Trigonometric Functions

Content Area: **Math**
Course(s):
Time Period: **MP4**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
In Unit 4, Students expand previous understandings of trigonometric functions. Students learn the trigonometric ratios and use them to find missing side lengths of triangles. They learn to graph trigonometric functions and identify the key features of the graphs. Students also learn methods to solve problems using trigonometric functions.	In this unit students continue to develop their understanding of functions. Functions help students to analyze the relationships between quantities. Skills related to functions allow students to determine how different aspects of a problem are related and how those relationships can be manipulated to achieve a desired results. The skills developed in this unit are also foundational skills for work in upper level mathematics courses.

NJSLS

F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
F.BF.B.4	Find inverse functions.
F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
G.GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
G.GPE.A.2	Derive the equation of a parabola given a focus and directrix.
G.GPE.A.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F.TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
G.SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in

	the triangle, leading to definitions of trigonometric ratios for acute angles.
G.SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
F.TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Standards for Mathematical Practice

1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure
8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• For any right triangle, the ratios of the sides are always the same for a given angle θ. These ratios define the six basic trigonometric functions: sine, cosine, secant, cosecant, tangent, and cotangent. • An angle in standard position has a vertex at the origin and an initial side along the positive x-axis. Reference angles on the unit circle are used to extend trigonometric ratios to use angles greater than 90°. • Reference angles and reference triangles are used to find and evaluate the six trigonometric functions on the unit circle. • Periodic functions are functions that repeat on a pattern of y-values at regular intervals. The sine function $y = a \sin bx$ and the cosine function $y = a \cos bx$ are periodic functions that have an amplitude of $\lvert a \rvert$ and a frequency of $b/2\pi$ • The tangent, cotangent, secant, and cosecant functions can be defined using ratios or reciprocals of the sine and cosine functions.	• How can ratios of lengths of sides within right triangles help determine other lengths and angle measures in the triangles? • How can we extend the trigonometric ratios to angles greater than 90°? • How is the unit circle related to trigonometric functions? • How can you identify key features of sine and cosine functions? • How do key features of one trigonometric function relate to key features of other trigonometric functions? • How can you find and use translations of graphs of trigonometric functions?

The graphs of these functions are periodic, have no amplitude, and have vertical asymptotes. • The parameters *a, b, c* and *d* represent values that transform the functions $y= a \sin(x-c) +d$ and $y= a \cos b(x-c) +d$. |a| represents the amplitude of the function. The period of the function is $2\pi/b$, c represents a horizontal shift; and d represents a vertical shift.	

Instructional Focus

Learning Targets

- Use special triangles to determine trigonometric ratios geometrically.
- Use trigonometric functions and the Pythagorean Theorem to find missing side lengths.
- Identify and explain trigonometric identities.
- Find the measures of an engle in standard position and its reference angle.
- Use radian measure on the unit circle to find arc length
- Convert between degrees and radians.
- Use reference angles and triangles to evaluate trigonometric function and their reciprocal functions
- Use the Pythagorean Identify to find the sine, cosine, and quadrant of an angle
- Graph and identify the features of sine and cosine functions
- Find and interpret the average rate of change of a periodic function over a specified interval
- Compare key features of different periodic functions
- Describe and compare key features of the graphs of trigonometric functions
- Graph functions of the form $f(x) - a \tan bx$ and relate the graph of the function to the graph of the parent function
- Identify how changing the parameters of the sine or cosine function affects the graph of the function
- Use trigonometric functions to model situations with specified amplitude, frequency, and midline

Prerequisite Skills

- Right triangle trigonometry, with advanced students working with laws of sines and cosines. connect the idea of functions with trigonometry and see sine, cosine and tangent values as functions of angle values input in radians. Connections are made such as the cosine of an angle equaling the sine of its complement as well as to the Geometry Standards involving radian measures.

- Relationship between the sine and cosine values for a particular angle. The fact that the sum of the squares of these values always equals 1 provides a unique way to view trigonometry through the lens of geometry.

Common Misconceptions

- Students may believe that there is no need for radians if one already knows how to use degrees. Students need to be shown a rationale for how radians are unique in terms of finding function values in trigonometry since the radius of the unit circle is 1.
- Students may also believe that all angles having the same reference values have identical sine, cosine and tangent values. They will need to explore in which quadrants these values are positive and negative.
- Students may believe that there is no connection between the Pythagorean Theorem and the study of trigonometry.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Trigonometric Ratios • Unit Circle	• Transformations of Functions (Algebra I/ Topic 3) • Side Lengths of Right Triangles (Geometry)	• IXL • Khan Academy • Delta Math

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests • Unit Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources

• Savvas EnVision Algebra 2 • Pacing Guide	• IXL • Delta Math • Desmos • Khan Academy

Career Readiness, Life Literacies, and Key Skills

9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
9.4.12.GCA.1	Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others (e.g., SL.11-12.1., HS-ETS1-1, HS-ETS1-2, HS-ETS1-4, 6.3.12.GeoGI.1, 7.1.IH.IPERS.6, 7.1.IL.IPERS.7, 8.2.12.ETW.3).
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.

Interdisciplinary Connections

SL.PE.11–12.1.A	Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas.
SL.PE.11–12.1.B	Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g., student developed rubrics), and establish individual roles as needed.
SL.PE.11–12.1.C	Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
SL.PE.11–12.1.D	Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task.
9-12.HS-ETS1-4.5.1	Use mathematical models and/or computer simulations to predict the effects of a design solution on systems and/or the interactions between systems.
9-12.HS-ETS1-2.6.1	Design a solution to a complex real-world problem, based on scientific knowledge, student-generated sources of evidence, prioritized criteria, and tradeoff considerations.
9-12.HS-ETS1-3.6.1	Evaluate a solution to a complex real-world problem, based on scientific knowledge, student-generated sources of evidence, prioritized criteria, and tradeoff considerations.