# Polynomial and Rational Functions

Content Area: **Math**
Course(s):
Time Period: **MP2**
Length: **45**
Status: **Published**

## Unit Overview

| Unit Summary | Unit Rationale |
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| Unit two focuses on extending students' previous knowledge of polynomials. Students identify key features of polynomial functions and interpret graphs of polynomial functions. They will learn methods to add, subtract, multiply and divide polynomial expressions. Students will use polynomial identities to multiply and factor polynomial expressions, use multiple theorems as tools to understand the roots of polynomial functions, and transform the graphs from cubic or quartic parent functions. Unit two also extends students' prior knowledge of polynomial functions to rational functions. They will identify key features of the graphs of rational functions and learn methods of solving rational equations. | In this unit students continue to develop their understanding of functions. Functions help students to analyze the relationships between quantities. Skills related to functions allow students to determine how different aspects of a problem are related and how those relationships can be manipulated to achieve a desired results. The skills developed in this unit are also foundational skills for work in upper level mathematics courses. |

## NJSLS

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| MATH.9-12.A.APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. |
| MATH.9-12.F.BF.A.1.b | Combine standard function types using arithmetic operations. |
| MATH.9-12.A.APR.B.2 | Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$. |
| MATH.9-12.A.APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. |
| MATH.9-12.A.APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. |
| MATH.9-12.A.APR.C.5 | Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle. |

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| MATH.9-12.F.BF.B.3 | Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$(both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. |
| MATH.9-12.A.APR.D.6 | Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. |
| MATH.9-12.A.APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. |
| MATH.9-12.A.CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. |
| MATH.9-12.A.CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. |
| MATH.9-12.A.REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |
| MATH.9-12.N.CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. |
| MATH.9-12.A.REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. |
| MATH.9-12.F.IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. |
| MATH.9-12.F.IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| MATH.9-12.F.IF.C.7.c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. |
| MATH.9-12.F.IF.C.7.d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. |
| MATH.9-12.A.REI.D.11 | Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. |
| MATH.9-12.F.IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). |
| MATH.9-12.A.SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. |

## Standards for Mathematical Practice

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| MATH.K-12.1 | Make sense of problems and persevere in solving them |
| MATH.K-12.2 | Reason abstractly and quantitatively |
| MATH.K-12.3 | Construct viable arguments and critique the reasoning of others |
| MATH.K-12.4 | Model with mathematics |
| MATH.K-12.5 | Use appropriate tools strategically |

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| MATH.K-12.6 | Attend to precision |
| MATH.K-12.7 | Look for and make use of structure |
| MATH.K-12.8 | Look for and express regularity in repeated reasoning |

## Unit Focus

| Enduring Understandings | Essential Questions |
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| • A polynomial Function is a function whose rule is either a monomial or a sum of monomials. The key features of the graph of a polynomial function - such as its end behavior, intercepts, and turning points - can be used to sketch a graph of the function.<br>• Just as with real numbers, the properties of operations can be used to add, subtract, and multiply polynomials. Polynomial functions can be used to represent and compare real world situations.<br>• Polynomial identities and the Binomial Theorem are helpful tools for efficiently rewriting expressions and describing mathematical relationships.<br>• Polynomial expressions can be divided by linear factors using long division or synthetic division. The Remainder Theorem is used to determine the remainder of a division problem.<br>• The zeros of a polynomial function can be determined by using factoring or synthetic division. The zeros of a function can be used to sketch its graph.<br>• Theorems such as the Rational Root Theorem, the Fundamental Theorem of Algebra and the Conjugate Root Theorems are helpful tools for determining the roots of a polynomial function.<br>• Polynomial functions are categorized as even, odd, or neither. Even functions are symmetric about the y-axis, and for all x in the domain, $f(-x) = -f(x)$<br>• The reciprocal function is used to model inverse variation, which is a proportional relationship between two variables such that when one variable increases the other variable decreases.<br>• A rational function is any function $R(x) = P(x)/Q(x)$ where $P(x)$ and $Q(x)$ are polynomial functions. The domain of a rational function is all real numbers except | • How do key features of a polynomial function help you to sketch its graph?<br>• How do you add, subtract, and multiply polynomials?<br>• How do you use polynomial identities to rewrite expressions efficiently?<br>• How can you divide polynomials?<br>• How are the zeros of a polynomial function related to an equation and graph of the function?<br>• How are the roots of a polynomial equation related to the coefficients and degree of the polynomial?<br>• How are symmetry and transformations represented in the graph and equation of a polynomial function?<br>• How are inverse variation and reciprocal functions related and represented?<br>• How can you graph a rational function?<br>• How does understanding operations with fractions help you to multiply and divide rational expressions?<br>• How do you rewrite rational expressions to find sums and differences?<br>• How can you solve rational equations and identify extraneous solutions? |

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| any x-values for which Q(x) equals zero.<br>• Rational expressions form a system similar to the system of rational numbers and can be multiplied and divided by applying the properties of operations as they apply to rational expressions.<br>• The properties of operations used to add and subtract rational numbers can be applied to adding and subtracting rational exrpessions.<br>• Rational equations contain a rational expression and can be solved by multiplying each side of the equation by a common denominator to eliminate the fractions. Any solution that is excluded from the domain of the original equation is extraneous | |

## Instructional Focus

**Learning Targets**

Learners will..

- Graph polynomial functions and show the key features of the graph
- Predict end behavior of polynomial functions by interpreting the leading coefficients and degrees.
- Sketch graphs showing key features, given a verbal description
- Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations.
- Compare a polynomial function represented algebraically with one represented graphically
- Prove polynomial identities and used them to multiply and factor polynomials
- Expand binomials using the Binomial Theorem and coefficients determined by Pascal's Triangle
- Divide polynomial expressions using long division
- Use synthetic division to rewrite rational expressions
- Identify zeros of a function by factoring or using synthetic division
- Use the zeros of a polynomial function to sketch its graph
- Extend polynomial theorems and identities to find the real and complex solutions of polynomial equations
- Write polynomial functions using conjugates
- Recognize even and odd functions from their graphs and algebraic equations
- Identify the effect on the graphs of cubic and quartic functions of replacing $f(x)$ with $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$
- Use inverse variation to write and graph the reciprocal function
- Identify the effect of transformations on the graph of the reciprocal function and define the effects of $h$ and $k$ on the function $f(x)= 1/(x-h) +k$.
- Graph rational functions by identifying asymptotes and end behavior

- Rewrite simple rational expressions in different forms using long division
- Use the structure or rational expressions to rewrite simple rational expressions in different forms
- Understand that rational expressions form a system analogous to the system of rational numbers and use that understanding to multiply and divide rational expressions.

**Prerequisite Skills**

- Graph quadratic functions
- Write equations of parabolas
- Solve quadratic equations by graphing
- Solve quadratic equations by factoring
- Find the number and type of solutions

**Common Misconceptions**

- Students may have difficulty sketching graphs with an appropriate level of accuracy.
- Students may find it difficult to determine which model should be used to represent a given situation.
- Students may struggle with accuracy while performing arithmetic operations on polynomials (ie: (a+b)2 does not equal a2 + b2) or with factoring.
- Students may have difficulty analyzing graphical models that do not fit in the grapher's standard window

## Spiraling For Mastery

| Current Unit Content/Skills | Spiral Focus | Activity |
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| • Operations with Polynomials<br>• Polynomial Identities<br>• Graphing Polynomial Functions<br>• Factoring Polynomials<br>• Graphing Rational Functions<br>• Solving Rational Equations | • Transformations of Functions (Algebra I)<br>• Multiplication with Exponents (Algebra I)<br>• Simplifying Polynomials (Algebra I)<br>• Zeros (Topic 2)<br>• Complex Numbers (Topic 2)<br>• Solving Equations (Algebra I)<br>• Operations with Polynomials (Topic 3)<br>• Graphing Polynomials (Topic 3) | • IXL<br>• Khan Academy<br>• Delta Math |

## Assessment

| Formative Assessment | Summative Assessment |
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| • Homework<br>• Lesson Checks<br>• MathXL<br>• Quizzes<br>• Exit Tickets<br>• Lesson Reflections<br>• Performance Tasks | • Topic Tests<br>• Unit Benchmark (Link-It) |

## Resources

| Key Resources | Supplemental Resources |
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| • Savvas EnVision Algebra 2<br>• Pacing Guide | • IXL<br>• Delta Math<br>• Desmos<br>• Khan Academy |

## Career Readiness, Life Literacies, and Key Skills

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| CRP.K-12.CRP2 | Apply appropriate academic and technical skills. |
| CRP.K-12.CRP4 | Communicate clearly and effectively and with reason. |
| CRP.K-12.CRP6 | Demonstrate creativity and innovation. |
| CRP.K-12.CRP8 | Utilize critical thinking to make sense of problems and persevere in solving them. |
| CRP.K-12.CRP11 | Use technology to enhance productivity. |
| CRP.K-12.CRP12 | Work productively in teams while using cultural global competence. |

## Interdisciplinary Connections

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| ELA.SL.PE.11–12.1 | Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. |
| ELA.SL.PE.11–12.1.A | Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas. |
| ELA.SL.PE.11–12.1.C | Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives. |
| ELA.SL.PE.11–12.1.D | Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task. |
| ELA.SL.ES.11–12.3 | Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used. |
| ELA.SL.PI.11–12.4 | Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience. |
| 9-12.HS-ETS1-2 | Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering. |
| 9-12.HS-ETS1-1.1.1 | Analyze complex real-world problems by specifying criteria and constraints for successful solutions. |