Trigonometric Functions

Content Area: **Math**
Course(s):
Time Period: **MP3**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
This unit focuses on trigonometry, emphasizing the relationships between angles and sides in triangles, especially right triangles. Students will learn trigonometric ratios like sine, cosine, and tangent, applying them to find unknown sides and angles. The unit circle will be explored to understand the exact values of trigonometric functions and their periodic nature. Students will also work with angle measures in degrees and radians, graph trigonometric functions, and analyze transformations. Real-world applications, such as arc length and sector area, will be investigated alongside concepts like linear and angular speed. Through problem-solving and discussions, students will build essential skills for advanced mathematics and appreciate trigonometry's relevance in various fields.	The rationale for this unit on trigonometry lies in the fundamental role that trigonometric concepts play in both mathematics and real-world applications. Understanding trigonometry equips students with essential tools for solving problems involving angles and distances, which are critical in fields such as physics, engineering, architecture, and computer science. Mastering trigonometric ratios and their relationships fosters analytical thinking and problem-solving skills that are applicable beyond the classroom. Additionally, grappling with the unit circle and the periodic nature of trigonometric functions enhances students' mathematical reasoning and prepares them for more advanced topics in calculus and beyond. This unit not only builds on prior knowledge but also encourages students to appreciate the interconnectedness of mathematical concepts, ultimately promoting a deeper understanding of the world around them through the lens of mathematics.

NJSLS

F.TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
G.SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
F.TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for □/3, □/4 and □/6, and use the unit circle to express the values of sine, cosine, and tangent for □ – □, □ + □, and 2□ – □ in terms of their values for □, where □ is any real number.
F.TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric

	functions.
G.SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
F.TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
G.SRT.D.9	Derive the formula □ = 1/2 □□ sin(□) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.D.10	Prove the Laws of Sines and Cosines and use them to solve problems.
G.SRT.D.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Standards for Mathematical Practice

1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure
8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• The six trigonometric ratios provide a foundation for understanding relationships in right triangles and can be applied to solve various mathematical problems. • Evaluating trigonometric ratios accurately requires proficiency in using calculators and understanding how to apply the correct functions. • Trigonometric ratios can be utilized to solve for unknown sides and angles in triangles, emphasizing the practical applications in real-world scenarios. • The ability to apply trigonometric concepts enables problem-solving in diverse fields	• How do the six trigonometric ratios of a right triangle relate to its angles and sides, and how can they be applied in problem-solving? • What strategies can we use to evaluate trigonometric ratios effectively with a calculator, and how do we ensure accuracy in our calculations? • In what ways can trigonometric ratios help us solve for unknown angles and sides in various types of triangles? • How can we apply trigonometric concepts to solve real-world problems in fields such as physics and engineering?

such as physics, engineering, and architecture. • The concept of angle measure extends beyond 180° and includes negative angles, allowing for a more comprehensive understanding of rotational movement. • Identifying reference angles and coterminal angles enhances the understanding of angle relationships and their applications in trigonometric contexts. • Angles in standard position allow for consistency in identifying and communicating about angles on the coordinate plane. • Understanding radian measure and converting between degrees and radians is essential for working with trigonometric functions and the unit circle. • The 16-point unit circle provides a visual and analytical tool for finding exact values of trigonometric functions, enriching comprehension of periodic behavior. • The ability to calculate arc length and area of a sector connects trigonometry to real-world applications, such as in circular motion and engineering design. • Understanding the relationship between linear speed and angular speed is crucial for analyzing motion in circular paths. • Defining trigonometric ratios in the coordinate plane expands the application of these concepts to a wider range of mathematical problems. • Graphing sine, cosine, and tangent functions reveals their periodic nature and the impact of transformations on their characteristics. • The ability to determine if an equation is an identity requires a deep understanding of trigonometric properties and their interrelationships. • Transforming trigonometric functions allows	• How does the definition of angle measure extend to include negative angles and angles greater than 180°, and why is this important? • Why is it important to understand reference angles and coterminal angles, and how do they affect the calculation of trigonometric ratios? • How does representing angles in standard position facilitate easier communication and calculations in trigonometry? • What is the significance of radian measure in trigonometry, and how do we convert between degrees and radians effectively? • How does the unit circle serve as a tool for understanding the values of trigonometric functions, and what insights does it provide about periodic behavior? • How do we calculate arc length and the area of a sector, and what real-world situations can these calculations apply to? • How are linear speed and angular speed related, and how can we use this relationship to analyze motion in circular paths? • How can we define and apply trigonometric ratios in the coordinate plane to solve problems involving angles and distances? • What key characteristics of sine, cosine, and tangent functions can we identify through graphing, and how do transformations affect these graphs? • What criteria do we use to determine whether an equation is an identity, and why is this concept important in trigonometry? • How can we analyze the effects of transformations on trigonometric functions, and what do these transformations reveal about their properties?

for the exploration of amplitude, period, vertical shifts, and phase shifts, enriching the understanding of function behavior.	

Instructional Focus

Learning Targets

- Define the six trigonometric ratios of an acute angle in terms of a right angle.
- Evaluate trigonometric ratios, using triangles and on a calculator.
- Solve triangles using trigonometric ratios.
- Solve applications using triangles.
- Use a rotating ray to extend the definition of angle measure to negative angles and angles greater than 180°.
- Find reference angles and coterminal angles.
- Name and draw angles in standard position.
- Define radian measure and convert angle measures between degrees and radians.
- Use the 16-point unit circle.
- Find exact values using the unit circle.
- Find arc length and area of a sector.
- Find linear speed and angular speed.
- Define trigonometric ratios in the coordinate plane.
- Define the trigonometric functions in terms of the unit circle.
- Develop basic trigonometric identities.
- Graph the sine, cosine, and tangent functions.
- State all values in the domain of a basic trigonometric function that correspond to a given value of the range.
- Graph transformations of the sine, cosine and tangent graphs.

- State the period and amplitude (if any) given the graph of a sine, cosine, or tangent function.
- Use the period and amplitude (if any) to sketch the graph of a sine, cosine, or tangent function.
- State the period, amplitude, vertical shift, and phase shift given the function rule or graph of a sine or cosine function.
- Use graphs to determine whether an equation could possibly be an identity.

Prerequisite Skills

- Students should have a solid understanding of basic geometric concepts, including properties of triangles, angles, and the Pythagorean theorem.
- Students should be familiar with different types of angles (acute, obtuse, right) and how to measure angles in degrees.
- Students need to know how to plot points and interpret coordinates in the Cartesian coordinate system.
- A foundational understanding of functions, including domain, range, and function notation, is essential.
- Proficiency in algebraic manipulation, including solving equations and working with inequalities, is necessary for solving trigonometric equations and identities.
- Students should grasp the concept of ratios and proportions, as these are fundamental to understanding trigonometric ratios.
- Familiarity with graphing linear functions will help students understand how to graph trigonometric functions and transformations.
- An understanding of square roots and basic radical expressions will assist in working with trigonometric ratios and solving problems.
- A basic introduction to circular functions may be beneficial, particularly in understanding the connection between angles and their corresponding points on the unit circle.
- Students should know how to use scientific or graphing calculators effectively, including how to switch between degree and radian mode.

Common Misconceptions

- Students often struggle with the difference between radians and degrees, leading to errors when converting between the two or misinterpreting angle measures.
- Some students may confuse sine, cosine, and tangent ratios, often mixing up which sides of the triangle correspond to each ratio (opposite, adjacent, or hypotenuse).
- Students might mistakenly apply trigonometric ratios without recognizing that these ratios specifically apply to right triangles, leading to incorrect conclusions in non-right triangles.

- Some students may not grasp the importance of the unit circle, which can hinder their understanding of how trigonometric functions behave and how to find exact values.
- Students may not fully understand that trigonometric functions are periodic, leading to confusion about their behavior and how to find solutions over different intervals.
- When learning about transformations of trigonometric functions, students may struggle to accurately apply shifts, stretches, and reflections, often resulting in incorrect graphs.
- Students might misunderstand the concept of reference angles, leading to errors when calculating trigonometric values for angles in different quadrants.
- Some students may not consider the domain and range of trigonometric functions, which can lead to misunderstandings about their possible values and behavior.
- Students may struggle to differentiate between linear speed and angular speed, leading to errors in problems involving circular motion.
- Some students may find it challenging to grasp the concept of trigonometric identities, often believing that they can manipulate them in the same way as standard algebraic expressions.
- Students may assume that sine, cosine, and tangent are always positive without recognizing that their signs change based on the quadrant in which the angle lies.
- When solving problems involving trigonometric ratios, students may misapply the Pythagorean theorem, especially when dealing with non-right triangles.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity

• Right-Triangle Trigonometry
• Evaluating trigonometric ratios
• Evaluating trigonometric ratios of special angles
• Solving right triangles
  ○ Missing side
  ○ Missing angle
	• Revisit basic geometric properties, particularly those related to triangles, such as the Pythagorean theorem and the properties of right triangles. This foundation will help students understand the derivation of trigonometric ratios.
	• Draw on prior knowledge of algebraic manipulation, including solving equations
		• iXL Diagnostic Assessment
		• iXL Problems
		• Delta Math

• Using angles of elevation and depression to solve triangles.
• Using indirect measurement
• Coterminal angles
• Arc length
• Radian measure
• Radian measure of special angles
• Introduce the 16-point Unit circle
• Convert between radians and degrees
• Find linear and angular speed
• Define trigonometric ratios on the coordinate plane.
• Trigonometric functions and the Unit Circle
• Domain and range of sine and cosine functions
• Exact values of trigonometric functions
• Reference angles
• Basic trig identities
  ○ Quotient
  ○ Reciprocal
  ○ Pythagorean
  ○ Periodicity
  ○ Negative angle
  ○ Identities involving
	and working with inequalities. This will be critical when students tackle trigonometric equations and identities.
	• Reinforce students' familiarity with the coordinate system, including plotting points and understanding the relationship between coordinates and angles. This will aid in comprehending the unit circle and graphing trigonometric functions.
	• Recall the concept of functions, including domain and range, from earlier units. Emphasizing the characteristics of trigonometric functions will help students understand their behavior and applications.
	• Utilize prior experiences with graphing linear equations to help students visualize and analyze the graphs of sine, cosine, and tangent functions, as well as their transformations.
	• Revisit the concept of ratios and proportions from previous math units, as this understanding is essential for grasping trigonometric ratios and their applications in various contexts.	

pi - t • Graphs of sine and cosine functions • Graphs of tangent functions • Period of a function • Amplitude of a function • Phase Shifts of a function • Combined transformations		

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	Part 1 Assessment - Right Triangle Trigonometry Part 2 Assessment - The Unit Circle Benchmark 3 (Linkit) AP Benchmark 3 (Linkit)

Resources

Key Resources	Supplemental Resources
Pre-Calculus: A Graphing Approach, Holt, Rinehart and Winston 2007, Chapters 6 and 7	iXL Delta Math Desmos Activity Builder Desmos Graphing Calculator Explorations

	Khan Academy Teacher made Worksheets APSI Resources for AP Precalculus

Career Readiness, Life Literacies, and Key Skills

9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
9.4.12.IML.1	Compare search browsers and recognize features that allow for filtering of information.
9.4.12.IML.3	Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

Interdisciplinary Connections

RL.CR.11–12.1	Accurately cite strong and thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what a literary text says explicitly and inferentially, as well as interpretations of the text; this may include determining where the text leaves matters uncertain.
W.AW.11–12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9-12.HS-PS3-2	Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as a combination of energy associated with the motion of particles (objects) and energy associated with the relative position of particles (objects).
9-12.HS-PS3-5.2.1	Cause and effect relationships can be suggested and predicted for complex natural and human designed systems by examining what is known about smaller scale mechanisms within the system.