Triangles

Content Area: **Math**
Course(s):
Time Period: **MP2**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
This unit begins by focusing on the properties of parallel lines and the angle relationships formed when parallel lines are cut by a transversal. Students will then examine how these angle relationships can help prove whether or not lines are parallel, the relationships between parallel lines and triangle angles, and the relationships between the slopes of parallel and perpendicular lines. Further into the unit students will focus on congruence and transformations resulting in congruent figures. This will include various triangles and define congruence theorems that prove triangles are congruent given congruent angles and sides of triangles. Finally, the unit ends with focusing on concurrent points found in a triangle using perpendicular bisectors, angle bisectors, medians, and altitudes. Students will also examine the relationships of the angle measures and side lengths within a triangle, and the angle measure and side lengths of two triangles.	Understanding the properties of parallel lines and the resulting angle relationships when intersected by a transversal is a fundamental concept in geometry. This knowledge serves as the foundation for more complex geometric concepts such as proving line parallelism and understanding the properties of polygons. By mastering these relationships, students will be equipped with essential tools for solving a wide range of geometric problems and proofs. Congruence is a core concept in geometry that allows students to understand when two figures are identical in shape and size. Understanding and applying congruence theorems is crucial for solving geometric proofs and real-world problems. By focusing on the specific criteria that make triangles congruent, students gain a deeper comprehension of geometric relationships and are better prepared to apply these principles to more complex scenarios. Examining the properties of triangles provides students with insights into the fundamental nature of geometric shapes. The study of concurrent points like the centroid, incenter, circumcenter, and orthocenter deepens students' understanding of triangle properties and their applications. Additionally, exploring the relationships between angles and sides within and between triangles prepares students for more advanced topics in geometry and trigonometry. This phase ensures that students can analyze and solve complex geometric problems, both in theoretical and applied contexts.

NJSLS

MATH.9-12.G.C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
MATH.9-12.G.CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MATH.9-12.G.CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
MATH.9-12.G.CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
MATH.9-12.G.CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
MATH.9-12.G.CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
MATH.9-12.G.CO.C.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
MATH.9-12.G.CO.C.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
MATH.9-12.G.CO.C.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
MATH.9-12.G.GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
MATH.9-12.G.MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MATH.9-12.G.MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MATH.9-12.G.SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision

MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• When parallel lines are cut by a transversal, the special angle pairs that are formed are congruent, supplementary, or both. • Pairs of congruent or supplementary angles formed when two lines are cut by a transversal can be used to prove parallel lines. • The sum of the measures of the interior angles is 180°, and the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles. • Two parallel lines have equal slopes. The slopes of perpendicular lines are negative multiplicative reciprocals of each other. • An isosceles triangle has congruent base angles and legs. The angle bisector of the vertex angle of an isosceles triangle is also the perpendicular bisector of the base. An equilateral triangle is also equiangular. • If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, the triangles are congruent. If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent. If two triangles are congruent, then all corresponding angles and sides of the two triangles are congruent. • Two triangles are congruent if two pairs of corresponding angles and the included sides are congruent. Two polygons are congruent if they can be divided into corresponding congruent triangular regions. • Two triangles can be proven congruent without showing that all corresponding parts are congruent. Right triangles can be proven congruent using one pair of right angles, a	• What angle relationships are created when parallel lines are intersected by a transversal? • What angle relationships can be used to prove that two lines intersected by a transversal are parallel? • What is true about the interior and exterior angles measures of a triangle? • How do the slopes of the lines that are parallel to each other compare? • How do the slopes of lines that are perpendicular to each other compare? • How are the side lengths and angle measures related in isosceles triangles and in equilateral triangles? • How are SAS and SSS used to show that two triangles are congruent? • How are ASA and AAS used to show that two triangles are congruent? • What minimum criteria are needed to show that right triangles are congruent? • What is the relationship between a segment and the points on its perpendicular bisector? • What is the relationship between an angle and the points on a bisector? • What are the properties of the perpendicular bisectors in a triangle? • What are the properties of the angle bisectors in a triangle? • What are the properties of the medians in a triangle?

pair of hypotenuses, and a pair of legs. • The perpendicular bisector of a segment contains all the points that are equidistant from the endpoints of the segment, and an angle bisector contains all the points that are equidistant from the sides of the angle. • The perpendicular bisectors of the sides of a triangle are concurrent at its circumcenter. The angle bisectors of a triangle are concurrent at its incenter. • The medians of a triangle are concurrent at its centroid. The lines containing the altitudes of a triangle are concurrent at its orthocenter. • The lengths of the sides of a triangle are related to the measures of the angles in the triangle. The sum of the lengths of two sides of a triangle is greater than the length of the third side. • When two triangles have two pairs of congruent sides, the longer third side is opposite the larger included angle and the shorter third side is opposite the smaller included angle.	• What are the properties of the altitudes in a triangle? • What are some relationships between the sides and angles of any triangle? • When two triangles have two pairs of congruent sides, how are the third pair of sides and the pairs of angles opposite the third pair of sides related?

Instructional Focus

Learning Targets

- Define parallel lines using the undefined terms point and line.
- Prove theorems about lines and angles.
- Use theorems to find the measures of angles formed by parallel lines and a transversal.
- Prove that two lines cut by a transversal are parallel using the converses of parallel line angle relationship theorems.
- Use properties of parallel lines and transversals to solve real-world problems.
- Use lines constructed parallel to another line to solve problems and prove theorems.

- Use the sum of the angles measured in a triangle to solve problems.
- Show that two lines in the coordinate plane are parallel by comparing their slopes, and solve problems.
- Show that two lines in the coordinate plane are perpendicular by comparing their slopes, and use that information to solve problems.
- Use properties of and theorems about isosceles and equilateral triangles to solve problems.
- Identify congruent triangles using properties of isosceles and equilateral triangles.
- Prove triangle congruence by SAS and SSS criteria and use triangle congruence to solve problems.
- Understand that corresponding parts of congruent triangles are congruent and use CPCTC to prove theorems and solve problems.
- Prove that two triangles are congruent using ASA and AAS criteria and apply ASA to solve problems.
- Prove that when all corresponding sides and angles of two polygons are congruent, the polygons are congruent.
- Prove and use the Hypotenuse-Leg Theorem.
- Use congruence criteria for triangles to solve problems and to prove relationships in geometric figures.
- Prove the Perpendicular Bisector Theorem, the Angle Bisector Theorem, and their converses.
- Use the Perpendicular Bisector Theorem to solve problems.
- Use the Angle Bisector Theorem to solve problems.
- Prove that the point of concurrency of the perpendicular bisectors of a triangle, called the circumcenter, is equidistant from the vertices.
- Prove that the point of concurrency of the angle bisectors of a triangle, called the incenter, is equidistant from the sides.
- Identify special segments in triangles and understand theorems about them.
- Find and use the point of concurrency of the medians of a triangle to solve problems and prove relationships in triangles.
- Find the point of concurrency of the altitudes of a triangle.
- Prove that the side lengths of a triangle are related to the angle measures of the triangle.
- Use the angle measures of a triangle to compare the side lengths of the triangle.

- Use the Triangle Inequality Theorem to determine if three given side lengths will form a triangle and to find a range of possible side lengths for a third side given two side lengths.
- Prove the Hinge Theorem and use the Hinge Theorem to compare side lengths.
- Prove the Converse of the Hinge Theorem and use the Converse of the Hinge Theorem to compare angle measures.

Prerequisite Skills

- Finding the length of a segment
- Vertical angles
- Slope of a line
- Basic geometry vocabulary
- Types of angles
- Triangle Classification
- Triangle Angle Sum
- Knowledge of Radii and Circles
- Finding the midpoint
- Perpendicular Bisectors

Common Misconceptions

- Students might think that parallel lines always have a transversal intersecting them.
- Some students believe that all angles formed by a transversal crossing parallel lines are equal.
- Students might think corresponding angles are always on the same side of the transversal.
- A misunderstanding of the definition might lead some students to think that parallel lines can eventually intersect if extended far enough.
- Students might think that transversals can only intersect parallel lines.
- Students might believe that parallel lines must be horizontal or vertical.
- Students may confuse congruent triangles with similar triangles, thinking that corresponding angles being equal is enough for congruence.
- Students might incorrectly apply a congruence theorem, such as using SSA (side-side-angle) to prove congruence.
- Students may not correctly identify the included angle in the SAS (side-angle-side) theorem or the

included side in the ASA (angle-side-angle) theorem.

- Students sometimes assume that because two triangles look congruent, they must be congruent without sufficient evidence.
- Students might think that any pair of corresponding sides or angles can be used to prove congruence.
- Students might struggle with triangles that are not in the standard or expected orientation.
- Some students might not realize that the HL theorem only applies to right triangles.
- Some students might think all triangles have the same concurrent points.
- Students somethings think the centroid is always the midpoint.
- Students may think the incenter, circumcenter, and orthocenter are always inside the triangle.
- Students may believe that angle bisectors and perpendicular bisectors are the same.
- Sometimes students think medians, altitudes, and angle bisectors are synonymous.
- Students may think that the sum of two sides can be equal to the third side or greater than or equal to.
- Students may think that any 3 positive lengths can form a triangle.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Understanding transversals • Angle pair relationships • Identifying theorems with angle pair relationships • Parallel lines and triangles • Triangle angle sum theorem • Slopes of parallel and perpendicular lines • Isosceles and Equilateral	• Understanding these basic undefined terms is essential for defining other geometric concepts such as parallel lines. • Complementary and Supplementary angles foundational for understanding angle relationships formed by parallel lines and transversals. • Understanding that vertical angles are congruent.	• IXL • Math Diagnostic and Intervention System Activities

triangles • Proving congruence using triangle congruence theorems. • Using right triangles to apply congruence. • Relating bisectors and equidistance • Finding circumcenter and incenter. • Finding the centroid and orthocenter. • Relating longest side to largest angle • Understanding the triangle inequality theorem • Comparing measures between two triangles	• Proficiency in solving linear equations is required for finding unknown angle measures and proving geometric theorems. • Knowledge of how to calculate and interpret the slope of a line is essential for determining if lines in the coordinate plane are parallel or perpendicular. • Familiarity with isosceles, equilateral, and right triangles is necessary for proving congruence and using triangle properties in problem-solving. • Distance and Midpoint formulas • Skills in basic constructions using a compass and straightedge are essential for various proofs and problem-solving tasks in geometry.	

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests • Unit 2 Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
Savvas Envision Geometry Pacing Guide	iXL Delta Math Desmos Khan Academy Math Medic Teacher Made worksheets

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Interdisciplinary Connections

ELA.RL.CR.9–10.1	Cite a range of thorough textual evidence and make relevant connections to strongly support analysis of multiple aspects of what a literary text says explicitly and inferentially, as well as including determining where the text leaves matters uncertain.
ELA.W.AW.9–10.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient textual and non-textual evidence.
SOC.6.2.12.HistoryCC.2.b	Explore the factors that laid the foundation for the Renaissance (i.e., Asian and Islamic, Ancient Greek and Roman innovations).
9-12.HS-ETS1-2.6.1	Design a solution to a complex real-world problem, based on scientific knowledge, student-generated sources of evidence, prioritized criteria, and tradeoff considerations.