Circles and Polygons

Content Area: **Math**
Course(s):
Time Period: **MP4**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
This unit begins by focusing on the interior and exterior angles of polygons. Students will then move into quadrilaterals, examining properties of kites and trapezoids, and then the properties and condition of parallelograms and special parallelograms. The unit will continue with an examination of arc length, sector area, segment area, and an introduction to radians as a unit of angle measure. Students will then examine properties of tangents, chords, and inscribed angles. Finally, students learn about the properties of angles, arcs, and segment lengths that are formed when two lines intersect inside or outside of a circle.	This unit begins with the study of interior and exterior angles of polygons, fundamental for understanding geometric properties and solving basic problems. Mastery of these concepts is crucial for advancing to topics like congruence and similarity, providing a strong foundation in geometry. Next, the unit explores specific quadrilaterals such as kites, trapezoids, and parallelograms. Understanding their unique properties is essential for distinguishing different shapes and applying this knowledge in fields like engineering and architecture. Additionally, students learn about arc length, sector area, segment area, and radians, bridging geometry and trigonometry and preparing them for advanced mathematics. The final section covers tangents, chords, inscribed angles, and relationships formed by intersecting lines in circles. This comprehensive study enhances spatial reasoning and problem-solving skills, crucial for scientific and engineering applications. By integrating these concepts, the unit ensures a robust understanding of geometry, beneficial for both academic and real-world contexts.

NJSLS

MATH.9-12.G.C.A.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
MATH.9-12.G.C.A.4	Construct a tangent line from a point outside a given circle to the circle.
MATH.9-12.G.C.B.5	Derive using similarity the fact that the length of the arc intercepted by an angle is

	proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
MATH.9-12.G.CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MATH.9-12.G.CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
MATH.9-12.G.CO.C.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
MATH.9-12.G.SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• The sum of the exterior angles of a polygon is 360°, regardless of the number of sides. The sum of the interior angles of a polygon is 180° (n - 2), where n is the number of sides. • Diagonals of a kite are perpendicular, and one diagonal bisects the other. In isosceles trapezoids, diagonals are congruent. The length of the midsegment of a trapezoid is half the sum of the base lengths. • In a parallelogram, consecutive angles are supplementary, opposite angles are congruent, opposite sides are congruent, and diagonals bisect each other. • A quadrilateral with two pairs of congruent	• How does the number of sides in convex polygons relate to the sums of the measure of the exterior and interior angles? • How are the diagonals and angle measure related in kites and trapezoids? • What are the relationships of the sides, the angles, and the diagonals of a parallelogram? • Which properties determine whether a quadrilateral is a parallelogram? • What properties of rhombuses, rectangles, and squares differentiate them from other parallelograms?

opposite sides, or one pair of congruent parallel sides, or diagonals bisecting each other is a parallelogram. A quadrilateral with an angle supplementary to both of its consecutive angles, or two pairs of opposite congruent angles is a parallelogram. • The diagonals of a rhombus are perpendicular, bisect each other, and bisect opposite angles. They form four congruent triangles. In a rectangle, the diagonals are congruent. Squares have properties of rhombuses and rectangles. • A parallelogram with perpendicular diagonals or diagonals that bisect angles is a rhombus. A parallelogram with congruent diagonals is a rectangle. A parallelogram with perpendicular congruent diagonals or with congruent diagonals and a diagonal that bisects angles is a square. • Arcs are classified as minor arcs or major arcs depending on whether they are smaller or larger than a semicircle. The length of an arc is a portion of the circumference proportional to the corresponding central angle. The area of a sector of a circle is a portion of the area of the circle proportional to the central angle. The area of a segment of a circle is the area of the corresponding sector minus the area of the corresponding triangle. • A line that is tangent to a circle intersects the circle at exactly one point and is perpendicular to the radius to that point. If two segments are tangent to the same circle and have a common endpoint exterior to the circle, the segments are congruent. • In a circle or congruent circles, two chords are congruent if the corresponding central angles are congruent or if the chords intercept congruent arcs. The perpendicular bisector of a chord is a diameter of the circle. • In a circle, the measure of an inscribed angle is one-half of the measure of its intercepted arc. In a circle, the measure of an angle formed by a chord and a tangent to the circle is one-half of the measure of its intercepted	• Which properties of the diagonals of a parallelogram help you classify a parallelogram? • How are arc length and sector area related to circumference and area of a circle? • How is a tangent line related to the radius of a circle at the point of tangency? • How are chords related to their central angles and intercepted arcs? • How is the measure of an inscribed angle related to its intercepted arc? • How are the measures of angles, arcs, and segments formed by intersecting secant lines related?

arc. • When two secants intersect inside a circle, the measure of the angle formed is half the sum of the intercepted arcs. When secants or tangents intersect outside a circle, the measure of the angle formed is half the difference of the intercepted arcs. When secants or tangents intersect inside or outside a circle, the product of the distances from the point of intersection to the points on the circle is the same for both lines.	

Instructional Focus

Learning Targets

- Show that the sum of the exterior angles of a polygon is 360° and use that to solve problems.
- Show that the sum of the interior angles of a polygon is the product of 180°and two less than the number of sides, and use that to solve problems.
- Use properties of the diagonals of a kite to solve problems.
- Use properties of isosceles trapezoids to solve problems.
- Use the relationship between the lengths of the bases and the midsegment of a trapezoid to solve problems.
- Show that the consecutive angles of a parallelogram are supplementary and opposite angles are congruent.
- Show that opposite sides of a parallelogram are congruent.
- Show that diagonals of a parallelogram bisect each other.
- Demonstrate that a quadrilateral is a parallelogram based on its sides and diagonals.
- Demonstrate that a quadrilateral is a parallelogram based on its angles.
- Prove that the diagonals of rhombuses are perpendicular bisectors of each other and angle bisectors of the angles of the rhombus.
- Prove that the diagonals of a rectangle are congruent.

- Use properties of rhombuses, rectangles, and squares to solve problems.
- Identify rhombuses, rectangles, and squares by the characteristics of diagonals of parallelograms.
- Calculate the length of an arc when the central angle is given in degrees or radians.
- Calculate the area of sectors and segments of circles.
- Identify lines that are tangent to a circle using angle measures and segment lengths.
- Solve problems involving tangent lines.
- Prove and apply relationships between chords, arcs, and central angles.
- Find lengths of chords given the distance from the center of the circle and use this information to solve problems.
- Identify and apply relationships between the measures of inscribed angles, arcs, and central angles.
- Identify and apply the relationships between an angle formed by a chord and a tangent to its intercepted arc.
- Recognize and apply angle relationships formed by secants and tangents intersecting inside and outside a circle.
- Recognize and apply segment length relationships formed by secants and tangents intersecting inside and outside a circle.

Prerequisite Skills

- Types of angles
- Triangle properties
- Triangle angle sum
- Complementary and supplementary angles
- Adjacent and vertical angles
- Parallel lines and transversals
- Properties of perpendicular lines
- Basic knowledge of quadrilaterals from previous grades
- Basic knowledge of parts of a circle including radius, diameter, circumference and center.
- Understanding the relationship between radius and diameter.
- Understanding pi

Common Misconceptions
• Students might incorrectly think that the sum of the interior angles of any polygon is always the same, rather than understanding it depends on the number of sides. • Students may not realize that the sum of the exterior angles of any polygon is always 360°, regardless of the number of sides. • Students might mix up properties of different quadrilaterals, such as thinking all quadrilaterals have the same angle properties. • Believing that the sum of the interior angles of quadrilaterals can vary. • Students might not realize that opposite sides of a parallelogram are both parallel and equal in length. • Misunderstanding the properties of diagonals in parallelograms, such as thinking they are always equal in length. • Assuming the sum of the angles in a triangle can be more or less than 180°. • Students might incorrectly apply properties of angles formed by a transversal intersecting parallel lines to non-parallel lines. • Misidentifying shapes, such as confusing a rhombus with a square. • Students may think that all chords are diameters. • Students may believe that the longest arc is always 180°. • Students may think that tangle lines always intersect the circle. • Some students may think that the area of a sector is the same as the area of a segment. • Students may believe that radians and degrees are interchangeable without conversion. • Students may think that secant and tangent lines are the same.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Understanding polygon angle sums • Analyzing kites and	• Understanding of basic geometric terms such as points, lines, planes, and angles.	• IXL • Math Diagnostic and Intervention System

trapezoids • Understanding properties of parallelograms • Determining whether a quadrilateral is a parallelogram • Understanding properties of special parallelograms • Determining whether a parallelogram is a special parallelogram. • Extending circumference and area • Understanding tangent lines • Understanding chords • Applying inscribed angles • Analyzing intersecting secants	• Familiarity with postulates and theorems related to parallel and perpendicular lines. • Knowledge of the different types of triangles and their properties. • Triangle inequality theorem and the relationships between angles and sides of triangles. • Congruence and similarity criteria for triangles (SSS, SAS, ASA, AAS, HL). • Complementary, supplementary, vertical, and adjacent angles. • Angle sum properties in polygons, especially triangles (sum of interior angles equals 180°) • Formulas for calculating the perimeter, area, and volume of various geometric figures. • Use of algebra to solve geometric problems, such as setting up and solving equations. • Equations of lines and relationships between slopes of parallel and perpendicular lines. • Understanding of transformations such as translations, rotations, reflections, and dilations. • Use of deductive reasoning to prove geometric statements.	Activities

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests • Unit 4 Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
Savvas Envision Geometry Pacing Guide	iXL Delta Math Desmos Khan Academy Math Medic Teacher Made worksheets

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Interdisciplinary Connections

ELA.RL.CR.9–10.1	Cite a range of thorough textual evidence and make relevant connections to strongly support analysis of multiple aspects of what a literary text says explicitly and inferentially, as well as including determining where the text leaves matters uncertain.
ELA.W.AW.9–10.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient textual and non-textual evidence.
VPA.1.4.12.A.CS3	Artistic styles, trends, movements, and historical responses to various genres of art evolve over time.
9-12.HS-ESS1-4.ESS1.B.1	Kepler's laws describe common features of the motions of orbiting objects, including their elliptical paths around the sun. Orbits may change due to the gravitational effects from, or collisions with, other objects in the solar system.