Limits, Continuity and Differentiation

Content Area: **Math**
Course(s):
Time Period: **MP1**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
This unit introduces high school students to the foundational concepts of calculus and its applications. This unit begins with exploring limits, where students learn to evaluate how functions behave as they approach particular points or infinity. Through hands-on activities and graphical analysis, students will deepen their understanding of continuity and the significance of one-sided limits. As the unit progresses, the focus shifts to derivatives, emphasizing the instantaneous rate of change and its geometric interpretation as the slope of a tangent line. Students will apply differentiation rules to find and interpret derivatives of various functions, including polynomial, trigonometric, and exponential functions. The unit also addresses common misconceptions, helping students develop a clear and accurate understanding of these essential calculus concepts. By the end of the unit, students will have developed critical thinking and problem-solving skills, enabling them to confidently tackle real-world applications of limits and derivatives and get them ready to analyze functions with differentiation.	The unit highlights the importance of these foundational concepts in the broader context of mathematics and its applications in various fields. Understanding limits is essential for grasping how functions behave in different situations, which is a critical building block for further calculus and higher mathematics studies. Limits provide insight into function behavior near specific points and lay the groundwork for the concept of continuity, which is vital for analyzing the stability of systems in real-world scenarios. The introduction of derivatives is equally significant, as they represent a function's instantaneous rate of change and have practical applications in physics, engineering, economics, and many other disciplines. By learning to differentiate various types of functions, students gain tools to model and analyze real-life situations, from calculating velocity to optimizing resources. This unit fosters critical thinking and problem-solving skills, encouraging students to apply mathematical reasoning to complex problems. By addressing common misconceptions and reinforcing prior knowledge, the unit aims to build confidence in students as they transition into more advanced topics in calculus. Ultimately, the rationale for this unit is to equip students with essential mathematical skills and concepts that will serve as a foundation for their future studies and professional endeavors, ensuring they are prepared for the challenges of higher education and the next part of differentiation.

AP Standards - Calculus

MA.9-12.I.B.1	An intuitive understanding of the limiting process
MA.9-12.I.B.2	Calculating limits using algebra
MA.9-12.I.B.3	Estimating limits from graphs or tables of data

MA.9-12.I.C.1	Understanding asymptotes in terms of graphical behavior
MA.9-12.I.C.2	Describing asymptotic behavior in terms of limits involving infinity
MA.9-12.I.C.3	Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)
MA.9-12.I.D.1	An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
MA.9-12.I.D.2	Understanding continuity in terms of limits
MA.9-12.I.D.3	Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)
MA.9-12.II.A.1	Derivative presented graphically, numerically, and analytically
MA.9-12.II.A.2	Derivative interpreted as an instantaneous rate of change
MA.9-12.II.A.3	Derivative defined as the limit of the difference quotient
MA.9-12.II.A.4	Relationship between differentiability and continuity
MA.9-12.II.B.1	Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
MA.9-12.II.B.2	Tangent line to a curve at a point and local linear approximation
MA.9-12.II.B.3	Instantaneous rate of change as the limit of average rate of change
MA.9-12.II.B.4	Approximate rate of change from graphs and tables of values
MA.9-12.II.F.1	Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
MA.9-12.II.F.2	Derivative rules for sums, products, and quotients of functions

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• Limits provide a way to understand how functions behave as they approach specific values, revealing insights into continuity and discontinuity. • The concept of one-sided limits helps to	• What does it mean for a function to approach a limit? • How do one-sided limits help us understand the behavior of functions?

analyze functions at points of interest and is crucial for understanding piecewise-defined functions. • Recognizing infinite limits allows students to understand asymptotic behavior and the nature of functions as they approach infinity, which is important in real-world applications. • A function is continuous at a point if the limit exists at that point and equals the function value, highlighting the interconnectedness of limits and continuity. • The properties of limits provide systematic methods for evaluating limits algebraically, enabling students to simplify complex expressions. • Graphs of functions can visually represent limits, helping students to intuitively understand approaching behavior and the significance of limits in calculus. • This theorem illustrates that if a function is continuous over an interval, it takes on every value between its endpoints, reinforcing the importance of limits in understanding function behavior. • The derivative represents the instantaneous rate of change of a function, providing critical insights into motion and change in various contexts. • The derivative at a point corresponds to the slope of the tangent line, illustrating how calculus connects geometric and algebraic interpretations. • A function must be continuous at a point to be differentiable there, emphasizing the relationship between these two fundamental concepts in calculus. • Derivatives are used to solve real-world problems, such as optimizing processes and analyzing motion, connecting mathematics to practical applications. • The sign and value of the derivative provide information about a function's increasing or	• What is the relationship between limits and continuity? • How can we evaluate limits using algebraic properties? • In what ways do infinite limits and limits at infinity affect the behavior of functions? • How does the Intermediate Value Theorem illustrate the importance of limits in function behavior? • How can graphical representations enhance our understanding of limits? • What is the significance of the derivative as an instantaneous rate of change? • How does the derivative relate to the slope of the tangent line? • What conditions must be met for a function to be differentiable at a point? • How can derivatives be applied to solve optimization problems? • What information can we learn about the behavior of a function from its derivative? • How can graphical and numerical methods assist in estimating derivatives? • What are the fundamental rules of differentiation, and how do they facilitate the computation of derivatives?

decreasing behavior, as well as identifying local maxima and minima. • Students can estimate derivatives graphically and from tables, reinforcing the idea that calculus concepts can be approached from multiple perspectives. • Mastery of the basic rules of differentiation (product rule, quotient rule) allows for the effective and efficient computation of derivatives for a wide variety of functions.	

Instructional Focus

Learning Targets

- Interpret rate of change at an instant in terms of average rates of change.
- Evaluate the limit, if it exists, by graphical and numerical methods.
- Find one-sided limits and general limits.
- Use properties of limits to evaluate graphically, numerically, and algebraically.
- Evaluate limits analytically, including substitution, cancelation, rationalization, and special trig rules.
- Evaluate limits of exponential and trigonometric functions.
- Apply the definition of continuity to determine whether a function is continuous at a point.
- Compare and contrast inginit limits and limits at infinity.
- Use properties of infinite limits to find asymptotes and describe function behavior.
- Understand and apply the Intermediate Value Theorem
- Interpret rates of change over an interval or at a simple point by graphical and numerical methods.
- Write equations of tangent lines and normal lines.
- Use limit definition of the derivative, proper notation to find the general form of a derivative.
- Analyze function behavior using the derivative.

- Use the tangent line to investigate function behavior.
- Estimate derivatives graphically and from tables.
- Understand the relationship between differentiability and continuity.
- Find derivatives use basic rules of differentiation for polynomial, power, sine, cosine, tangent, exponential and logarithmic functions.
- Apply the product and quotient rules.
- Solve basic motion problems involving position, speed, velocity and acceleration.

Prerequisite Skills

- Proficiency in simplifying algebraic expressions, including factoring, expanding, and combining like terms.
- Ability to recognize and interpret various types of functions (linear, quadratic, polynomial, rational, exponential, and logarithmic) and their properties.
- Skills in plotting functions on a coordinate plane and understanding their graphical behavior, including intercepts, asymptotes, and continuity.
- Familiarity with the rules of exponents, including operations with fractional exponents and simplification of radical expressions.
- Understanding the fundamental trigonometric functions (sine, cosine, tangent) and their relationships, as well as their values at key angles.
- Knowledge of the concept of slope as a measure of rate of change, including the ability to calculate slope between two points on a line.
- Ability to write the equation of a line in slope-intercept form ($y = mx + b$) and point-slope form ($y - y_1 = m(x - x_1)$).
- Familiarity with the basic concept of limits, including evaluating simple limits and understanding their graphical representations.
- Basic skills in using graphing calculators or software to visualize functions and their behavior, which aids in understanding limits and derivatives.
- Ability to approach mathematical problems analytically and logically, enabling students to tackle complex calculus concepts effectively.

Common Misconceptions

- Students may think that the limit of a function as it approaches a point is always equal to the function's value at that point, ignoring cases of discontinuity.
- Some students might overlook the importance of one-sided limits and assume that limits from both

sides will always yield the same result.

- Students may struggle with the concept that limits can approach infinity, believing that limits must always be finite values.
- Students often misinterpret vertical asymptotes as limits that don't exist, rather than recognizing that the limit can approach infinity at those points.
- Some students may not grasp that the limit is concerned with the behavior of the function in the neighborhood around a point, not just at the point itself.
- Students might confuse the concepts of continuity and limits, thinking that if a limit exists, the function is continuous without understanding the conditions for continuity.
- Students may misapply limit laws, assuming they are universally applicable without considering the conditions under which they hold true.
- Students may not fully understand that the derivative represents the slope of the tangent line at a point, thinking instead of it as a simple average rate of change over an interval.
- Some students might believe that the instantaneous rate of change can be calculated by simply taking the average rate of change over a small interval.
- Students may think that if a function is continuous, it must also be differentiable, not realizing the necessity of a smooth change for differentiability.
- Students might overlook the importance of critical points when finding local maxima and minima, failing to check for points where the derivative is zero or undefined.
- Students often misapply the rules of differentiation (product rule, quotient rule, chain rule) due to a misunderstanding of when and how to use them.
- Some students may assume that all functions have derivatives at every point, not recognizing that there are functions (like absolute value) that have points of non-differentiability.
- Students might think that derivatives alone can provide a complete picture of the function's behavior without considering other factors such as second derivatives or the function's overall context.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Concept of instantaneous rate of change • Understanding limits	• Mastery of algebraic manipulation, including factoring, expanding, and simplifying expressions, is	• iXL Diagnostic Assessment

graphically and numerically
• Finding one-sided limits
• Properties of limits
• Finding limits by analytic methods
  ○ Cancelation
  ○ Rationalization
  ○ substitution
• Limits of transcendental functions
• Limits and continuity
  ○ Definition of continuity
• Infinite limits and limits at infinity
  ○ Asymptotes and end behavior
• Intermediate Value Theorem
• Rates of Change
• The Tangent Line Problem
• Tangent lines and the derivative
• Differentiability and continuity
• Basic differentiation rules
• Product Rule
• Quotient Rule
• Velocity problems with differentiation
	essential for evaluating limits and calculating derivatives.
	• A deep understanding of functions, including their types (linear, quadratic, polynomial, rational, exponential, and logarithmic) and their characteristics (domain, range, intercepts, and asymptotes) will be crucial when analyzing limits and derivatives.
	• Skills in graphing functions and interpreting their behavior (including identifying intercepts, asymptotes, and continuity) are important for visualizing limits and understanding the geometric interpretation of derivatives.
	• Concepts from previous courses regarding rates of change, such as average rate of change, set the stage for the transition to instantaneous rates of change represented by derivatives.
	• Knowledge of trigonometric functions and their properties will be valuable when working with derivatives, especially when applying the derivatives of sine, cosine, and other trigonometric functions.
	• Introductory exposure to limits from pre-calculus courses, including understanding how functions behave as they
		• iXL Problems
		• Delta Math

	approach specific points, prepares students for more advanced limit concepts in calculus. • Previous discussions about continuous functions and the conditions for continuity will support students in understanding the relationship between limits and differentiability. • Familiarity with graphing calculators or software from previous math courses will assist students in visualizing functions and their derivatives, enhancing their understanding of calculus concepts.	

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	Mid Unit Assessment - Limits Benchmark 1 (Linkit) AP Benchmark 1(Linkit)

Resources

Key Resources	Supplemental Resources
Larson, R., & Edwards, B. (2010). *Calculus* (9th ed.). Brooks/Cole.	iXL

• Chapters 1 and 2 Calculus Online Textbook - Openstax	Delta Math Math Medic AP Classroom Desmos Activity Builder Desmos Graphing Calculator Explorations Khan Academy APSI Resources Teacher made Worksheets

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Interdisciplinary Connections

VA.9-12.1.5.12prof.Cn10	Synthesizing and relating knowledge and personal experiences to create products.
CS.9-12.AP	Algorithms & Programming
SOC.9-12.1.2.2	Relate current events to the physical and human characteristics of places and regions.
9-12.HS-PS2	Motion and Stability: Forces and Interactions