Integration and its Applications

Content Area:	**Math**
Course(s):	
Time Period:	**MP3**
Length:	**45**
Status:	**Published**

Unit Overview

Unit Summary	Unit Rationale
In this unit, we explore integrals and their applications. We start with the basics of definite and indefinite integrals, understanding them through limits and Riemann sums. The Fundamental Theorem of Calculus is a crucial focus, linking differentiation and integration. Students learn various integration techniques, such as substitution and integration by parts, and apply these to solve complex problems. Real-world applications include finding areas under curves, volumes of solids of revolution, and solving motion problems. This unit also revisits concepts from previous calculus units, such as limits, continuity, and differentiation, reinforcing and building on these foundations. By the end, students will have a solid grasp of integrals and be able to apply them in various mathematical and practical contexts.	The rationale, focusing on integrals and their applications, is to equip students with essential mathematical tools foundational for advanced calculus studies and numerous real-world applications. Integrals are pivotal in understanding and solving problems related to areas, volumes, and rates of change. This unit builds on the student's prior knowledge of limits and differentiation, reinforcing these concepts while introducing new techniques for solving more complex problems. By mastering integrals, students better understand the Fundamental Theorem of Calculus, which connects differentiation and integration into a cohesive framework. The skills acquired in this unit are critical for higher-level mathematics and fields such as physics, engineering, economics, and any discipline that requires quantitative analysis. This unit aims to develop students' problem-solving abilities and enhance their analytical thinking, preparing them for future academic and professional endeavors.

AP Standards - Calculus

MA.9-12.6.1.CHA-4.A.1	The area of the region between the graph of a rate of change function and the x axis gives the accumulation of change.
MA.9-12.6.1.CHA-4.A.2	In some cases, accumulation of change can be evaluated by using geometry.
MA.9-12.6.1.CHA-4.A.3	If a rate of change is positive (negative) over an interval, then the accumulated change is positive (negative).
MA.9-12.6.1.CHA-4.A.4	The unit for the area of a region defined by rate of change is the unit for the rate of change multiplied by the unit for the independent variable.
MA.9-12.7.1.FUN-7.A.1	Differential equations relate a function of an independent variable and the function’s derivatives.
MA.9-12.8.1.CHA-4.B.1	The average value of a continuous function f over an interval $[a, b]$ is $1/(b-a)$ ∫ (from a to b) $f(x)dx$.

MA.9-12.8.2.CHA-4.C.1	For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
MA.9-12.8.3.CHA-4.D.1	A function defined as an integral represents an accumulation of a rate of change.
MA.9-12.8.3.CHA-4.D.2	The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
MA.9-12.8.3.CHA-4.E.1	The definite integral can be used to express information about accumulation and net change in many applied contexts.
MA.9-12.8.4.CHA-5.A.1	Areas of regions in the plane can be calculated with definite integrals.
MA.9-12.8.5.CHA-5.A.2	Areas of regions in the plane can be calculated using functions of either x or y.
MA.9-12.8.6.CHA-5.A.3	Areas of certain regions in the plane may be calculated using a sum of two or more definite integrals or by evaluating a definite integral of the absolute value of the difference of two functions.
MA.9-12.8.7.CHA-5.B.1	Volumes of solids with square and rectangular cross sections can be found using definite integrals and the area formulas for these shapes.
MA.9-12.8.8.CHA-5.B.2	Volumes of solids with triangular cross sections can be found using definite integrals and the area formulas for these shapes.
MA.9-12.8.8.CHA-5.B.3	Volumes of solids with semicircular and other geometrically defined cross sections can be found using definite integrals and the area formulas for these shapes.
MA.9-12.8.9.CHA-5.C.1	Volumes of solids of revolution around the x- or y-axis may be found by using definite integrals with the disc method.
MA.9-12.8.11.CHA-5.C.3	Volumes of solids of revolution around the x- or y-axis whose cross sections are ring shaped may be found using definite integrals with the washer method.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• Riemann Sums (Left, Right, and Midpoint) and the Trapezoidal Rule are methods to approximate the area under a curve. • Integrals can be expressed as the limit of sums, connecting the concept of sequences	• How can Riemann Sums and the Trapezoidal Rule be used to estimate the area under a curve, and in what contexts are these approximations useful? • What is the relationship between sequences,

and series with integration. • The Fundamental Theorem of Calculus connects differentiation and integration, allowing for the evaluation of definite integrals using antiderivatives. • Definite integrals can be calculated using geometric areas and properties, providing exact values for the accumulation of quantities. • Constructing the general antiderivative involves finding a function whose derivative is the given function. • Recognizing patterns and using substitution can simplify the process of finding indefinite and definite integrals. • Integrating transcendental functions, including inverse trigonometric functions, requires specific rules and techniques. • The average value of a function over an interval can be found using definite integrals, providing a measure of the function's overall behavior. • Integration can be used to analyze position, velocity, and acceleration functions, providing a comprehensive understanding of motion. • The volume of solids of revolution can be found using the disc and washer methods, as well as by revolving solids about lines other than the axes.	series, and integrals, and how can we express an integral as a limit of sums? • How does the Fundamental Theorem of Calculus connect differentiation and integration, and what are its implications for solving real-world problems? • What are the methods for calculating definite integrals, and how can areas and properties of definite integrals be used in these calculations? • How can we construct the general antiderivative of a function, and what role does this play in solving differential equations and understanding integration? • How can pattern recognition and change of variables simplify the process of finding indefinite and definite integrals? • What specific techniques are used to integrate transcendental functions, including inverse trigonometric functions, and why are these techniques important? • How can definite integrals be applied to find the average value of a function over an interval, and what are the practical implications of this concept? • In what ways can integration be utilized to analyze position, velocity, and acceleration functions, and how does this analysis apply to real-world motion? • How can we find the volume of a solid of revolution using methods such as the disc and washer methods, and how does revolving a solid about a line other than an axis affect the calculation?

Instructional Focus

Learning Targets

- Use Left, Right, and Midpoint Riemann Sums and Trapezoidal Rule to estimate area under a curve
- Apply the properties of sequences to write integrals as a limit of right-hand sums.
- Use the Fundamental Theorem of Calculus to analyze functions, find displacement, total distance, and more.
- Calculate a definite integral using areas and properties of definite integrals
- Construct the general antiderivative of a function through discovery and check approach.
- Use the Fundamental Theorem of Calculus to find the definite integral of a function over an interval.
- Use pattern recognition and change of variables to find indefinite and definite integrals
- (+) Use rules of integration for transcendental functions, including inverse trigonometric rules.
- Apply definite integrals to problems involving the average value of a function over an interval.
- Use properties of integraphs and accumulation to analyze position, velocity, and acceleration functions.
- Describe integration as an accumulation process.
- Find the area of a region between two curves using integration.
- Find the volume of a solid of revolution with known cross sections.
- Find the volume of a solid of revolution using the disc and washer methods.
- Find the volume of a solid by revolving about a line other than an axis.

Prerequisite Skills

- Students should have a strong grasp of limits, including how to evaluate limits algebraically and graphically.
- Knowledge of what it means for a function to be continuous at a point and over an interval.
- Proficiency in finding the derivative of a function, understanding the definition of the derivative, and interpreting it as a rate of change.
- Understanding of how to use derivatives to find and interpret critical points, local and absolute extrema, and points of inflection.
- Familiarity with the concept of an antiderivative and basic antiderivative rules.
- Proficiency in algebraic manipulation and trigonometric identities.
- Knowledge of how to graph various types of functions and interpret their graphical behavior.

• Basic understanding of sequences and series, including arithmetic and geometric series.
Common Misconceptions
• Students often think that the integral always represents the area under a curve, without considering the possibility of negative areas. • Students sometimes confuse definite integrals (which have limits of integration and yield a numerical value) with indefinite integrals (which represent a family of functions and include a constant of integration). • When finding antiderivatives, students often forget to include the constant of integration, (C). • Students may incorrectly apply the Fundamental Theorem of Calculus by not properly evaluating the antiderivative at the upper and lower limits of integration. • Students sometimes switch the limits of integration or integrate in the wrong order. • Students often struggle with the substitution method, particularly in choosing the correct substitution and changing the limits of integration. • Students may confuse the average value of a function over an interval with the average of the function's values at specific points. • Students may incorrectly set up integrals for finding volumes of solids of revolution using the disc and washer methods. • Students often forget to include units when interpreting the results of integrals in real-world applications.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Accumulations of change • Approximate areas with Riemann Sums • Define a definite integral • Discuss errors in Riemann Sums • Understand summation	• Understanding the behavior of functions as they approach specific points or infinity; continuity of functions. • Techniques for finding the derivative of a function; interpretation of the derivative as an instantaneous rate of	• iXL Diagnostic Assessment • iXL Problems • Delta Math

notation • Writing area as a limit • Use accumulations functions to discuss the Fundamental Theorem of Calculus • Definite Integral Notation • Properties of definite integrals • Define an antiderivative • Discuss the difference between an Indefinite integral and a definite integral • Find the antiderivative • Solve differential equations subject to given conditions • Discuss the Fundamental Theorem of Calculus and Definite integrals • Procedure for using U-substitution • Integration by substitution • Mean Value Theorem for Integrals • Average value of a function • Discuss the Net Change Theorem and the difference between total distance and displacement • Finding the area between two curves • Finding volume with cross sections	change. • Using derivatives to find critical points, local and absolute extrema, and points of inflection; solving optimization problems. • Finding basic antiderivatives and understanding their relationship to original functions. • Understanding arithmetic and geometric series, convergence, and divergence. • Simplifying expressions, solving equations, and using trigonometric identities. • Sketching and analyzing graphs of various functions, identifying key features such as intercepts, asymptotes, and intervals of increase/decrease. • Developing logical thinking, methodical approaches, and reasoning skills in mathematical contexts. • Chain rule in differentiation • Area and volume calculations	

• Use the disc and washer method to find the volume		

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	Mid Unit Assessment - Integration Benchmark 3 (Linkit) AP Benchmark 3 (Linkit)

Resources

Key Resources	Supplemental Resources
Larson, R., & Edwards, B. (2010). *Calculus* (9th ed.). Brooks/Cole. • Chapters 4 and 7 Calculus Online Textbook - Openstax	iXL Delta Math Math Medic AP Classroom Desmos Activity Builder Desmos Graphing Calculator Explorations Khan Academy APSI Resources Teacher made Worksheets

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Interdisciplinary Connections

CS.9-12.AP	Algorithms & Programming
9-12.HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
9-12.HS-LS2	Ecosystems: Interactions, Energy, and Dynamics
9-12.HS-PS2	Motion and Stability: Forces and Interactions