AP Statistics Unit 2 - Probability and Distributions

Content Area: **Math**
Course(s):
Time Period: **MP2**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
In this unit, students will explore the foundational role of probability and randomness in statistical inference. Beginning with probability concepts, students will identify patterns in data that raise questions suitable for investigation using statistical methods. They will use simulations and probability rules to estimate and calculate the likelihood of various outcomes, including compound events, complements, unions, and intersections. Students will distinguish between mutually exclusive and independent events and calculate related probabilities, including conditional probabilities. Building on probability, students will analyze discrete random variables and their probability distributions. They will represent distributions graphically and calculate key parameters such as the mean (expected value) and standard deviation. This understanding will extend to the effect of linear transformations and combinations of random variables, preparing students to interpret these values in context. Students will then deepen their understanding of probability distributions through the binomial and geometric models. They will use simulation and formal calculation methods to determine probabilities, understand when each model is appropriate, and interpret parameters such as the number of trials, probability of success, and expected number of trials until success. In the next section, students transition into sampling distributions, which form the theoretical basis for inference. Students will explore how statistics vary across samples from the same population and use	Understanding probability and sampling distributions is fundamental to developing statistical reasoning and conducting meaningful data analysis. This unit serves as the mathematical and conceptual bridge between descriptive statistics and inferential statistics, empowering students to make predictions and decisions under uncertainty. By examining how random events behave over the long run and how sample statistics vary, students build the tools necessary for interpreting data beyond what is immediately observable. Probability equips students with the language and models to quantify randomness, analyze risk, and evaluate the likelihood of outcomes in both theoretical and real-world contexts. Learning about discrete and special probability distributions, including binomial and geometric, allows students to explore structured patterns in randomness and apply those patterns to practical problems such as quality control, medical testing, and decision-making. The concept of sampling distributions underpins statistical inference by explaining how sample statistics behave across repeated samples. This knowledge helps students understand variability in data, assess estimator reliability, and determine the appropriateness of using a normal model. Through simulation, visualizations, and analytical methods, students will engage with both conceptual and computational approaches, developing fluency in probabilistic reasoning. This unit supports students in becoming critical thinkers who can distinguish between random variation and meaningful patterns, laying the groundwork for the confidence intervals and

simulations to estimate sampling distributions. They will analyze sampling distributions of sample proportions, sample means, and differences between proportions or means, assessing whether a normal model can be used based on sample size and population conditions. Students will calculate and interpret parameters of sampling distributions (such as means and standard deviations), evaluate whether estimators are unbiased, and understand the variability inherent in sample statistics. By understanding the distribution of sample statistics, students will gain insight into the reliability of statistical estimates and set the stage for confidence intervals and hypothesis testing in future units. This unit emphasizes both conceptual understanding through simulation and formal computation, helping students make sense of variability and develop strong probabilistic reasoning—crucial for effective statistical thinking and decision-making.	hypothesis tests that follow in later units. The skills acquired are essential not only for success on the AP exam but also for real-world applications in fields such as science, business, healthcare, and public policy.

NJSLS

MA.9-12.4.1.VAR-1.F.1	Patterns in data do not necessarily mean that variation is not random.
MA.9-12.4.2.UNC-2.A.1	A random process generates results that are determined by chance.
MA.9-12.4.2.UNC-2.A.2	An outcome is the result of a trial of a random process.
MA.9-12.4.2.UNC-2.A.3	An event is a collection of outcomes.
MA.9-12.4.2.UNC-2.A.4	Simulation is a way to model random events, such that simulated outcomes closely match real-world outcomes. All possible outcomes are associated with a value to be determined by chance. Record the counts of simulated outcomes and the count total.
MA.9-12.4.2.UNC-2.A.5	The relative frequency of an outcome or event in simulated or empirical data can be used to estimate the probability of that outcome or event.
MA.9-12.4.2.UNC-2.A.6	The law of large numbers states that simulated (empirical) probabilities tend to get closer to the true probability as the number of trials increases.
MA.9-12.4.3.VAR-4.A.1	The sample space of a random process is the set of all possible non-overlapping outcomes.
MA.9-12.4.3.VAR-4.A.2	If all outcomes in the sample space are equally likely, then the probability an event E will occur is defined as the fraction: (number of outcomes in event E)/(total number of outcomes in sample space).
MA.9-12.4.3.VAR-4.A.3	The probability of an event is a number between 0 and 1, inclusive.
MA.9-12.4.3.VAR-4.A.4	The probability of the complement of an event E, E' or E^c , (i.e., not E) is equal to $1 - P(E)$.

MA.9-12.4.3.VAR-4.B.1	Probabilities of events in repeatable situations can be interpreted as the relative frequency with which the event will occur in the long run.
MA.9-12.4.4.VAR-4.C.1	The probability that events A and B both will occur, sometimes called the joint probability, is the probability of the intersection of A and B, denoted $P(A \cap B)$.
MA.9-12.4.4.VAR-4.C.2	Two events are mutually exclusive or disjoint if they cannot occur at the same time. So $P(A \cap B) = 0$.
MA.9-12.4.5.VAR-4.D.1	The probability that event A will occur given that event B has occurred is called a conditional probability and denoted $P(A\|B) = P(A \cap B)/P(B)$.
MA.9-12.4.5.VAR-4.D.2	The multiplication rule states that the probability that events A and B both will occur is equal to the probability that event A will occur multiplied by the probability that event B will occur, given that A has occurred. This is denoted $P(A \cap B) = P(A) \cdot P(B\|A)$.
MA.9-12.4.6.VAR-4.E.1	Events A and B are independent if, and only if, knowing whether event A has occurred (or will occur) does not change the probability that event B will occur.
MA.9-12.4.6.VAR-4.E.2	If, and only if, events$A$ and B are independent, then $P(A\|B) = PA$, $P(B\|A) = P(B)$, and $P(A \cap B) = P(A) \cdot P(B)$.
MA.9-12.4.6.VAR-4.E.3	The probability that event A or event B (or both) will occur is the probability of the union of A and B, denoted $P(A \cup B)$.
MA.9-12.4.6.VAR-4.E.4	The addition rule states that the probability that event A or event B or both will occur is equal to the probability that event A will occur plus the probability that event B will occur minus the probability that both events A and B will occur. This is denoted $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
MA.9-12.4.7.VAR-5.A.1	The values of a random variable are the numerical outcomes of random behavior.
MA.9-12.4.7.VAR-5.A.2	A discrete random variable is a variable that can only take a countable number of values. Each value has a probability associated with it. The sum of the probabilities over all of the possible values must be 1.
MA.9-12.4.7.VAR-5.A.3	A probability distribution can be represented as a graph, table, or function showing the probabilities associated with values of a random variable.
MA.9-12.4.7.VAR-5.A.4	A cumulative probability distribution can be represented as a table or function showing the probability of being less than or equal to each value of the random variable.
MA.9-12.4.7.VAR-5.B.1	An interpretation of a probability distribution provides information about the shape, center, and spread of a population and allows one to make conclusions about the population of interest.
MA.9-12.4.8.VAR-5.C.1	A numerical value measuring a characteristic of a population or the distribution of a random variable is known as a parameter, which is a single, fixed value.
MA.9-12.4.8.VAR-5.C.2	The mean, or expected value, for a discrete random variable X is $\mu_x = \Sigma x_i \cdot P(x_i)$.
MA.9-12.4.8.VAR-5.C.3	The standard deviation for a discrete random variable X is $\sigma_x = \sqrt{[(x_i - \mu_x)^2 \cdot P(x_i)]}$.
MA.9-12.4.8.VAR-5.D.1	Parameters for a discrete random variable should be interpreted using appropriate units and within the context of a specific population.
MA.9-12.4.9.VAR-5.E.1	For random variables X and Y and real numbers a and b, the mean of $aX + bY$ is $a\mu_x + b\mu$[subscript y].
MA.9-12.4.9.VAR-5.E.2	Two random variables are independent if knowing information about one of them does not change the probability distribution of the other.
MA.9-12.4.9.VAR-5.E.3	For independent random variables X and Y and real numbers a and b, the mean of $aX + bY$ is $a\mu_x + b\mu$[subscript y] and the variance of $aX + bY$ is $a^2\sigma^2_x + b^2\sigma^2$[subscript y].
MA.9-12.4.9.VAR-5.F.1	For $Y = a + bX$, the probability distribution of the transformed random variable, Y, has the same shape as the probability distribution for X, so long as $a > 0$ and $b > 0$. The mean of Y is μ[subscript y] $= a + b\mu_x$. The standard deviation of Y is σ[subscript y] $= \|b\|\sigma_x$.
MA.9-12.4.10.UNC-3.A.1	A probability distribution can be constructed using the rules of probability or estimated

	with a simulation using random number generators.
MA.9-12.4.10.UNC-3.A.2	A binomial random variable, X, counts the number of successes in n repeated independent trials, each trial having two possible outcomes (success or failure), with the probability of success p and the probability of failure $1 - p$.
MA.9-12.4.10.UNC-3.B.1	The probability that a binomial random variable, X, has exactly x successes for n independent trials, when the probability of success is p, is calculated as $P(X = x)$ = (n over x) $p^x (1 - p)^{n - x}$, x = 0, 1, 2, . . . , n. This is the binomial probability function.
MA.9-12.4.11.UNC-3.C.1	If a random variable is binomial, its mean, μ_x, is np and its standard deviation, σ_x, is √np(1 − p).
MA.9-12.4.11.UNC-3.D.1	Probabilities and parameters for a binomial distribution should be interpreted using appropriate units and within the context of a specific population or situation.
MA.9-12.4.12.UNC-3.E.1	For a sequence of independent trials, a geometric random variable, X, gives the number of the trial on which the first success occurs. Each trial has two possible outcomes (success or failure) with the probability of success p and the probability of failure $1 - p$.
MA.9-12.4.12.UNC-3.E.2	The probability that the first success for repeated independent trials with probability of success p occurs on trial x is calculated as $P(X = x) = (1 - p)^{x - 1} p$, x = 1, 2, 3, ...This is the geometric probability function.
MA.9-12.4.12.UNC-3.F.1	If a random variable is geometric, its mean, μ_x, is $1/p$ and its standard deviation, σ_x is √(1 − p)/p.
MA.9-12.4.12.UNC-3.G.1	Probabilities and parameters for a geometric distribution should be interpreted using appropriate units and within the context of a specific population or situation.
MA.9-12.5.1.VAR-1.G.1	Variation in statistics for samples taken from the same population may be random or not.
MA.9-12.5.2.VAR-6.A.1	A continuous random variable is a variable that can take on any value within a specified domain. Every interval within the domain has a probability associated with it.
MA.9-12.5.2.VAR-6.A.2	A continuous random variable with a normal distribution is commonly used to describe populations. The distribution of a normal random variable can be described by a normal, or “bell-shaped,” curve.
MA.9-12.5.2.VAR-6.A.3	The area under a normal curve over a given interval represents the probability that a particular value lies in that interval.
MA.9-12.5.2.VAR-6.B.1	The boundaries of an interval associated with a given area in a normal distribution can be determined using z-scores or technology, such as a calculator, a standard normal table, or computer-generated output.
MA.9-12.5.2.VAR-6.C.1	Normal distributions are symmetrical and “bell-shaped.” As a result, normal distributions can be used to approximate distributions with similar characteristics.
MA.9-12.5.3.UNC-3.H.1	A sampling distribution of a statistic is the distribution of values for the statistic for all possible samples of a given size from a given population.
MA.9-12.5.3.UNC-3.H.2	The central limit theorem (CLT) states that when the sample size is sufficiently large, a sampling distribution of the mean of a random variable will be approximately normally distributed.
MA.9-12.5.3.UNC-3.H.3	The central limit theorem requires that the sample values are independent of each other and that n is sufficiently large.
MA.9-12.5.3.UNC-3.H.4	A randomization distribution is a collection of statistics generated by simulation assuming known values for the parameters. For a randomized experiment, this means repeatedly randomly reallocating/reassigning the response values to treatment groups.
MA.9-12.5.3.UNC-3.H.5	The sampling distribution of a statistic can be simulated by generating repeated random samples from a population.
MA.9-12.5.4.UNC-3.I.1	When estimating a population parameter, an estimator is unbiased if, on average, the value of the estimator is equal to the population parameter.

MA.9-12.5.4.UNC-3.J.1	When estimating a population parameter, an estimator exhibits variability that can be modeled using probability.
MA.9-12.5.4.UNC-3.J.2	A sample statistic is a point estimator of the corresponding population parameter.
MA.9-12.5.5.UNC-3.K.1	For independent samples (sampling with replacement) of a categorical variable from a population with population proportion, p, the sampling distribution of the sample proportion, $\hat{p}$, has a mean, $\mu_{\hat{p}} = p$ and a standard deviation, $\sigma_{\hat{p}} = \sqrt{(p(1-p))/n}$.
MA.9-12.5.5.UNC-3.K.2	If sampling without replacement, the standard deviation of the sample proportion is smaller than what is given by the formula above. If the sample size is less than 10% of the population size, the difference is negligible.
MA.9-12.5.5.UNC-3.L.1	For a categorical variable, the sampling distribution of the sample proportion, $\hat{p}$, will have an approximate normal distribution, provided the sample size is large enough: $np \geq 10$ and $n(1-p) \geq 10$.
MA.9-12.5.5.UNC-3.M.1	Probabilities and parameters for a sampling distribution for a sample proportion should be interpreted using appropriate units and within the context of a specific population.
MA.9-12.5.6.UNC-3.N.1	For a categorical variable, when randomly sampling with replacement from two independent populations with population proportions p_1 and p_2, the sampling distribution of the difference in sample proportions $\hat{p}_1 - \hat{p}_2$ has mean, μ[subscript $\hat{p}_1 - \hat{p}_2$] = $p_1 - p_2$ and standard deviation, σ[subscript $\hat{p}_1 - \hat{p}_2$] = $\sqrt{(p_1(1-p_1))/n_1 + (p_2(1-p_2))/n_2}$.
MA.9-12.5.6.UNC-3.N.2	If sampling without replacement, the standard deviation of the difference in sample proportions is smaller than what is given by the formula above. If the sample sizes are less than 10% of the population sizes, the difference is negligible.
MA.9-12.5.6.UNC-3.O.1	The sampling distribution of the difference in sample proportions $\hat{p}_1 - \hat{p}_2$ will have an approximate normal distribution provided the sample sizes are large enough: $n_1p_1 \geq 10$, $n_1(1-p_1) \geq 10$, $n_2p_2 \geq 10$, $n_2(1-p_2) \geq 10$.
MA.9-12.5.6.UNC-3.P.1	Parameters for a sampling distribution for a difference of proportions should be interpreted using appropriate units and within the context of a specific populations.
MA.9-12.5.7.UNC-3.Q.1	For a numerical variable, when random sampling with replacement from a population with mean μ and standard deviation, σ, the sampling distribution of the sample mean has mean μ[subscript $\bar{x}$] = μ and standard deviation σ[subscript $\bar{x}$] = $\sigma/\sqrt{n}$.
MA.9-12.5.7.UNC-3.Q.2	If sampling without replacement, the standard deviation of the sample mean is smaller than what is given by the formula above. If the sample size is less than 10% of the population size, the difference is negligible.
MA.9-12.5.7.UNC-3.R.1	For a numerical variable, if the population distribution can be modeled with a normal distribution, the sampling distribution of the sample mean, $\bar{x}$, can be modeled with a normal distribution.
MA.9-12.5.7.UNC-3.R.2	For a numerical variable, if the population distribution cannot be modeled with a normal distribution, the sampling distribution of the sample mean, $\bar{x}$, can be modeled approximately by a normal distribution, provided the sample size is large enough, e.g., greater than or equal to 30.
MA.9-12.5.7.UNC-3.S.1	Probabilities and parameters for a sampling distribution for a sample mean should be interpreted using appropriate units and within the context of a specific population.
MA.9-12.5.8.UNC-3.T.1	For a numerical variable, when randomly sampling with replacement from two independent populations with population means μ_1 and μ_2 and population standard deviations σ_1 and σ_2, the sampling distribution of the difference in sample means $\bar{x}_1 - \bar{x}_2$ has mean μ[subscript $(\bar{x}_1 - \bar{x}_2)$] = $\mu_1 - \mu_2$ and standard deviation, σ[subscript $(\bar{x}_1 - \bar{x}_2)$] = $\sqrt{(\sigma/n_1 + \sigma/n_2)}$.
MA.9-12.5.8.UNC-3.T.2	If sampling without replacement, the standard deviation of the difference in sample means is smaller than what is given by the formula above. If the sample sizes are less than 10% of the population sizes, the difference is negligible.
MA.9-12.5.8.UNC-3.U.1	The sampling distribution of the difference in sample means $(\bar{x}_1 - \bar{x}_2)$ can be modeled with

	a normal distribution if the two population distributions can be modeled with a normal distribution.
MA.9-12.5.8.UNC-3.U.2	The sampling distribution of the difference in sample means $\bar{x}_1 - \bar{x}_2$ can be modeled approximately by a normal distribution if the two population distributions cannot be modeled with a normal distribution but both sample sizes are greater than or equal to 30.
MA.9-12.5.8.UNC-3.V.1	Probabilities and parameters for a sampling distribution for a difference of sample means should be interpreted using appropriate units and within the context of a specific populations.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
Part 4: Probability, Random Variables, and Probability Distributions • Given that variation may be random or not, conclusions are uncertain. • Simulation allows us to anticipate patterns in data. • The likelihood of a random event can be quantified. • Probability distributions may be used to model variation in populations. • Probabilistic reasoning allows us to anticipate patterns in data. *Part 5: Sampling Distributions* • Given that variation may be random or not, conclusions are uncertain. • The normal distribution may be used to model variation. • Probabilistic reasoning allows us to anticipate patterns in data.	*Part 4: Probability, Random Variables, and Probability Distributions* • How can an event be both random and predictable? • About how many rolls of a fair six-sided die would we anticipate it taking to get three 1s? *Part 5: Sampling Distributions* • How likely is it to get a value this large just by chance? • How can we anticipate patterns in the values of a statistic from one sample to another?

Instructional Focus

Learning Targets

Part 4: Probability, Random Variables, and Probability Distributions

- Identify questions suggested by patterns in data.
- Estimate probabilities using simulation.
- Calculate probabilities for events and their complements.
- Interpret probabilities for events.
- Explain why two events are (or are not) mutually exclusive.
- Calculate conditional probabilities.
- Calculate probabilities for independent events and for the union of two events.
- Represent the probability distribution for a discrete random variable.
- Calculate parameters for a discrete random variable.
- Interpret parameters for a discrete random variable.
- Calculate parameters for linear combinations of random variables.
- Describe the effects of linear transformations of parameters of random variables.
- Estimate probabilities of binomial random variables using data from a simulation.
- Calculate probabilities for a binomial distribution.
- Calculate parameters for a binomial distribution.
- Interpret probabilities and parameters for a binomial distribution.
- Calculate probabilities for geometric random variables.
- Calculate parameters of a geometric distribution.
- Interpret probabilities and parameters for a geometric distribution.

Part 5: Sampling Distributions

- Identify questions suggested by variation in statistics for samples collected from the same population.
- Calculate the probability that a particular value lies in a given interval of a normal distribution.
- Determine the interval associated with a given area in a normal distribution.
- Determine the appropriateness of using the normal distribution to approximate probabilities for unknown distributions.
- Estimate sampling distributions using simulation.
- Explain why an estimator is or is not unbiased.
- Calculate estimates for a population parameter.
- Determine parameters of a sampling distribution for sample proportions.
- Determine whether a sampling distribution for a sample proportion can be described as approximately normal.
- Interpret probabilities and parameters for a sampling distribution for a sample proportion.
- Determine parameters of a sampling distribution for a difference in sample proportions.
- Determine whether a sampling distribution for a difference of sample proportions can be described as approximately normal.
- Interpret probabilities and parameters for a sampling distribution for a difference in proportions.

- Determine parameters for a sampling distribution for sample means.
- Determine whether a sampling distribution of a sample mean can be described as approximately normal.
- Interpret probabilities and parameters for a sampling distribution for a sample mean.
- Determine parameters of a sampling distribution for a difference in sample means.
- Determine whether a sampling distribution of a difference in sample means can be described as approximately normal.
- Interpret probabilities and parameters for a sampling distribution for a difference in sample means.

Prerequisite Skills

- Perform calculations involving fractions, decimals, and percentages.
- Rearrange and solve equations and formulas.
- Use and interpret variables in expressions.
- Recognize and interpret sets, subsets, and complements.
- Use logical reasoning to analyze compound events (e.g., "and", "or", "not").
- Understand and apply the fundamental counting principle.
- Calculate combinations and permutations in simple contexts.
- Understand probability as a measure between 0 and 1.
- Interpret simple probabilities as relative frequencies.
- Recognize the long-run relative frequency interpretation of probability.
- Read two-way tables and identify joint, marginal, and conditional frequencies.
- Use bar graphs and segmented bar charts to interpret categorical data.
- Conduct and interpret outcomes from simple simulations (e.g., using spinners, dice, coins, or random number generators).
- Understand how simulation can model randomness.
- Differentiate between a population and a sample.
- Recognize that sample statistics vary from sample to sample.
- Interpret and compare dotplots, histograms, and boxplots.
- Understand shape, center, variability, and outliers in distributions.
- Recognize the bell-shaped curve and its properties (symmetry, mean = median = mode).
- Know the empirical rule (68-95-99.7%).

- Calculate and interpret z-scores.
- Use normal distribution tables or technology to find areas under the curve.
- Conduct simulations of repeated sampling.
- Observe and describe patterns in sample statistics across multiple samples.
- Recognize sources of bias in sample collection.
- Understand that larger samples generally yield more stable estimates.

Common Misconceptions

- Confusing Disjoint (Mutually Exclusive) with Independent Events
  - Students often think disjoint events are also independent, when in fact mutually exclusive events are never independent (because the occurrence of one completely rules out the other).
- Misinterpreting “Random” as Meaning “Unpredictable”
  - Students may think randomness implies chaos or complete unpredictability, instead of understanding randomness as structured unpredictability with long-run patterns.
- Incorrect Use of Addition and Multiplication Rules
  - Students frequently use the addition rule when they should multiply, and vice versa. They often forget to subtract the intersection in the general addition rule.
- Over- or Underestimating Complementary Probabilities
  - Students may forget that the probability of an event plus the probability of its complement must equal 1, or misinterpret the complement in multi-step problems.
- Assuming All Events Are Equally Likely
  - Many students default to equal probabilities (like 1/2 or 1/6) when events clearly have different likelihoods.
- Believing in the "Gambler’s Fallacy"
  - Students mistakenly believe that past outcomes affect future independent events (e.g., a coin flip has to be heads next because it’s been tails five times).
- Misreading Probability Distributions
  - Students may misidentify the values of a random variable versus their associated probabilities, especially with non-uniform distributions.
- Incorrectly Applying Formulas for Mean and Standard Deviation of Random Variables
  - Students often confuse population vs. sample standard deviation or neglect squaring and

taking square roots properly.

- Forgetting When to Use Binomial or Geometric Models
  - Students might apply binomial or geometric formulas in contexts that don't meet the conditions (e.g., using binomial for non-fixed number of trials, or geometric when trials are not independent).
- Confusing Sample Statistics with Population Parameters
  - Students often mix up sample statistics (e.g., $\hat{p}$, $\bar{x}$) and population parameters (e.g., p, μ) or use them interchangeably.
- Assuming Sample Means or Proportions Are Always the Same
  - Some students incorrectly believe that sample statistics should always match the population parameter, failing to understand natural sampling variability.
- Not Understanding the Central Limit Theorem (CLT)
  - Students may think only normally distributed populations lead to normal sampling distributions, instead of recognizing that CLT applies to sample means with large enough samples regardless of shape.
- Assuming Larger Samples Always Reduce Bias
  - Students often confuse bias with variability, not realizing that a large sample reduces variability but won't fix bias in the data collection method.
- Misinterpreting the Standard Deviation of a Sampling Distribution
  - Students may mistake the standard deviation of the population for the standard deviation of the sampling distribution (standard error), and forget to divide by $\sqrt{n}$.
- Inappropriate Use of Normal Approximation
  - Students may apply the normal model to approximate distributions without checking conditions (e.g., $np \geq 10$ and $n(1 - p) \geq 10$ for proportions).
- Confusion About the Meaning of "Unbiased Estimator"
  - Students often misinterpret this to mean individual estimates are always accurate, rather than understanding it as a long-run average property.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity

• **4A Randomness Probability and Simulation**
  • Interpret probability as a long run relative frequency.
  • Estimate probabilities using simulation.
• **4B Probability Rules**
  • Give a probability model for a random process with equally likely outcomes and use it to find the probability of an event.
  • Calculate probabilities using the complement rule.
  • Use the addition rule for mutually exclusive events to find probabilities.
  • Use a two-way table to find probabilities.
  • Calculate probabilities with the general addition rule.
• **4C Conditional Probability and Independent Events**
  • Calculate conditional probabilities.
  • Determine whether two events are
	• **Connections to Unit 1**
	  ○ ***Data Displays and Interpretation***
	  Use dotplots, bar graphs, histograms, and two-way tables when organizing and displaying probability distributions and simulation outcomes.
	
	  Students revisit how to interpret shape, center, and variability in graphical representations of random variables.
	  ○ ***Describing and Comparing Distributions (Unit 1 & 2)***
	  Analyze distributions of random variables and sampling distributions by comparing center and spread.
	
	  Apply vocabulary such as mean, standard deviation, skew, and symmetric in new contexts involving probability models.
	  ○ ***Simulations***
	  Previously introduced in basic data contexts, simulations now help estimate probabilities and model sampling variability.
	
	  Reinforce use of random number generators, coin tosses, or spinners in modeling real-world probabilistic events.
	• **Spiral Focus within Unit:**
		• **Spiral Warm-Ups**
		  ○ Daily or weekly reviews of past concepts with a twist connecting to the current unit.
		    ▪ "Revisit a histogram of quiz scores: What's the shape, center, and spread? Now imagine this is a probability distribution. How do your interpretations change?"
		    ▪ "Given this two-way table, calculate joint, marginal, and conditional probabilities then determine if the events are independent."
		    ▪ "Find the z-score of a value. Now estimate the probability that a random variable is less than that value."
		• **Randomness Stations**
		  ○ Use simulations (manual or tech-based) to model key concepts and connect back to randomness, sampling, and distributions.
		    ▪ Use dice, spinners, or coins to simulate binomial or geometric distributions.
		    ▪ Use Desmos, GeoGebra, or random number generators to create sampling distributions for means and proportions.
		    ▪ Reflect: How is this similar to the simulations in Unit 1? What has changed?
		• **Error Analysis & Concept Sorting**
		  ○ Deepen understanding through critique and classification.
		    ▪ Error Analysis:
		      ▪ Give students flawed sample problems (e.g., misapplied multiplication rule, incorrect z-score usage).

independent. • Use the general multiplication rule to calculate probabilities. • Use the tree diagram to model a random process involving multiple states and to find probabilities. • Calculate probabilities using the multiplication rule for independent events. • Determine if it is appropriate to use the multiplication rule for independent events in a given setting. • **4D Introduction to Discrete Random Variables** • Calculate probabilities involving a discrete random variable. • Display the probability distribution of a discrete random variable with a histogram and describe its shape. • Calculate and interpret the mean, or expected value, or a discrete random variable.	○ ***Randomness, Probability, and Simulation*** Emphasize long-run relative frequency through simulations to connect the concept of probability to empirical frequency from earlier descriptive statistics. ○ ***Probability Rules*** Reinforce set notation and interpretation from two-way tables, previously introduced in categorical data analysis. Spiral earlier work with conditional and joint distributions into more formal application with probability models and addition rules. ○ ***Conditional Probability and Independence*** Build on earlier logic from Venn diagrams and two-way tables to introduce tree diagrams and the general multiplication rule. Re-emphasize interpretation of independence in real-life contexts (e.g., surveys or experiments). ○ ***Discrete Random Variables*** Leverage familiarity with distributions to now label and describe discrete probability distributions more	▪ Have them explain and correct the mistake—and link it back to earlier topics. ▪ Concept Sorting: ▪ Provide mixed cards (e.g., vocab, graphs, formulas, real-world situations). ▪ Students sort into “Probability Rules,” “Distributions,” “Sampling,” “Simulations,” etc. • **Cumulative Problem Sets or Mini Quizzes** ○ Blend current and past topics together in graded or practice form. ▪ Example: A quiz could include: • **Spiral Review Games** ○ Increase engagement while reviewing multiple topics. ▪ Jeopardy-style Review: Include categories like "Random Variables," "Normal Distributions," "Sampling vs. Population," "Simulation Station." ▪ Probability Board Games: Create board games where students solve spiraled problems to move forward. ▪ Escape Room or Digital Breakout: Puzzles require students to apply concepts from Units 1–5 to unlock combinations. • **Math Medic or Skew the Script Spiral Tasks** ○ Use applied tasks or activities from partner resources that

• Calculate and interpret the standard deviation of a discrete random variable. • **4E Transforming and Combining Random Variables** • Describe the effect of a linear transformation - adding or subtracting a constant and/or multiplying or dividing by a constant - on the probability distribution of a random variable. • Calculate the mean of a sum, difference, or other linear combination of random variables. • If appropriate, calculate the standard deviation of a sum, difference, or other linear combination of random variables. • **4F Binomial and Geometric Random Variables** • Determine whether a random variable has a binomial distribution. • Calculate and interpret probabilities involving binomial random variables.	formally. Connect histogram work with new focus on probability mass functions. ○ ***Transformations of Random Variables*** Connect to earlier transformations in unit conversions or z-scores to analyze linear combinations of random variables. ○ ***Binomial and Geometric Models*** Emphasize criteria (fixed n, success/failure) and connect to prior simulations or observed long-run frequencies of binary outcomes. Highlight real-world relevance and repetition of Bernoulli trials. ○ ***Normal Distributions, Revisited*** Reconnect to Unit 2's normal model work—emphasize standard normal calculations and revisit empirical rule and z-scores. Reinforce concepts of area under the curve and distribution shape. ○ ***What Is a Sampling Distribution?*** Connect back to variability and bias in data collection (Unit 3). Use prior understanding of	spiral in real-world contexts. ○ Modify existing Math Medic or Skew the Script lessons by inserting warm-up questions or extension problems that pull in skills from earlier units. • **Concept Mapping Across Units** ○ Visually connect major concepts from Units 1–5. ▪ Students build a concept map starting with current unit (e.g., "Sampling Distribution") and link back to: "Distribution shape", "Simulations", "Parameters vs. statistics", "Probability rules" • **Quick Writes / Reflective Journals** ○ Strengthen conceptual understanding and retrieval. ○ Prompt examples: ▪ "What's one thing about variability that you now see differently after studying sampling distributions?" ▪ "Compare the use of simulations in Unit 1 and Unit 4. What stayed the same? What changed?" • **AP Exam Free Response Spiral Practice (FRAPPYs)** ○ Practice real AP problems aligned to both current and past units. ▪ Choose problems that span multiple skills (e.g., describing distributions, interpreting simulation results, calculating probabilities). ▪ Use a gradual release: Individual → Group → Class Discussion. ▪ Include student reflection: "Which part felt familiar? From which earlier unit?"

<table>
<tr>
<td>

Find the mean and standard deviation of a binomial distribution. Interpret these values.
Calculate and interpret probabilities involving geometric random variables.
Find the mean and standard deviation of a geometric distribution. Interpret these values.

5A Normal Distributions, Revisited

Calculate probabilities and percentiles involving normal random variables.
Find probabilities involving a sum, a difference, or another linear combination of independent, normal random variables.

5B What is a Sampling Distribution?

Distinguish between a parameter and a statistic.
Create a sampling distribution using all possible samples from a small population.

</td>
<td>
samples vs. populations to distinguish statistics and parameters.

Sample Proportions and Sample Means
Emphasize how sampling distributions represent the foundation for statistical inference.

Reuse and deepen knowledge of mean, variability, and shape in new probabilistic settings.

</td>
<td></td>
</tr>
</table>

- Use the sampling distribution of a statistic to evaluate a claim about a parameter. - Determine if a statistic is an unbiased estimator of a population parameter. - Describe the relationship between sample size and the variability of an estimator. **5C Sample Proportions** - Calculate and interpret the mean and standard deviation of the sampling distribution of a sample proportion, p^. - Determine if the sampling distribution of p^ is approximately normal. - If appropriate, use a normal distribution to calculate probabilities involving sample proportions. - Describe the shape, center, and variability of the sampling distribution of a difference in sample proportions p_1^ - p_2^. **5D Sample Means**		

• Calculate and interpret the mean and standard deviation of the sampling distribution of a sample mean, x bar. • Determine if the sampling distribution of x bar is approximately normal. • If appropriate, use a normal distribution to calculate probabilities involving sample means. • Describe the shape, center, and variability of the sampling distribution of a difference in a sample means x_1 bar - x_2 bar.		

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks • AP Classroom Progress Checks	• Topic Tests • Unit 2 Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
AP Classroom AP Statistics CED	iXL Delta Math Desmos Khan Academy Math Medic Skew the Script Teacher Made worksheets Textbook - Starnes, D., & Tabor, J. (2024). *The Practice of Statistics for the AP Classroom* (7th ed.). Beford, Freeman & Worth.

Career Readiness, Life Literacies, and Key Skills

PFL.9.1.12.FI.3	Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g., college, retirement).
PFL.9.1.12.RM.1	Describe the importance of various sources of income in retirement, including Social Security, employer-sponsored retirement savings plans, and personal investments.
WRK.9.2.12.CAP.4	Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
WRK.9.2.12.CAP.6	Identify transferable skills in career choices and design alternative career plans based on those skills.
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.DC.6	Select information to post online that positively impacts personal image and future college and career opportunities.
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.
TECH.9.4.12.IML.3	Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).
TECH.9.4.12.IML.7	Develop an argument to support a claim regarding a current workplace or societal/ethical issue such as climate change (e.g., NJSLSA.W1, 7.1.AL.PRSNT.4).

Interdisciplinary Connections

MATH.9-12.S.ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional

	relative frequencies). Recognize possible associations and trends in the data.
SOC.6.1.12.GeoGI.1.a	Explain how geographic variations impacted economic development in the New World, and its role in promoting trade with global markets (e.g., climate, soil conditions, other natural resources).
MATH.9-12.S.IC.A.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
ELA.RL.CR.11–12.1	Accurately cite strong and thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what a literary text says explicitly and inferentially, as well as interpretations of the text; this may include determining where the text leaves matters uncertain.
MATH.9-12.S.IC.B.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
ELA.RI.MF.9–10.6	Analyze, integrate, and evaluate multiple interpretations (e.g., charts, graphs, diagrams, videos) of a single text or text/s presented in different formats (visually, quantitatively) as well as in words in order to address a question or solve a problem.
ELA.RI.AA.9–10.7	Describe and evaluate the argument and specific claims in an informational text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and reasoning.
ELA.W.AW.11–12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
SCI.HS-LS2-6	Evaluate the claims, evidence, and reasoning that the complex interactions in ecosystems maintain relatively consistent numbers and types of organisms in stable conditions, but changing conditions may result in a new ecosystem.
	Global economic activities involve decisions based on national interests, the exchange of different units of exchange, decisions of public and private institutions, and the ability to distribute goods and services safely.
SCI.HS-ETS1-4	Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.
CS.9-12.8.1.12.DA.5	Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
CS.9-12.8.2.12.ED.3	Evaluate several models of the same type of product and make recommendations for a new design based on a cost benefit analysis.