Rational, Exponential and Logarithmic Functions

Content Area: **Math**
Course(s):
Time Period: **MP2**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
In this unit, students will engage with the concept of functions, exploring various types such as rational, exponential, and logarithmic functions through graphical and algebraic methods. Key areas of focus include understanding domain and range, identifying asymptotes and holes, and manipulating exponents and radicals. Students will learn to apply transformations to function graphs and will investigate real-world applications by using mathematical modeling to analyze growth, decay, and data trends. By the end of the unit, students will have developed a strong foundation in functions and their applications, equipping them for further mathematical studies.	This unit is designed to provide students with a comprehensive understanding of functions, which are fundamental to advanced mathematics and various real-world applications. By exploring different types of functions, students will develop critical thinking and problem-solving skills essential for analyzing complex relationships and making informed decisions based on mathematical data. The focus on transformations, exponents, and logarithmic functions allows students to connect abstract concepts to practical scenarios, fostering a deeper appreciation for mathematics in everyday life. Additionally, this unit prepares students for future coursework in calculus and other advanced mathematical disciplines, ensuring they have the necessary skills to succeed in their academic and professional pursuits.

NJSLS

F.BF.B.3	Identify the effect on the graph of replacing □(□) by □(□) + □, □ □(□), □(□□), and □(□ + □) for specific values of □(both positive and negative); find the value of □ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
A.APR.D.6	Rewrite simple rational expressions in different forms; write □(□)/□(□) in the form □(□) + □(□)/□(□), where □(□), □(□), □(□), and □(□) are polynomials with the degree of □(□) less than the degree of □(□), using inspection, long division, or, for the more complicated examples, a computer algebra system.
A.APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational

	expression; add, subtract, multiply, and divide rational expressions.
F.BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.C.7.d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
F.IF.C.8.b	Use the properties of exponents to interpret expressions for exponential functions.
A.SSE.A.1.a	Interpret parts of an expression, such as terms, factors, and coefficients.
A.SSE.A.1.b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
F.LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to □□^{ct} = □ where □, □, and □ are numbers and the base □ is 2, 10, or □; evaluate the logarithm using technology.
A.SSE.B.3.c	Use the properties of exponents to transform expressions for exponential functions.

Standards for Mathematical Practice

1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure
8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• Rational functions involve ratios of polynomials, and analyzing their properties, including intercepts and asymptotes, is important for understanding their behavior and applications. • Functions can be represented in various forms (algebraic, graphical, tabular), with each form providing different insights into the function's behavior.	• How do different representations of a function (algebraic, graphical, tabular) enhance our understanding of its behavior? • What strategies can be used to determine the domain and range of rational functions, and why are these concepts important? • How do vertical and horizontal asymptotes influence the overall shape and behavior of a

- Determining the domain and range of rational functions allows for the prediction of possible values of the function.
- Vertical and horizontal asymptotes in rational functions indicate the limits of function values as inputs approach certain values.
- Holes in the graph of a function reflect values that are excluded from the domain, emphasizing the importance of factoring and simplifying rational expressions.
- Analyzing the end behavior of functions, including its relation to asymptotes, helps predict function behavior at extreme values.
- The laws of exponents, including rational and irrational exponents, facilitate the simplification of expressions.
- Simplifying expressions containing radicals and rational exponents highlights the connection between different forms of numbers.
- Transformations affect the graphs of exponential functions, enabling predictions about how changes in parameters alter the graph's shape and position.
- Exponential functions can model real-world situations, including growth and decay scenarios, reinforcing the relevance of mathematics in everyday life.
- The properties of common and natural logarithms enhance the ability to evaluate logarithmic expressions and solve logarithmic equations.
- The relationship between exponential and logarithmic equations allows for solving problems using both forms interchangeably.
- Creating and using exponential and logarithmic models helps solve practical problems, such as compound interest and radioactive decay.
- Modeling real data sets using power, exponential, logarithmic, and logistic

rational function's graph?

- What is the significance of holes in the graph of a function, and how do they affect the function's domain?
- How can analyzing the end behavior of a function assist in making predictions about its values at extreme inputs?
- How do the laws of exponents simplify calculations, and why are they fundamental in algebraic expressions?
- In what ways do radicals and rational exponents provide different perspectives on representing numbers?
- How do transformations of exponential functions impact their graphs, and what implications do these changes have in real-world contexts?
- How can exponential functions model real-world phenomena, and what are some examples of their applications?
- What properties of logarithms are essential for evaluating logarithmic expressions and solving equations, and how do they apply in various contexts?
- How can understanding the relationship between exponential and logarithmic equations help in solving mathematical problems?
- What are the steps involved in creating effective exponential and logarithmic models for real-world problems?
- How can different types of functions (power, exponential, logarithmic, logistic) be used to analyze and interpret real-world data trends?

functions showcases the versatility of mathematical concepts in analyzing trends and making predictions.	

Instructional Focus

Learning Targets

- Find the domain and range of a rational function.
- Find intercepts, vertical asymptotes, and horizontal asymptotes.
- Identify holes.
- Describe End Behavior.
- Define and apply rational and irrational exponents
- Simplify expressions containing radicals or rational exponents
- Graph and identify transformations of exponential functions.
- Use exponential functions to solve application problems.
- Evaluate common and natural logarithms with and without a calculator.
- Solve common and natural exponential and logarithmic equations by using an equivalent equation.
- Graph and identify transformations of common and natural logarithmic functions.
- Use properties and laws of logarithms to simplify and evaluate expressions.
- Solve exponential and logarithmic equations.
- Create and use exponential/logarithmic models to solve compound interest problems.
- Create and use exponential/logarithmic models to solve continuous compounding problems.
- Create and use exponential/logarithmic models to solve exponential growth and decay problems.
- Create and use exponential/logarithmic models to solve half-life and radioactive decay problems.
- Model real data sets with power, exponential, logarithmic, and logistic functions.

Prerequisite Skills

- Proficiency in simplifying algebraic expressions and solving linear equations.
- Familiarity with the concept of functions, including function notation and basic properties.
- Ability to plot points on a coordinate plane and understand the graphical representation of linear functions.
- Understanding the Cartesian coordinate system and the ability to identify key features of graphs, such as intercepts and slopes.
- Skill in factoring polynomials and recognizing common factors, which is essential for simplifying rational expressions.
- Ability to work with rational expressions, including adding, subtracting, multiplying, and dividing them.
- Understanding the laws of exponents and the ability to simplify expressions involving exponents and radicals.
- Proficiency in solving quadratic equations using various methods, including factoring, completing the square, and the quadratic formula.
- Familiarity with basic trigonometric functions and their properties, as they may be applicable in certain precalculus contexts.
- Ability to solve and graph linear and polynomial inequalities and understand their implications.
- Basic understanding of how to perform transformations on functions, such as translations and reflections.
- Familiarity with the basic characteristics of exponential functions and their graphs.
- Ability to interpret and analyze data sets, recognizing patterns and trends.

Common Misconceptions

- Misunderstandings about the concept of asymptotes can lead students to incorrectly identify the behavior of rational functions at these critical points.
- Students may confuse a function with a non-function; for example, not recognizing that a vertical line intersects a graph more than once, indicating it is not a function.
- Students often overlook restrictions on the domain, such as excluding values that make a denominator zero, leading to incorrect conclusions about the range.
- Some students may believe that asymptotes are part of the graph rather than lines that the graph approaches but never touches.
- Students may not recognize that holes indicate removable discontinuities and may assume that the function is defined at those points.
- Students might struggle to predict end behavior based solely on leading coefficients and degrees,

leading to incorrect assumptions about how functions behave at extreme values.

- Students often misunderstand the difference between exponential growth and linear growth, mistakenly thinking they will produce similar results over time.
- Some students may misapply the properties of logarithms or confuse logarithmic functions with exponential functions, leading to errors in solving equations.
- Students may have difficulty visualizing how transformations affect the graph of a function, leading to mistakes in predicting the new position or shape of the graph.
- Some students may struggle with the relationship between radicals and rational exponents, often misapplying the rules for simplification.
- Students may not recognize the inverse relationship between exponential and logarithmic functions, leading to confusion when switching between the two.
- Students may have difficulty interpreting data trends when using functions, often overlooking the significance of the model chosen for analysis.
- Students may struggle with the idea that the variable in a function can represent different quantities, leading to confusion when analyzing function behavior.
- Some students may overgeneralize certain properties of functions, such as assuming all exponential functions grow at the same rate or that all quadratic functions have the same shape.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Identify asymptotes • Find end behavior using limit notation • Graph rational functions • Factor and simplify rational expressions • Find removable and non-removable points of discontinuity. • Analyze rational functions	• Students will revisit their knowledge of simplifying expressions and solving equations, which is crucial for handling rational expressions. • Prior experiences with graphing equations will be instrumental as students learn to graph exponentials, and logarithms. • Understanding the	• iXL Diagnostic Assessment • iXL Problems • Delta Math

• Describe end behavior at horizontal and vertical asymptotes. • Solve equations involving fractions • Rational exponents • Rationalizing numerators and denominators • Graphs of exponential functions • Exponential growth and decay • The number e • Compound interest • Continuous compounding • Constructing exponential and logarithmic decay functions. • Radioactivity decay • Common Logarithms • Natural Logarithms • Graphing logarithmic functions • Properties of logarithms • Exponential and Logarithmic equations • Exponential, Logistic, Power and Logarithmic models	Cartesian coordinate system and properties of lines will provide a foundation for analyzing the graphs of various functions and their transformations. • Skills in interpreting data and recognizing trends will enhance students' ability to model real-world situations using appropriate functions, connecting mathematical theory to practical applications.	

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	Assessment Part 1 - Rational Functions Assessment Part 2 - Exponential Functions Benchmark 2 (Linkit) AP Benchmark 2 (Linkit)

Resources

Key Resources	Supplemental Resources
Pre-Calculus: A Graphing Approach, Holt, Rinehart and Winston 2007, Chapters 4 and 5	iXL Delta Math Desmos Activity Builder Desmos Graphing Calculator Explorations Khan Academy Teacher made Worksheets APSI Resources for AP Precalculus

Career Readiness, Life Literacies, and Key Skills

9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.1	Compare search browsers and recognize features that allow for filtering of information.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

Interdisciplinary Connections

RL.CR.11–12.1	Accurately cite strong and thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what a literary text says explicitly and inferentially, as well as interpretations of the text; this may include determining where the text leaves matters uncertain.
W.AW.11–12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9-12.HS-LS2	Ecosystems: Interactions, Energy, and Dynamics