Transformations, Similarity, and Right Triangles

Content Area: **Math**
Course(s):
Time Period: **MP3**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
This unit begins by focusing on transformations, moving from the definition of rigid motion to the rigid transformations: reflections, translations, and rotations. Students will examine how transformations can be combined to create new imaged and complete proofs, such as demonstrating that a composition of two or more rigid motions is also a rigid motion. Further into the unit, students will examine dilations and similarity transformations. These concepts are then applied to triangles; students examine the criteria for proving two triangles are similar, analyze similarity in right triangles and proportions in triangles. The unit will finish up with applying properties of similar right triangles to understand the Pythagorean Theorem, relationships in special right triangles, and trigonometric ratios. Students will apply what they have learned to various contextual problems using the angles of elevation and depression.	This unit is designed to give students a strong foundation in geometric transformations and similarity. Beginning with rigid motions—reflections, translations, and rotations—students will learn how these transformations preserve the shape and size of figures, helping them understand congruence. Exploring the composition of these motions will enhance their logical reasoning and proof-writing skills. Next, the unit transitions to dilations and similarity transformations, introducing scale factors and the preservation of angle measures. Students will apply these concepts to triangles, learning the criteria for proving similarity and analyzing proportions in triangles. These skills are essential for various applications, including architecture and engineering. The unit concludes with the properties of similar right triangles to understand the Pythagorean Theorem, special right triangles, and trigonometric ratios. By solving real-world problems involving angles of elevation and depression, students will see the practical utility of these concepts. This structured progression aims to develop their mathematical reasoning, proof skills, and problem-solving abilities, preparing them for advanced studies.

NJSLS

MATH.9-12.G.C.A.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

MATH.9-12.G.CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
MATH.9-12.G.CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
MATH.9-12.G.CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
MATH.9-12.G.CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
MATH.9-12.G.CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
MATH.9-12.G.CO.C.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
MATH.9-12.G.GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
MATH.9-12.G.SRT.A.1.a	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
MATH.9-12.G.SRT.A.1.b	The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
MATH.9-12.G.SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
MATH.9-12.G.SRT.A.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
MATH.9-12.G.SRT.B.4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
MATH.9-12.G.SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
MATH.9-12.G.SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
MATH.9-12.G.SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.
MATH.9-12.G.SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics

MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• Reflections are rigid motions across a line of reflection. Students will create an image given a primate and the line of reflection both on a coordinate plane and without the use of a coordinate plane. • A translation is a rigid motion that moves all points of the preimage the same distance in the same direction. A translation is the composition of two reflections. • Rotation is a rigid motion described by its center of rotation and angle of rotation. Any rotation can be described by two reflections whose lines of reflection meet at the center of rotation at half the angle of rotation. • Any composition of a rigid motion can be represented by a combination of at least two of the following: a translation, reflection, rotation, or glide reflection. • A figure that can be mapped onto itself using rigid motions is symmetric. Rotational symmetry uses rotation to map a figure onto itself, and reflection symmetry uses reflection to map a figure onto itself. • A dilation is a transformation that preserves angle measure but not length. The dilation of a figure is determined by the scale factor and center of dilation. Every distance from the center of dilation and every side length in a preimage are multiplied by the scale factor to find the corresponding distance and side length in the image. • A similarity transformation is a dilation combined with one or more rigid motions. In order for two figures to be similar, there must be a similarity transformation that maps one	• How are the properties of reflection used to transform a figure? • What are the properties of a translation? • What are the properties that identify a rotation? • How can rigid motions be classified? • How can you tell whether a figure is symmetric? • How does a dilation affect the side lengths and angle measures of a figure? • What makes a transformation a similarity transformation? • What is the relationship between a preimage and the image resulting from a similarity transformation? • How do similarity transformations determine the angle and side length conditions necessary for triangle similarity? • In a right triangle, what is the relationship between the altitude to the hypotenuse, triangle similarity, and the geometric mean? • When parallel lines intersect two transversals, what are the relationships among the lengths of the segments formed? • How are similarity in right triangles and the Pythagorean Theorem and its converse describe how the side lengths of right triangles are related? • How do trigonometric ratios relate angle

figure to the other. All circles are similar. • Two triangles are similar if a composition of rigid motions and dilation will map one triangle onto the other. Two pairs of congruent angles, or three pairs of sides with lengths that are in the same proportion, or two pairs of sides having congruent included angles with lengths that are in the same proportion, are sufficient to show that two triangles are similar. • The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the two segments into which the altitude divides the hypotenuse. The length of a leg of a right triangle is the geometric mean of the length of the hypotenuse and the segment of the hypotenuse adjacent to the leg. • A segment parallel to one side of a triangle divides the triangle into two similar triangles. If that segment connects the midpoints of two sides, the smaller triangle is in proportion 1 : 2 with the larger triangle. A segment that bisects an angle of a triangle divides the opposite side of the triangle into segments that are proportional to the adjacent sides. • The Pythagorean Theorem can be understood through the relationships between the similar triangles formed by the altitude to the hypotenuse. The length of the hypotenuse of a 45°-45°-90°triangle is 2 times the leg length. The length of the hypotenuse of a 30Â°-60°-90° triangle is twice the length of the shorter leg, and the length of the longer leg is 3 times the length of the shorter leg. • For any two right triangles with a given acute angle, the ratios of any two corresponding side lengths are equal. The ratio of the opposite side to the hypotenuse is the sine ratio, the ratio of the adjacent side to the hypotenuse is the cosine ratio, and the ratio of the opposite side to the adjacent side is the tangent ratio. • The ratios of the corresponding sides of right triangles are constant for right triangles with given base angles and are related to the base	measure of side lengths of right triangles? • How can trigonometry be used to solve real-world and mathematical problems?

angles. These relationships can be used to solve problems where side lengths, angle measures, or areas of triangles are desired.	

Instructional Focus

Learning Targets
- Find a reflected image and write a rule for a reflection.   - Define reflection as a transformation across a line of reflection with given properties and perform reflections on and off a coordinate grid.   - Translate a figure and write a rule of a translation.   - Find the image of a figure after a composition of rigid motions.   - Prove that a translation is a composition of two reflections.   - Rotate a figure and write a rule for a rotation.   - Prove that a rotation can be written as a composition of two reflections.   - Specify a sequence of transformations that will carry a given figure onto another.   - Use geometric descriptions of rigid motions to transform figures.   - Describe the rotations and/or reflections that carry a polygon onto itself.   - Predict the effect of a given rigid motion on a figure.   - Identity types of symmetry in a figure.   - Dilate figures on and off the coordinate plane.   - Understand how distances and lengths in a dilation are related to the scale factor and center of dilation.   - Understand that two figures are similar if there is a similarity transformation that maps one figure to the other.   - Identify a combination of rigid motions and dilation that maps one figure to a similar figure.   - Identify the coordinates of an image under a similarity transformation.

- Use dilations and rigid motions to prove triangles are similar.
- Prove and use the AA ~, SSS ~, and SAS ~ theorems to prove triangles are similar.
- Use similarity of right triangles to solve problems.
- Use length relationships of the sides of right triangles and an altitude drawn to the hypotenuse to solve problems.
- Use the Side-Splitter Theorem and the Triangle Midsegment Theorem to find lengths of sides and segments of triangles.
- Use the Triangle-Angle-Bisector Theorem to find lengths of sides and segments of triangles.
- Prove the Pythagorean Theorem using similar right triangles.
- Understand and apply the relationships between side lengths in 45°-45°-90° and 30°-60°-90° triangles.
- Define and calculate sine, cosine, and tangent ratios.
- Use trigonometric ratios to solve problems.
- Distinguish between angles of elevation and depression.
- Use trigonometric ratios to solve problems.

Prerequisite Skills

- Understanding points, lines and planes
- Types of angles
- Basic shapes and properties
- The coordinate plane
- Measuring angles
- Basic understanding of translations (slide), reflections (flip) and rotations (turn).
- Understanding of ratios, proportions, scale factors
- Pythagorean theorem

Common Misconceptions

- Students might think that translating a shape changes its size or orientation.
- Students often believe that reflecting a shape over a line will result in a rotated shape rather than a

mirrored one.

- Some students think that rotating a shape 90 degrees will result in resizing the shape.
- Students might think that if a shape is symmetrical, any line through the shape is a line of symmetry.
- Some students believe that a shape with rotational symmetry looks the same at any rotation.
- Students may think complex shapes cannot have symmetry.
- Some students believe dilations change the shape of a figure.
- Students might think that the center of dilation is always the origin.
- Students often believe similar figures are always the same size.
- Students may believe that if two figures are similar, their areas are proportional in the same way as their sides.
- Students may think that the geometric mean of two numbers is the same as their arithmetic mean.
- Students sometimes mix up the definitions of sine, cosine, and tangent. For example, they might incorrectly state that sine is opposite/adjacent instead of opposite/hypotenuse.
- Students may struggle to correctly identify the hypotenuse, especially in triangles that are not oriented in a standard way.
- Students might incorrectly set up proportions when solving for missing sides, leading to incorrect answers.
- Students often forget to set their calculators to the correct mode (degrees or radians) when performing trigonometric calculations.
- Incorrectly entering values into the calculator can lead to significant errors, especially with parentheses in complex expressions.
- Students might not understand that inverse trigonometric functions (like sin-1, cos-1, tan-1) are used to find angles, not side lengths.
- Students may incorrectly apply right triangle trigonometry concepts to non-right triangles.
- Difficulty in visualizing what the trigonometric ratios represent in terms of the triangle's sides.
- Confusion about what these angles represent and how to set up problems involving them.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Understanding reflections • Translations • Rotations • Using multiple transformations • Symmetry • Understand dilations • Understanding similarity in terms of similarity transformations • Proving triangles similar • Understanding similarity in right triangles • Applying proportions in triangles • Proving the Pythagorean Theorem and special right triangles. • Understanding trigonometric ratios • Angles of elevation and depression and applying trigonometry	• Identifying and measuring different types of angles (acute, right, obtuse, and straight). • Complementary and supplementary angles, vertical angles, and linear pairs. • Using the coordinate plane to plot points and understand the relationship between algebra and geometry. • Applying the distance formula to find the distance between two points in a coordinate plane. • Understanding and performing translations on the coordinate plane. • Performing reflections across the x-axis, y-axis, and other lines. • Rotating figures around a point, often the origin, by 90°, 180°, and 270°. • Classifying triangles by their sides and angles, understanding the properties of different types of triangles. • Identifying congruent figures and using congruence postulates and theorems (SSS, SAS, ASA, AAS). • Understanding the concept of similarity • Applying the Pythagorean	• IXL • Math Diagnostic and Intervention System Activities

	Theorem to find the lengths of sides in right triangles. • Solving linear equations and inequalities. • Solving proportions and using them to solve problems involving similar figures.	

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Test • Unit 3 Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
Savvas Envision Geometry Pacing Guide	iXL Delta Math Desmos Khan Academy Math Medic Teacher Made worksheets

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Interdisciplinary Connections

ELA.RL.CR.9–10.1	Cite a range of thorough textual evidence and make relevant connections to strongly support analysis of multiple aspects of what a literary text says explicitly and inferentially, as well as including determining where the text leaves matters uncertain.
ELA.W.AW.9–10.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient textual and non-textual evidence.
9-12.HS-LS2-1.3.1	students understand the significance of a phenomenon is dependent on the scale, proportion, and quantity at which it occurs. They recognize patterns observable at one scale may not be observable or exist at other scales, and some systems can only be studied indirectly as they are too small, too large, too fast, or too slow to observe directly. Students use orders of magnitude to understand how a model at one scale relates to a model at another scale. They use algebraic thinking to examine scientific data and predict the effect of a change in one variable on another (e.g., linear growth vs. exponential growth).
	Complex programs are designed as systems of interacting modules, each with a specific role, coordinating for a common overall purpose. Modules allow for better management of complex tasks.