Measurement

Content Area: **Math**
Course(s):
Time Period: **MP1**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
This unit opens by considering the relationship between the numbers of faces, vertices, and edges in polyhedrons, examining cross sections, and determining the three-dimensional figure formed by rotating a two-dimensional figure. Students then consider the volume of oblique solids by comparing the cross sections of oblique solids to corresponding right solids. Throughout the topic, students apply the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems. This unit continues to focus on the measurements and properties of line segments and angles. The rest of the unit introduces proofs. Students examine the nature of basic reasoning in both inductive and deductive forms, explore if-then statements, and write their first proofs.	Geometry is a branch of mathematics that provides critical tools for understanding the spatial world around us. The exploration of polyhedrons, cross sections, and three-dimensional figures is not only fundamental to mathematics but also to fields such as engineering, architecture, and various physical sciences. This unit aims to solidify students' understanding of geometric properties and measurements, providing them with the skills to analyze and solve complex problems involving three-dimensional figures and their respective volumes. This unit is designed to provide a thorough understanding of three-dimensional geometry and the foundational skills needed for geometric proofs. By integrating practical applications, collaborative learning, and critical thinking exercises, students will gain valuable skills that extend beyond the mathematics classroom. This unit not only prepares students for advanced mathematical studies but also equips them with problem-solving skills applicable to a wide range of disciplines and real-world scenarios.

NJSLS

MATH.9-12.G.CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MATH.9-12.G.CO.C.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.

MATH.9-12.G.CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
MATH.9-12.G.GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
MATH.9-12.G.GMD.A.2	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
MATH.9-12.G.GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
MATH.9-12.G.GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
MATH.9-12.G.GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
MATH.9-12.G.MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• The sum of the number of faces and vertices of a polyhedron equals two more than the number of edges. The cross section of a plane and a convex polyhedron is a polygon. A polygon rotated about an axis yields a three-dimensional figure with circular cross sections • Cavalieri's Principle states that two figures with the same height and the same cross-sectional area at every level have the same volume. • Cones and pyramids with the same height and	• How are three-dimensional figures and polygons related? • How does the volume of a prism or cylinder relate to a cross section parallel to its base? • How are the formulas for volume of a pyramid and volume of a cone alike? • How does the volume of a sphere relate to the volume of other solids? • How are the properties of segments and

the same area at every cross section have equal volume. • Apply Cavalieri's Principle to show that the volume of a hemisphere is equal to the volume of a cylinder of equal diameter and height minus the volume of a cone of equal diameter and height. • Find the volumes of objects composed of two or more solids by decomposing the solid into its component parts. • The sum of the lengths of all the sections of a segment is the length of the segment. The sum of the measures of the two smaller angles that form a larger angle is the measure of the larger angle. • The Midpoint Formula is used to find the midpoint of a segment in the coordinate plane and the Distance Formula is to find the length. • A construction is a geometric figure produced using only a straightedge and a compass. Constructions are useful tools for geometry. • Justify statements of a proof with definitions, postulates, theorems, and properties. • Indirect reasoning assumes that the statement to be proved is false, then shows that assumption leads to a contradiction.	angles used to determine their measures? • How are the midpoint and length of a segment on a coordinate plane determined? • How are a straightedge and compass used to make basic constructions? • How is deductive reasoning used to prove a theorem? • How can you conclude when valid reasoning leads to a contradiction?

Instructional Focus

Learning Targets
• Use Euler's Formula to calculate the number of vertices, faces, and edges in polyhedrons. • Describe cross sections of polyhedrons. • Describe rotations of polygons about an axis.

- Understand how the volume formulas for prisms and cylinders apply to oblique prisms and cylinders.
- Model three-dimensional figures as cylinders and prisms to solve problems.
- Understand how the volume formulas for pyramids and cones apply to oblique pyramids and cones.
- Model three-dimensional figures as pyramids and cones to solve problems.
- Use Cavalieri's Principle to show how the volume of a hemisphere is related to the volume of a cone and a cylinder.
- Calculate volumes and surface areas of spheres and composite figures.
- Use the Ruler and the Segment Addition Postulates.
- Use the Protractor and Angle Addition Postulates.
- Identify congruent segments and congruent angles.
- Use the midpoint formula to find the midpoint of a segment drawn on the coordinate plane.
- Use the distance formula to find the length of a segment drawn on the coordinate plane.
- Construct copies of segments and angles, perpendicular bisectors of segments and bisectors of angles.
- Apply construction to solve problems.
- Use deductive reasoning to prove geometric theorems about lines and angles.
- Understand and use indirect reasoning to prove or disprove propositions and theorems.
- Write indirect proofs.

Prerequisite Skills

- Volume formulas for prisms, cylinders, pyramids, cones and spheres.
- Definitions used in 8th grade; lines, segments, rays and angles.
- The absolute value of a number is the distance from 0 on the number line.
- Classifying angles by their measures.
- Identifying patterns and using inductive reasoning to form and test conjectures about patterns.
- Properties of real numbers

Common Misconceptions

- Euler's Formula is always edges plus 2

- Rotations polygons and visualizing the three-dimensional shape.
- The change in volume given the change in dimensions
- Prisms with a non rectangular base – students often confuse the height as the distance between the bases and not the height of the trapezoid.
- When finding the volume of composite figures, students may add or subtract incorrectly depending on what is being asked.
- Negative answers for distance, not using the absolute value.
- Using the midpoint formula, students may try to subtract the points in the specific coordinate instead of the x value of each coordinate.
- The difference between complements and supplements.
- The term "Prove" is a justified reason in a proof.
- When completing indirect proofs, students may confuse the term negation.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Examining properties of three-dimensional figures • Using cross sections to understand volume. • Applying volume • Understanding the elements of geometry • Copying line segments and angles • Finding a midpoint and a length • Writing proofs • Exploring indirect proofs	• Understanding basic arithmetic operations which are foundational for calculating measurements, such as lengths, areas, and volumes. • Using algebraic techniques to solve equations and manipulate formulas, which is essential when working with geometric formulas. • Recognizing and understanding the properties of basic shapes like triangles, squares, rectangles, and circles.	• IXL • Math Diagnostic and Intervention System Activities

	• Knowledge of how to measure lengths, areas, and volumes of basic geometric shapes. • Ability to plot points on the coordinate plane. • Understanding of slope, distance between points, and midpoint, which are crucial when dealing with geometric problems on the coordinate plane.	

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests • Unit 1 Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
Savvas Envision Geometry Pacing Guide	iXL Delta Math Desmos Khan Academy Math Medic Teacher Made worksheets

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Interdisciplinary Connections

ELA.RL.CR.9–10.1	Cite a range of thorough textual evidence and make relevant connections to strongly support analysis of multiple aspects of what a literary text says explicitly and inferentially, as well as including determining where the text leaves matters uncertain.
ELA.W.AW.9–10.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient textual and non-textual evidence.
VPA.1.3.12.D.1	Synthesize the elements of art and principles of design in an original portfolio of two- and three-dimensional artworks that reflects personal style and a high degree of technical proficiency and expressivity.
VPA.1.3.12.D.CS5	Two- and three-dimensional artworks can be rendered culturally specific by using the tools, techniques, styles, materials, and methodologies that are germane to a particular cultural style.