Rational Exponents, Radical, Exponential and Logarithmic Functions

Content Area: **Math**
Course(s):
Time Period: **MP3**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
In Unit 3, students expand their knowledge of radical functions. Students understand properties of rational exponents and radicals. They learn methods to graph radical functions, solve radical equations, and combine functions. Students identify inverses of functions and learn to write the equations of inverse functions. This unit also focuses on extending previous understandings of exponential functions. In this unit students will identify the key features of exponential functions. They will understand logarithms and their properties. Students will also learn how to solve exponential and logarithmic equations.	In this unit students continue to develop their understanding of functions. Functions help students to analyze the relationships between quantities. Skills related to functions allow students to determine how different aspects of a problem are related and how those relationships can be manipulated to achieve a desired results. The skills developed in this unit are also foundational skills for work in upper level mathematics courses.

NJSLS

MATH.9-12.N.RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MATH.9-12.F.BF.A.1.b	Combine standard function types using arithmetic operations.
MATH.9-12.N.RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MATH.9-12.F.BF.A.1.c	Compose functions.
MATH.9-12.S.ID.B.6.a	Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MATH.9-12.F.BF.B.4.b	Verify by composition that one function is the inverse of another.
MATH.9-12.F.BF.B.4.c	Read values of an inverse function from a graph or a table, given that the function has an

	inverse.
MATH.9-12.F.BF.B.4.d	Produce an invertible function from a non-invertible function by restricting the domain.
MATH.9-12.F.BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
MATH.9-12.A.CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MATH.9-12.A.REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MATH.9-12.A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
MATH.9-12.F.IF.C.8.b	Use the properties of exponents to interpret expressions for exponential functions.
MATH.9-12.A.SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.
MATH.9-12.F.LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MATH.9-12.F.LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
MATH.9-12.A.SSE.B.3.c	Use the properties of exponents to transform expressions for exponential functions.
MATH.9-12.F.LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• Rational exponents and radicals represent the number of roots a polynomial has. The roots of a polynomial are used to simplify expressions and solve equations. • The properties of integer exponents can be applied to terms with rational exponents, ad well as to radicals. The properties of exponents and radicals can be used to rewrite radical expressions. When rewriting radical expressions, like radicals, which have the same index, can be added and subtracted. • The function $g(x) = a\sqrt[n]{(x-h)} + k$ represents the transformation of the parent radical function $f(x) = \sqrt[n]{x}$, where a stretches or compresses the graph vertically, h translates the graph horizontally, and k translates the graph vertically. • Solving equations that include radicals or rational exponents is similar to solving rational equations. • Functions can be combined by operations (+, -, ×, ÷) and by composition. The results of the operation or composition can be described as a single function. The domain of the result may be different from the domains of the original functions. • The inverse of a function is found by exchanging the roles of the independent and dependent variables. Composition can be used to verify that two functions are inverses. • The rate of exponential growth or decay is the ratio between two consecutive output values in an exponential function. • Exponential models are useful in representing situations in which the rate increases by the same percent for each period of time and for interpreting problems that involve compound interest. Exponential regression can be used to generate exponential models for real-world contexts. • A logarithmic function is the inverse of an exponential function. Logarithms are found by determining that the exponent must be applied to the base to yield a given result. • The inverse relationship between exponential and logarithmic functions reveals key features of the graphs of both functions. Logarithmic functions can be used to model several real-	How are exponents and radicals used to represent roots of real numbers? • How can properties of exponents and radicals be used to rewrite radical expressions? • How can you use what you know about transformations of functions to graph radical functions? • How can you solve equations that include radicals or rational exponents? • How do you combine, multiply, divide, and compose functions, and how do you find the domain of the resulting function? • How can you find the inverse of a function and verify the two functions are inverses?

world situations. • Properties of Logarithms can be used to rewrite logarithmic expressions and to evaluate logarithms by changing the base • Some exponential equations can be solved by rewriting both sides with a common base. For others, rewriting the equation using logarithms and applying properties of logarithms, is a more efficient method. • A geometric sequence is a sequence of numbers in which terms are related to the previous term by a common ratio, *r.* A geometric series is the sum of a certain number of terms in a geometric sequence.	

Instructional Focus

Learning Targets
Learners will.. • Find all real *n*th roots of a number • Evaluate expressions with rational exponents • Use *n*th roots to solve equations by rewriting expressions using the properties of exponents • Use the properties of exponents and radicals to identify ways to rewrite radical expressions • Interpret radical expressions that represent a quantity in terms of its context • Graph radical functions, including square root and cube root functions. • Identify the effect of transformations on the key features of the graphs of radical functions • Solve radical equations in one variable • Explain how extraneous solutions may arise when solving radical equations • Solve radical inequalities and apply the solution within a real-world context • Combine functions by addition, subtraction, multiplication, or division, and identify the domain of the result • Compose functions, specifying the order in which the functions are applied and describing the domain of the composite function • Use tables, graphs, and equations to represent the inverse of a relation. • Write an equation for the inverse of a function by restricting the domain • Verify that one function is the inverse of another, using composition. • Interpret key features of exponential finctions represented by graphs, tables, and equations. • Graph transformations of exponential functions showing intercepts and end behavior • Rewrite exponential functions to identify rates • Interpret the parameters of an exponential function within the context of compound interest problems • Construct exponential models given two points or by using regression

- Understand the inverse relationship between exponents and logarithms
- Use logarithms to solve exponential models
- Evaluate logarithms using technology

Prerequisite Skills

- Evaluate expressions
- Simplify expressions
- Combining like terms
- Adding, subtracting, and multiplying polynomials
- Properties of exponents

Common Misconceptions

- Incorrectly calculating *n*th roots.
- Viewing roots as factors.
- Incorrectly applying the properties of exponents using the base rather than the exponents
- Graphing translated functions without using the parent function
- Incorrectly identifying the independent and dependent variables in a given situation
- Incorrectly identifying the quantity that is being raised to a power
- Confusing the reciprocal of a value for the value itself
- incorrectly evaluating expressions when composing and combining functions.
- believing that zero of a function is not in the domain of a combination of functions.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Graphing and Solving Radical Equations • Inverse Functions	• Square Roots and Cube Roots (Algebra I) • Graphing Rational	• IXL • Khan Academy

• Exponential and Logarithmic Functions • Exponential and Logarithmic Equations • Exponential and Logarithmic Models	Functions (Topic 4) • Checking for Extraneous Solutions (Topic 4) • Properties of Exponents (Algebra I) • Transformations of Functions (Algebra I) • Inverse Functions (Algebra I)	• Delta Math

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests • Unit Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
• Savvas EnVision Algebra 2 • Pacing Guide	• IXL • Delta Math • Desmos • Khan Academy

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Interdisciplinary Connections

ELA.SL.PE.11–12.1.A	Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas.
ELA.SL.PE.11–12.1.B	Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g., student developed rubrics), and establish individual roles as needed.
ELA.SL.PE.11–12.1.C	Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
ELA.SL.PE.11–12.1.D	Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task.
9-12.HS-ETS1-4.5.1	Use mathematical models and/or computer simulations to predict the effects of a design solution on systems and/or the interactions between systems.
9-12.HS-ETS1-3.6.1	Evaluate a solution to a complex real-world problem, based on scientific knowledge, student-generated sources of evidence, prioritized criteria, and tradeoff considerations.
9-12.HS-ETS1-3.ETS1.B.1	When evaluating solutions, it is important to take into account a range of constraints, including cost, safety, reliability, and aesthetics, and to consider social, cultural, and environmental impacts.