Quadratic, Polynomial, Rational Functions

Content Area: **Math**
Course(s):
Time Period: **MP1**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
In this unit, students will focus on the essential concepts of functions and their characteristics. They will learn to identify functions, find their domains, and analyze graphs to determine key features such as domain, range, local maxima and minima, and intervals of increase or decrease. The unit will cover quadratic functions, including their three forms, vertices, intercepts, and conversions between forms. Students will also explore transformations of parent functions, as well as forming sum, difference, product, quotient, and composite functions while identifying their domains. Additionally, the unit introduces inverse relations and functions, allowing students to find and verify inverses. The study of polynomials will involve defining, dividing, and applying the Remainder Theorem, along with identifying rational zeros and using the Factor Theorem. Finally, students will examine the properties of polynomial functions, including continuity, end behavior, and local extrema, as well as analyze rational functions for intercepts and asymptotes. This unit provides a solid foundation for understanding functions and their behaviors.	The rationale for this unit lies in its critical role in establishing a strong mathematical foundation for students as they prepare for more advanced concepts in mathematics, science, and engineering. Understanding functions and their properties is essential, as they serve as the building blocks for various mathematical models and real-world applications. By exploring the characteristics of different types of functions, including quadratic and polynomial functions, students develop analytical skills that enable them to interpret and manipulate mathematical relationships effectively. The focus on graph analysis fosters a deeper comprehension of how changes in equations affect their graphical representations, enhancing critical thinking and problem-solving abilities. Additionally, the introduction of inverse functions and transformations promotes flexibility in thinking, allowing students to approach problems from multiple perspectives. Overall, this unit is designed to engage students and equip them with the necessary tools to navigate more complex mathematical topics, ensuring their success in future mathematical endeavors.

NJSLS

A.APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are

	closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$.
F.BF.A.1.b	Combine standard function types using arithmetic operations.
A.APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
F.BF.A.1.c	Compose functions.
F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
F.BF.B.4.b	Verify by composition that one function is the inverse of another.
A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
F.IF.C.8.a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Standards for Mathematical Practice

1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision

7	Look for and make use of structure
8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• A function is a specific type of relation that assigns exactly one output for each input, which is fundamental for analyzing mathematical relationships. • The domain defines the set of possible inputs for a function, while the range represents the outputs, both of which are crucial for understanding function behavior. • The graphical representation of a function provides valuable insights into its properties, including key features such as intercepts, maxima, minima, and intervals of increase or decrease. • Quadratic functions can be expressed in multiple forms, and understanding these forms facilitates easier analysis, conversion, and graphing of their characteristics. • Functions can be combined through operations such as addition, subtraction, multiplication, division, and composition, which expands the toolkit for solving complex mathematical problems. • Inverse relations effectively undo the effects of functions, and understanding how to find and verify inverses is essential for exploring relationships in various contexts. • Polynomials exhibit specific behaviors and characteristics, including continuity and end behavior, which can be systematically analyzed to understand their graphs and zeros. • Understanding how transformations affect the graphs of parent functions is essential for predicting changes in shape, position, and orientation. • The interplay between algebraic expressions,	• What makes a relation a function, and how can we determine if a relation is a function? • How do we identify the domain and range of different functions, and why are they important? • How can the graph of a function help us understand its key features, such as intercepts and maxima or minima? • What are the different forms of quadratic functions, and how do we convert between them to analyze their characteristics effectively? • How can we combine functions through addition, subtraction, multiplication, division, and composition, and what do these operations reveal about their relationships? • How do we find the inverse of a function, and why is it important to understand inverse relationships? • What unique features do polynomials have, and how can we use these features to analyze their graphs and zeros? • How do transformations such as shifts, reflections, and stretches change the graphs of parent functions? • How do algebraic expressions, graphs, and real-world situations connect to enhance our understanding of mathematics? • How do piecewise functions model real-world scenarios, and what are the advantages of using them? • Why is it important to identify local maxima and minima in functions, and how can we

graphical representations, and real-world applications illustrates the interconnectedness of mathematical concepts. • Piecewise-defined functions allow for modeling situations that require different rules for different parts of their domains, enhancing flexibility in mathematical modeling. • Recognizing local maxima and minima in functions is important for optimization and real-world applications, such as profit maximization or resource allocation. • Understanding the concepts of continuity and points of discontinuity in functions is vital for analyzing their behavior and predicting changes. • The end behavior of functions provides insights into how they behave as inputs approach extreme values, which is crucial for graphing and understanding limits. • Analyzing and interpreting functions graphically and algebraically fosters critical thinking and problem-solving skills that are applicable across various disciplines.	apply this knowledge in real-life situations? • What is the significance of continuity in functions, and how do points of discontinuity affect their behavior? • How can we determine the end behavior of a function, and why is this information useful in graphing and analysis? • How can analyzing and interpreting functions both graphically and algebraically improve our problem-solving skills in mathematics and beyond?

Instructional Focus

Learning Targets
• Determine whether a relation is a function. • Find the domain of functions. • Evaluate piecewise-defined and greatest integer functions. • Determine whether a graph represents a function. • Analyze graphs to determine domain and range, local maxima and minima, inflection points and intervals where they are increasing, decreasing, concave up, and concave down.

- Define three forms for quadratic functions.
- Find the vertex and intercepts of a quadratic function and sketch its graph.
- Convert one form of a quadratic function to another.
- Transform graphs of parent functions.
- Form sum, difference, product, and quotient functions and find their domains.
- Form composite functions and find their domains.
- Define inverse relations and functions.
- Find inverse relations from tables, graphs, and equations.
- Determine whether an inverse relation is a function.
- Verify inverses using composition.
- (+) Find the average rate of change of a function over an interval.
- Define a polynomial.
- Divide polynomials.
- Apply the Remainder Theorem, and the connections between remainders and factors.
- Determine the maximum number of zeros of a polynomial.
- Find all rational zeros of a polynomial function.
- Use the Factor Theorem.
- Factor a polynomial completely.
- Find lower and upper bounds of zeros.
- Recognize the shape of basic polynomial functions.
- Describe the graph of a polynomial function.
- Identify properties of general polynomial functions: Continuity, End Behavior, Intercepts, Local Extrema, Points of Inflection.
- Identify complete graphs of polynomial functions.
- Sketch complete graphs of polynomial functions.

Prerequisite Skills

- Proficiency in performing operations with integers, fractions, and decimals, including addition,

subtraction, multiplication, and division.

- Ability to solve one-variable linear equations and inequalities, and to understand the concepts of equality and balance.
- Familiarity with graphing linear equations on the coordinate plane and interpreting slope and intercepts.
- Knowledge of the Cartesian coordinate system, including how to identify and plot points using ordered pairs.
- Basic skills in factoring polynomials, including recognizing common factors and using methods like grouping and the difference of squares.
- Ability to substitute values into algebraic expressions and simplify them accordingly.
- Proficient use of the order of operations (PEMDAS) to simplify mathematical expressions correctly.
- Familiarity with the properties of exponents and the ability to simplify expressions involving exponents and radicals.
- Skills in translating real-world situations into mathematical expressions and equations.
- An initial understanding of the concept of a function, including input-output relationships and function notation.
- Ability to evaluate and interpret absolute value expressions and inequalities.
- Basic experience in graphing quadratic functions and understanding their general shape (parabola).
- Skills in recognizing and analyzing patterns in numerical and algebraic contexts.
- Familiarity with graphing calculators or graphing software to assist in visualizing functions and their transformations.
- Understanding of basic statistics concepts, such as mean, median, and mode, can be helpful in analyzing data sets related to functions.

Common Misconceptions

- Students may believe that all relations are functions, not recognizing that a function must assign exactly one output for each input.
- Many students struggle to accurately determine the domain and range of a function, often overlooking restrictions such as division by zero or square roots of negative numbers.
- Students might misinterpret the graphical features of functions, confusing maxima and minima or failing to identify key points such as intercepts.
- Some students may not understand the significance of different forms of quadratic functions (standard, vertex, and factored forms) and how to convert between them.

- Students may inaccurately apply operations to functions, such as misunderstanding how to combine functions algebraically or misapplying function composition.
- Many students struggle with the concept of inverse functions, believing that they are simply the opposite of a function without understanding the process of finding them.
- Students may not fully grasp the end behavior of polynomials, leading to incorrect assumptions about how the graph behaves as values approach infinity.
- Students often struggle to visualize how transformations (shifts, reflections, stretches) affect the graphs of parent functions, leading to confusion in sketching graphs.
- Some students may find it difficult to work with piecewise functions, often misunderstanding how to evaluate them at specific points within their domains.
- Students may mistakenly believe that all functions are continuous, failing to recognize the significance of points of discontinuity.
- Students might overlook the importance of critical points in determining the behavior of functions, such as local maxima and minima, which can affect their understanding of optimization.
- Some students may forget to apply the correct order of operations when evaluating functions, leading to incorrect results.
- When graphing functions, students may misplace points or fail to accurately reflect the characteristics of the function based on its equation.
- Students may struggle to connect algebraic representations of functions with their graphical interpretations, leading to a fragmented understanding of the material.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Domain and range • Determine whether a relation is a function • Vertical Line test • Intervals of increase, decrease or constant. • Locate relative extrema	• Reinforcement of operations with integers, fractions, and rational expressions, ensuring fluency in manipulating algebraic expressions. • Returning to the skills of solving one-variable linear equations, which are foundational for	• iXL Diagnostic Assessment • iXL Problems • Delta Math

• Continuity of functions • Identify asymptotes • Find end behavior using limit notation • Analyze properties of functions • Graph functions using horizontal and vertical shifts • Graph functions using compressions and stretches • Graph functions using reflections about the x-axis or y-axis • Perform operations on functions • Find a composition of functions • Verify functions are inverses of each other • Recognize and graph quadratic functions. • Describe properties of quadratic functions. • Interchange different forms of quadratic functions. • Basic characteristics of a polynomial • Find zeros, multiplicity of zeros, extrema, and end behavior • Estimate points of inflection. • Graph rational functions • Factor and simplify	understanding more complex functions. • Revisiting the concepts of graphing linear equations, including slope and y-intercept, to lay the groundwork for more advanced graphing tasks. • Reinforcing knowledge of the Cartesian coordinate system and the ability to plot points accurately. • Building on previous experiences with factoring polynomials, including recognizing common factors and applying different factoring methods. • Refreshing skills in substituting values into algebraic expressions and simplifying them, which is essential for function evaluation. • Reinforcing the use of the order of operations (PEMDAS) in evaluating mathematical expressions. • Revisiting properties of exponents and the simplification of radical expressions, which are crucial for understanding polynomial and rational functions. • Reintroducing the concept of functions, including input-output relationships, function notation, and the distinction between different types of functions. • Revisiting the analysis of	

rational expressions • Find removable and non-removable points of discontinuity. • Analyze rational functions • Describe end behavior at horizontal and vertical asymptotes. • Solve equations involving fractions	numerical patterns and sequences, which can relate to function behavior and transformations. • Building on prior knowledge of quadratic functions to deepen understanding of their properties and graphing techniques. • Utilizing skills from coordinate geometry, such as distance and midpoint formulas, to analyze geometric relationships within functions. • Applying basic statistics concepts, such as mean and median, to understand data representation through functions. • Reinforcing problem-solving techniques used in previous courses to tackle complex mathematical situations. • Encouraging the use of technology tools, such as graphing calculators, that students have previously used in other math courses. • Drawing connections from previous units where students modeled real-world situations with mathematical expressions and functions. • Encouraging students to articulate their mathematical reasoning and solutions, a skill developed in earlier courses. • Revisiting the concept of	

	relationships between variables, which is foundational for exploring functions in depth. • Enhancing visualization skills from geometry and earlier algebra courses to better understand the graphical representation of functions. • Continuing to develop critical thinking skills applied to mathematical problems, building on their experiences from previous units and courses.	

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	Part 1 Assessment - Quadratics Part 2 Assessment - Polynomials Benchmark 1 (Linkit)

Resources

Key Resources	Supplemental Resources
Pre-Calculus: A Graphing Approach, Holt, Rinehart and Winston 2007, Chapters 3 and 4	iXL Delta Math Desmos Activity Builder

	Desmos Graphing Calculator Explorations Khan Academy Teacher made Worksheets APSI Resources for AP Precalculus

Career Readiness, Life Literacies, and Key Skills

9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.IML.1	Compare search browsers and recognize features that allow for filtering of information.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

Interdisciplinary Connections

DA	Data & Analysis
AP	Algorithms & Programming
9-12.HS-LS2	Ecosystems: Interactions, Energy, and Dynamics
9-12.HS-PS2	Motion and Stability: Forces and Interactions