12.9.4.VAR-7.L.1.d.i	If the observed distribution is skewed, n should be greater than 30.
MA.9-12.9.4.VAR-7.L.1.d.ii	If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.
MA.9-12.9.5.VAR-7.M.1	The distribution of the slope of a regression model assuming all conditions are satisfied and the null hypothesis is true (null distribution) is a t-distribution.
MA.9-12.9.5.VAR-7.M.2	For simple linear regression when random sampling from a population for the response that can be modeled with a normal distribution for each value of the explanatory variable, the sampling distribution of $t = (b - \beta)/SE$[subscript b] has a t-distribution with degrees of freedom equal to $n - 2$. When testing the slope in a simple linear regression model with one parameter, the slope, the test for the slope has $df = n - 1$.
MA.9-12.9.5.DAT-3.M.1	An interpretation of the p-value of a significance test for the slope of a regression model should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population slope is equal to the particular value stated in the null hypothesis.
MA.9-12.9.5.DAT-3.N.1	A formal decision explicitly compares the p-value to the significance α. If the p-value ≤ α, then reject the null hypothesis, $H_0 : \beta = \beta_0$. If the p-value > α, then fail to reject the null hypothesis.
MA.9-12.9.5.DAT-3.N.2	The results of a significance test for the slope of a regression model can serve as the statistical reasoning to support the answer to a research question about that sample.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others

MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	**Essential Questions**
Part 6: Inference for Categorical Data: Proportions • Given that variation may be random or not, conclusions are uncertain. • An interval of values should be used to estimate parameters, in order to account for uncertainty. • The normal distribution may be used to model variation. • Significance testing allows us to make decisions about hypotheses within a particular context. • Probabilities of Type I and Type II errors influence inference. *Part 7: Inference for Quantitative Data: Proportions* • Given that variation may be random or not, conclusions are uncertain. • The *t*-distribution may be used to model variation. • An interval of values should be used to estimate parameters, in order to account for uncertainty. • Significance testing allows us to make decisions about hypotheses within a particular context. *Part 8: Inference for Categorical Data: Chi-Square* • Given that variation may be random or not, conclusions are uncertain. • The chi-square distribution may be used to model variation. • Significance testing allows us to make decisions about hypotheses within a particular context. *Part 9: Inference for Quantitative Data: Slopes*	*Part 6: Inference for Categorical Data: Proportions* • When can we use a normal distribution to perform inference calculations involving population proportions? • How can we narrow the width of a confidence interval? • If the proportion of subjects who experience serious side effects when taking a new drug is smaller than the proportion of subjects who experience serious side effects when taking a placebo, how can we determine if the difference is statistically significant? *Part 7: Inference for Quantitative Data: Proportions* • How do we know whether to use a *t*-test or a *z*-test for inference with means? • How can we make sure that samples are independent? • Why is it inappropriate to accept a hypothesis as true based on the results of statistical inference testing? *Part 8: Inference for Categorical Data: Chi-Square* • How does increasing the degrees of freedom influence the shape of the chi-square distribution? • Why is it inappropriate to use statistical inference to justify a claim that there is no association between variables? *Part 9: Inference for Quantitative Data: Slopes* • How can there be variability in slope if the slope statistic is uniquely determined for a line of best fit? • When is it appropriate to perform inference about the slope of a population regression line

• Given that variation may be random or not, conclusions are uncertain. • An interval of values should be used to estimate parameters, in order to account for uncertainty. • The *t*-distribution may be used to model variation. • Significance testing allows us to make decisions about hypotheses within a particular context.	based on sample data? • Why do we not conclude that there is no correlation between two variables based on the results of a statistical inference for slopes?

Instructional Focus

Learning Targets

Part 6: Inference for Categorical Data: Proportions

- Identify questions suggested by variation in the shapes of distributions of samples taken from the same population.
- Identify an appropriate confidence interval procedure for a population proportion.
- Verify the conditions for calculating confidence intervals for a population proportion.
- Determine the margin of error for a given sample size and an estimate for the sample size that will result in a given margin of error for a population proportion.
- Calculate an appropriate confidence interval for a population proportion.
- Calculate an interval estimate based on a confidence interval for a population proportion.
- Interpret a confidence interval for a population proportion.
- Justify a claim based on a confidence interval for a population proportion.
- Identify the relationships between sample size, width of a confidence interval, confidence level, and margin of error for a population proportion.
- Identify the null and alternative hypotheses for a population proportion.
- Identify an appropriate testing method for a population proportion.
- Verify the conditions for making statistical inferences when testing a population proportion.
- Calculate an appropriate test statistic and *p*-value for a population proportion.
- Interpret the *p*-value of a significance test for a population proportion.
- Justify a claim about the population based on the results of a significance test for a population proportion.
- Identify Type I and Type II errors.
- Calculate the probability of a Type I and Type II errors.
- Identify factors that affect the probability of errors in significance testing.
- Interpret Type I and Type II errors.
- Identify an appropriate confidence interval procedure for a comparison of population proportions.
- Verify the conditions for calculating confidence intervals for a difference between population proportions.

- Calculate an appropriate confidence interval for a comparison of population proportions.
- Calculate an interval estimate based on a confidence interval for a difference of proportions.
- Interpret a confidence interval for a difference of proportions.
- Justify a claim based on a confidence interval for a difference of proportions.
- Identify the null and alternative hypotheses for a difference of two population proportions.
- Identify an appropriate testing method for the difference of two population proportions
- Verify the conditions for making statistical inferences when testing a difference of two population proportions.
- Calculate an appropriate test statistic for the difference of two population proportions.
- Interpret the *p*-value of a significance test for a difference of population proportions.
- Justify a claim about the population based on the results of a significance test for a difference of population proportions.

Part 7: Inference for Quantitative Data: Proportions

- Identify questions suggested by probabilities of errors in statistical inference.
- Describe *t*-distributions.
- Identify an appropriate confidence interval procedure for a population mean, including the mean difference between values in matched pairs.
- Verify the conditions for calculating confidence intervals for a population mean, including the mean difference between values in matched pairs.
- Determine the margin of error for a given sample size for a one-sample *t*-interval.
- Calculate an appropriate confidence interval for a population mean, including the mean difference between values in matched pairs.
- Interpret a confidence interval for a population mean, including the mean difference between values in matched pairs.
- Justify a claim based on a confidence interval for a population mean, including the mean difference between values in matched pairs.
- Identify the relationships between sample size, width of a confidence interval, confidence level, and margin of error for a population mean.
- Identify an appropriate testing method for a population mean with unknown σ , including the mean difference between values in matched pairs.
- Identify the null and alternative hypotheses for a population mean with unknown σ , including the mean difference between values in matched pairs.
- Verify the conditions for the test for a population mean, including the mean difference between values in matched pairs.
- Calculate an appropriate test statistic for a population mean, including the mean difference between values in matched pairs.
- Interpret the *p*-value of a significance test for a population mean, including the mean difference between values in matched pairs.
- Justify a claim about the population based on the results of a significance test for a population mean.
- Identify an appropriate confidence interval procedure for a difference of two population means.
- Verify the conditions to calculate confidence intervals for the difference of two population means.
- Determine the margin of error for the difference of two population means.
- Calculate an appropriate confidence interval for a difference of two population means.
- Interpret a confidence interval for a difference of population means.
- Justify a claim based on a confidence interval for a difference of population means.
- Identify the effects of sample size on the width of a confidence interval for the difference of two means.

- Identify an appropriate selection of a testing method for a difference of two population means.
- Identify the null and alternative hypotheses for a difference of two population means.
- Verify the conditions for the significance test for the difference of two population means.
- Calculate an appropriate test statistic for a difference of two means.
- Interpret the *p*-value of a significance test for a difference of population means.
- Justify a claim about the population based on the results of a significance test for a difference of two population means in context.

Part 8: Inference for Categorical Data: Chi-Square

- Identify questions suggested by variation between observed and expected counts in categorical data.
- Describe chi-square distributions.
- Identify the null and alternative hypotheses in a test for a distribution of proportions in a set of categorical data.
- Identify an appropriate testing method for a distribution of proportions in a set of categorical data.
- Calculate expected counts for the chi-square test for goodness of fit.
- Verify the conditions for making statistical inferences when testing goodness of fit for a chi-square distribution.
- Calculate the appropriate statistic for the chi-square test for goodness of fit.
- Determine the *p*-value for chi-square test for goodness of fit significance test.
- Interpret the *p*-value for the chi-square test for goodness of fit.
- Justify a claim about the population based on the results of a chi-square test for goodness of fit.
- Calculate expected counts for two-way tables of categorical data.
- Identify the null and alternative hypotheses for a chi-square test for homogeneity or independence.
- Identify an appropriate testing method for comparing distributions in two-way tables of categorical data.
- Verify the conditions for making statistical inferences when testing a chi-square distribution for independence or homogeneity.
- Calculate the appropriate statistic for a chi-square test for homogeneity or independence.
- Determine the *p*-value for a chi-square significance test for independence or homogeneity.
- Interpret the *p*-value for the chi-square test for homogeneity or independence.
- Justify a claim about the population based on the results of a chi-square test for homogeneity or independence.

Part 9: Inference for Quantitative Data: Slopes

- Identify questions suggested by variation in scatter plots.
- Identify an appropriate confidence interval procedure for a slope of a regression model.
- Verify the conditions to calculate confidence intervals for the slope of a regression model.
- Determine the given margin of error for the slope of a regression model.
- Calculate an appropriate confidence interval for the slope of a regression model.
- Interpret a confidence interval for the slope of a regression model.
- Justify a claim based on a confidence interval for the slope of a regression model.
- Identify the effects of sample size on the width of a confidence interval for the slope of a regression model.
- Identify the appropriate selection of a testing method for a slope of a regression model.
- Identify appropriate null and alternative hypotheses for a slope of a regression model.
- Verify the conditions for the significance test for the slope of a regression model.
- Calculate an appropriate test statistic for the slope of a regression model.

- Interpret the *p*-value of a significance test for the slope of a regression model.
- Justify a claim about the population based on the results of a significance test for the slope of a regression model.

Prerequisite Skills

- Understanding sampling distributions and the concept of sampling variability
- Distinguishing between parameters and statistics
- Interpreting z-scores and normal distributions
- Using probability to model chance behavior and make predictions
- Understanding simulation methods to approximate sampling distributions
- Calculating and interpreting standard deviation, standard error, and mean
- Classify variables as categorical or quantitative
- Construct and interpret one-variable and two-variable frequency tables
- Understand and apply the binomial setting and conditions
- Calculate and interpret sample proportions
- Know how to use normal approximation to the binomial distribution
- Use technology or formulas to find z-scores and areas under the normal curve
- Work with quantitative data: organize, describe, and interpret distributions
- Understand the t-distribution and how it differs from the normal distribution
- Calculate sample means, sample standard deviations, and standard errors
- Use formulas and/or technology to compute one-sample and paired-sample statistics
- Understand how to conduct and interpret exploratory data analysis (dot plots, boxplots, histograms)
- Familiarity with matched pairs and independent groups in experimental design
- Construct and interpret two-way tables
- Understand expected vs. observed counts
- Compute row and column totals and understand marginal and joint distributions
- Identify whether categorical variables are independent or related
- Use basic probability rules (multiplication, addition, conditional probability)

- Understand the concept of degrees of freedom
- Construct and interpret scatterplots
- Understand and interpret the least-squares regression line and its components
- Interpret correlation (r) and coefficient of determination (r^2)
- Distinguish between explanatory and response variables
- Understand residuals and how to assess linearity
- Identify and interpret slope, y-intercept, and standard deviation of the residuals (s)

Common Misconceptions

- Misinterpreting the Confidence Level:
  - Students often believe that a 95% confidence level means "there is a 95% chance that the true parameter is in this one interval."
- Misunderstanding Margin of Error:
  - Students may think the margin of error includes all types of error, including bias or data collection errors.
- Confusing Width of Interval with Confidence Level:
  - Students assume a wider interval always indicates better results or more precision.
- Misusing the Interval for Individual Predictions:
  - Students might try to use the confidence interval to predict an individual outcome.
- P-Value Misinterpretation:
  - Students often interpret the p-value as the probability that the null hypothesis is true.
- Wrong Conclusions from p-values:
  - Students believe a small p-value proves the alternative hypothesis is correct.
- Confusing Statistical Significance with Practical Significance:
  - Students may assume a statistically significant result always means the finding is important.
- Failing to Reject = Proving the Null:
  - Students assume that failing to reject the null hypothesis confirms it is true.
- Reversing Error Types:

- Students confuse Type I and Type II errors or forget which is which.

- Forgetting Context in Errors:
    - Students describe errors generically without applying them to the context of the problem.
- Expected vs. Observed Counts Confusion:
    - Students mistakenly switch expected and observed values when calculating the test statistic.
- Assuming Independence Without Checking Conditions:
    - Students may skip verifying the random, independence, or expected cell count conditions.
- Interpreting Chi-Square as Directional:
    - Students incorrectly interpret chi-square tests as having a direction (e.g., "greater than").
- Confusing Correlation and Causation:
    - Students often assume a significant slope proves a cause-and-effect relationship.
- Misinterpreting the Slope:
    - Students might say the slope is the "rate of change of y" without tying it to the context.
- Ignoring Conditions for Regression Inference:
    - Students forget to verify linearity, independence, normality of residuals, equal variance, and randomness.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• **6A Confidence Intervals: The Basics** ○ Interpret a confidence interval in context. ○ Use a confidence interval to make a decision about the value of a	• **Connections to Unit 1** ○ ***Exploring & Describing Data*** ▪ Describing distributions: shape, center, spread, and unusual features ▪ Constructing and interpreting graphs	• **Spiral Warm-Ups or Exit Tickets** ○ Daily review of prior units (data displays, simulation, design, probability) ▪ Graph construction ▪ Interpreting sampling distributions ▪ Writing

parameter. ○ Interpret a confidence level in context. • **6B Confidence Intervals for a Population Proportion** ○ Check the conditions for calculating a confidence interval for a population proportion. ○ Calculate a confidence interval for a population proportion. ○ Construct and interpret a one-sample z interval for a proportion. ○ Describe how the sample size and confidence level affect the margin of error. ○ Determine the sample size required to obtain a confidence interval for a population proportion with a specified margin of error. • **6C Significance Tests: The Basics** ○ State appropriate hypotheses for a significance test about a population parameter. ○ Interpret a P-value in context. ○ Make an	(dotplots, histograms, boxplots, scatterplots) ▪ Identifying and interpreting categorical and quantitative variables ○ ***Analyzing Two-Variable Data*** ▪ Interpreting linear relationships through correlation and least-squares regression ▪ Making predictions and understanding residuals ▪ Evaluating strength and direction of associations • **Connections to Unit 2** ○ ***Randomness, Probability & Simulation*** ▪ Long-run relative frequency and variability in sampling ▪ Using probability models to determine likelihoods of sample results ▪ Understanding Type I and Type II errors through simulation and probability ○ ***Sampling & Experimental Design*** ▪ Random	hypotheses in context ▪ Determining if inference conditions are met 5-minute spiral tasks to review: ▪ Tie them to current inference topic (e.g., “What display would help verify conditions for t-procedures?”) • **Error Type Scenarios (Inference + Probability)** ○ Type I/II errors, simulation, probability ▪ Create real-world scenarios (medical testing, court cases, product recalls). ▪ Students identify hypotheses, possible errors, and consequences of each. ▪ Add probability elements: "Given a test with a 5% false positive rate..." ▪ Extension: Use simulations (e.g., cards or dice) to estimate error rates. • **Confidence Interval Matching Cards** ○ Confidence intervals, sampling variability, estimation ▪ Prepare cards with different confidence intervals, sample sizes, and interpretations. ▪ Students match confidence levels with appropriate widths and conclusions. ▪ Include intervals for proportions, means, and regression slopes. ▪ Challenge Round: Introduce incorrect interpretations for error analysis and discussion. • **Inference Simulation Stations** ○ Random sampling, sampling distributions, conditions for inference

appropriate conclusion for a significance test. • **6D Significance Tests for a Population Proportion** ○ Check the conditions for performing a test about a population proportion. ○ Calculate the standardized test statistic and P-value for a test about a population proportion. ○ Perform a one-sample z test for a proportion. ○ Interpret a Type I error and a Type II error in context and give a consequence of each type of error. ○ Interpret the power of a significance test and describe which factors affect the power of a test. • **6E Confidence Intervals for a Difference in Population Proportions** ○ Check the conditions for calculating a confidence interval about a difference between two population proportions. ○ Calculate a confidence	selection vs. random assignment ▪ Recognizing bias and ensuring representative samples ▪ Understanding independence and the impact of confounding variables ○ ***Sampling Distributions*** ▪ Variability of statistics from sample to sample ▪ Understanding the shape, center, and spread of sampling distributions ▪ Central Limit Theorem (CLT) and standard error	▪ Set up stations with different simulation-based inference tasks: ▪ Simulating sampling variability with coins or dice ▪ Bootstrapping confidence intervals with resampling ▪ Visualizing t-distributions with small sample data ▪ Students rotate in small groups, collecting results and reflecting on conditions. • **Graph Interpretation Gallery Walk** ○ Distributions, residuals, linear regression, normality ▪ Post graphs around the room (boxplots, scatterplots, residual plots, histograms). ▪ Students analyze each graph: Are conditions for inference met? What inference could be done? ▪ Include prompts about shape, outliers, independence, linearity, etc. • **Mini-Case Studies Across Topics** ○ Study design, inference type selection, hypothesis testing, confidence intervals ▪ Present brief case studies with real or mock data. ▪ Students:

interval for a difference between two population proportions. • **6F Significance Tests for a Difference in Population Proportions** ○ State appropriate hypotheses for performing a test about a difference between two population proportions. ○ Check the conditions for performing a test about a difference between two population proportions. ○ Calculate the standard test statistic and P - value for a test about a difference between two population proportions. • **7A Confidence Intervals for a Population Mean or Mean Difference** ○ Determine the critical value t* for calculating a confidence interval for a population mean. ○ Check the conditions for calculating a confidence interval for a population mean. ○ Calculate a confidence interval for a population mean.		▪ Identify whether it's a proportion or mean situation ▪ Choose confidence interval or test ▪ Write hypotheses ▪ Justify the method and interpret results in context • **Peer Teaching: "Which Procedure?" Challenges** ○ All inference types, condition checking, test/interval identification ▪ In pairs or groups, one student presents a problem. Others must: ▪ Identify the type of inference ▪ State assumptions ▪ Sketch or outline an appropriate solution ▪ Rotate roles to reinforce all inference models (1-prop z, 2-mean t, chi-square, regression). • **P-Value Debates** ○ Hypothesis testing, significance, p-value interpretation ▪ Pose controversial or ambiguous study claims (e.g., "Teens who use social media are less

- Construct and interpret a one-sample t interval for a mean. - In the special case of paired data, construct and interpret a confidence interval for a population mean difference. • **7B Significance Tests for a Population Mean or Mean Difference** - State appropriate hypotheses and check the conditions for performing a test about a population mean. - Calculate the standard test statistic and P-value for a test about a population mean. - Perform a one-sample t test for a mean. - In the special case of paired data, perform a significance test about a population mean difference. • **7C Confidence Intervals for a Difference in Population Means** - Check the conditions for calculating a confidence interval for a difference between two population means.		happy.") - Provide summary statistics and p-values. - One side argues to reject the null; the other argues not to reject. - Bonus: Include Type I/II error consequences in the debates • **AP Exam Free Response Spiral Practice (FRAPPYs)** - Practice real AP problems aligned to both current and past units. - Choose problems that span multiple skills (e.g., describing distributions, interpreting simulation results, calculating probabilities). - Use a gradual release: Individual → Group → Class Discussion. - Include student reflection: "Which part felt familiar? From which earlier unit?"

○ Calculate a confidence interval for a difference between two population means. • **7D Significance Tests for a Difference in Population Means** ○ State appropriate hypotheses and check the conditions for performing a test about a difference between two population means. ○ Calculate the standardized test statistic and P-value for a test about a difference between two population means. ○ Perform a two-sample t test for a difference in needs. • **8A Chi-Square Tests for Goodness** ○ State appropriate hypotheses for a test about the distribution of a categorical variable. ○ Calculate expected counts for a test about the distribution of a categorical variable. ○ Check the conditions for a test about the distribution		

of a categorical variable. • **8B Chi-Square Tests for Independence or Homogeneity** ○ State appropriate hypotheses for a test about the relationship between two categorical variables. ○ Calculate expected counts for a test about the relationship between two categorical variables. ○ Calculate the test statistic and p-value of a test about the relationship between two categorical variables. ○ Perform a chi-square test for Independence. ○ Perform a chi-square test for homogeneity. ○ Distinguish between a chi-square test for independence and a chi-square test for homogeneity. • **9A Confidence Intervals for the Slope of a Population Regression Line** ○ Describe the sampling distribution of the sample slope b.		

○ Check the conditions for calculating a confidence interval for the slope β of a population regression line. ○ Calculate a confidence interval for the slope of a population regression line. • **9B Significance Tests for the Slope of a Population Regression Line** ○ State appropriate hypotheses and check the conditions for a test about the slope of a population regression line. ○ Calculate the standardized test statistic and P-value for a test about the slope of a population regression line.		

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests • Unit 3 Benchmark (Link-It)

• AP Classroom Progress Checks	

Resources

Key Resources	Supplemental Resources
AP Classroom AP Statistics CED Pacing Guide	iXL Delta Math Desmos Khan Academy Math Medic Skew the Script Teacher Made worksheets Textbook - Starnes, D., & Tabor, J. (2024). *The Practice of Statistics for the AP Classroom* (7th ed.). Beford, Freeman & Worth.

Career Readiness, Life Literacies, and Key Skills

WRK.9.2.12.CAP.4	Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
WRK.9.2.12.CAP.21	Explain low-cost and low-risk ways to start a business.
TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.CT.3	Enlist input from a variety of stakeholders (e.g., community members, experts in the field) to design a service learning activity that addresses a local or global issue (e.g., environmental justice).
TECH.9.4.12.IML.5	Evaluate, synthesize, and apply information on climate change from various sources appropriately (e.g., 2.1.12.CHSS.6, S.IC.B.4, S.IC.B.6, 8.1.12.DA.1, 6.1.12.GeoHE.14.a, 7.1.AL.PRSNT.2).
TECH.9.4.12.IML.7	Develop an argument to support a claim regarding a current workplace or societal/ethical issue such as climate change (e.g., NJSLSA.W1, 7.1.AL.PRSNT.4).

Interdisciplinary Connections

ELA.RL.CR.11–12.1	Accurately cite strong and thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what a literary text says explicitly and inferentially, as well as interpretations of the text; this may include determining where the text leaves matters uncertain.
ELA.W.IW.11–12.2	Write informative/explanatory texts (including the narration of historical events, scientific procedures/experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
ELA.SL.PI.11–12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SOC.6.1.12.EconET.14.b	Analyze economic trends, income distribution, labor participation (i.e., employment, the composition of the work force), and government and consumer debt and their impact on society.
SCI.HS-LS2-6	Evaluate the claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.
SCI.HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.