Proportional Relationships and Linear Functions

Content Area: **Math**
Course(s):
Time Period: **MP3**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
This unit focuses on proportional relationships, unit rates, and linear functions. Students will compute unit rates associated with ratios of fractions and learn to identify and represent proportional relationships through equivalent ratios using tables and graphs. They will also explore the constant of proportionality in various contexts. Students will apply their understanding to solve multistep ratio and percent problems, while interpreting linear models by examining slopes and intercepts within real-world scenarios. They will create equations representing relationships between quantities and graph these equations accurately. Additionally, the unit emphasizes the properties of functions, including function notation and evaluation. Students will derive equations for linear relationships, understand slope as a rate of change, and compare different functions represented in various forms. They will construct functions to model linear relationships, determining rate of change and initial values.	The rationale for this unit is rooted in the importance of understanding proportional relationships, unit rates, and linear functions as foundational concepts in mathematics. By focusing on these topics, students develop critical analytical and problem-solving skills essential for real-world applications. Mastering the computation of unit rates and the identification of proportional relationships allows students to make informed decisions in various contexts, such as finance, science, and everyday life. Furthermore, interpreting linear models and understanding the significance of slope and intercepts helps students connect mathematical concepts to real-world data, fostering a deeper comprehension of how mathematics describes relationships in their environment. The ability to create and graph equations enhances their understanding of how different quantities relate to each other, while exploring functions and their properties lays the groundwork for future mathematical learning.

NJSLS

7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
7.RP.A.2	Recognize and represent proportional relationships between quantities.
7.RP.A.2.a	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
7.RP.A.2.b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.A.2.c	Represent proportional relationships by equations.
7.RP.A.2.d	Explain what a point (□, □) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, □) where □ is the unit rate.
7.RP.A.3	Use proportional relationships to solve multistep ratio and percent problems.
S.ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
A.CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
8.EE.B.6	Use similar triangles to explain why the slope □ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation □ = □□ for a line through the origin and the equation □ = □□ + □ for a line intercepting the vertical axis at □.
F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If □ is a function and □ is an element of its domain, then □(□) denotes the output of □ corresponding to the input □. The graph of □ is the graph of the equation □ = □(□).
F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
8.F.A.2	Compare properties (e.g., rate of change, intercepts, domain and range) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
8.F.A.3	Interpret the equation □ = □□ + □ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
A.REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (□, □) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

F.LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standards for Mathematical Practice

1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure
8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• Understanding unit rates allows me to compare different quantities and make informed choices in real-life situations. • Proportional relationships are fundamental in math and help me understand how two quantities change together. • Tables are useful tools for organizing data and identifying patterns in equivalent ratios. • Graphs visually represent relationships and can provide insights into data trends and behaviors. • The constant of proportionality is a key concept that helps me understand the strength of relationships between quantities. • Ratios and percentages are tools for solving a wide range of problems in everyday life, from budgeting to cooking. • Slope is an important concept that describes how steep a line is and indicates the rate of change between two variables. • The y-intercept represents the starting point of a relationship and is essential in	• How do unit rates help us make comparisons and informed decisions in everyday situations? • What characteristics define a proportional relationship, and why are they important in mathematics? • In what ways do tables help us identify patterns and relationships in ratios? • How can graphs provide insights into the relationships between different quantities? • Why is the constant of proportionality significant when analyzing relationships between variables? • How can we use ratios and percentages to solve real-world problems effectively? • What does slope tell us about the relationship between two variables, and how can we interpret it? • How does the y-intercept affect the overall meaning of a linear equation? • Why is it important to write equations that

understanding linear equations. • Writing equations helps me express relationships mathematically and solve for unknown values. • Graphing linear equations allows me to visualize how two quantities interact and predict outcomes. • Functions are a way to describe relationships between variables, and understanding them is crucial for advanced math. • Function notation provides a concise way to represent and evaluate relationships in mathematics. • Evaluating functions helps me find outputs based on specific inputs, reinforcing the concept of cause and effect in relationships. • Comparing different functions allows me to see how they behave and how they can be used in various contexts. • Creating functions to model real-life situations helps me apply math to understand and solve problems in my everyday life.	represent relationships between quantities? • What can we learn about two quantities by graphing their linear equations? • How do functions help us understand relationships between variables in mathematics? • What is the purpose of function notation, and how does it streamline our work with functions? • How does evaluating a function for specific inputs illustrate the concept of cause and effect? • What can we discover by comparing different types of functions and their behaviors? • How can creating functions to model real-life situations enhance our problem-solving abilities in everyday life?

Instructional Focus

Learning Targets
• Find unit rates for different measurements. • Explain what a proportional relationship is. • Use tables to show equivalent ratios. • Create graphs to represent proportional relationships. • Find the constant of proportionality in a problem. • Solve problems that involve ratios and percentages.

- Understand and use slope in a graph.
- Identify the y-intercept in a linear equation.
- Write equations for relationships between two quantities.
- Graph linear equations on a coordinate plane.
- Recognize the definition of a function.
- Use function notation correctly.
- Find the output of a function for given inputs.
- Compare different types of functions.
- Create functions to model real-life situations and explain their meaning.

Prerequisite Skills

- Students should be comfortable working with fractions, including addition, subtraction, multiplication, and division.
- Students need to understand what ratios are and how to compare two quantities.
- Proficiency in addition, subtraction, multiplication, and division of whole numbers and decimals is essential.
- Students should know how to calculate percentages and understand their meaning in different contexts.
- Students should have prior experience plotting points on a coordinate plane and interpreting simple graphs.
- Students should be familiar with the concept of variables and how they can represent unknown quantities in equations.
- An understanding of simple algebraic expressions and equations, including how to solve for an unknown variable, is beneficial.
- Students should have some exposure to the concept of linear relationships and how they can be represented graphically

Common Misconceptions

- Students may struggle to differentiate between ratios and fractions, thinking they are the same when they serve different purposes.
- Some students might incorrectly assume that all relationships are proportional, not recognizing when a relationship is not constant.
- Students may think that the slope of a line only represents steepness, not understanding that it also

represents the rate of change between two variables.

- Students might struggle with the concept of the y-intercept, often misinterpreting its significance in a linear equation.
- Some students may think that unit rates always relate to prices or costs, not realizing they can apply to various situations and measurements.
- Students might have trouble plotting points accurately on a graph, leading to misrepresentations of the relationships they are trying to show.
- Students may mistakenly believe that all functions must be linear, not recognizing that functions can take many different forms.
- Students might solve mathematical problems without considering the real-world context, leading to incorrect interpretations of their answers.
- Some students may struggle with function notation, not realizing that it simply represents a relationship between inputs and outputs.
- Students might overlook the importance of units when calculating unit rates or solving problems, leading to confusion in their answers.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Define Ratios and Rates • Find Unit Rates • Compare relationships using ratios/rates/unit rates • Create equivalent ratios • Use equivalent ratios to compare relationships • Use equivalent ratios to determine if a relationship is proportional • Recognize that the graph of a proportional relationship is a straight line through	• Use rational numbers when calculating unit rates, ratios, and solving proportional relationships. • Build on knowledge of algebraic expressions to write and simplify expressions related to linear relationships and proportional reasoning. • Previous experiences with solving equations will be essential as students learn to formulate and solve equations that describe proportional relationships	• iXL problems • iXL Diagnostic Assessment • Delta Math

the origin • Identify if graphs represent proportional relationships • Write the equation of a line that models direct variation • Understand that the value of k is the same as the unit rate and slope of the line • Recognize that the graph of a proportional relationship is a straight line through the origin • Identify if graphs represent proportional relationships • Write the equation of a line that represents direct variation from the graph • Recognize that the point (1, k) represents the unit rate • Calculate the unit rate from any point (x,y) on the graph • Explain what points on the graph of a proportional relationship represent in terms of the context of the situation • Recognize the relationship between slope and unit rate where appropriate • Calculate the rate of change between two points in a table or on a graph • Interpret how changes in slope effect the line on a graph • Interpret the slope of a line in the context of a real	and linear functions. • Understanding inequalities will allow students to compare quantities and express relationships that involve constraints. • Understanding of plotting rational numbers on a number line will be essential when they graph points on a coordinate plane to represent linear equations and proportional relationships. • Building on prior knowledge of functions, students will recognize how to identify and evaluate functions in various forms, including linear functions. • Familiarity with the order of operations will be crucial when students simplify expressions and solve equations that arise from proportional relationships and functions. • Understanding properties such as the distributive property and combining like terms will aid students in manipulating expressions and solving equations. • Prior lessons on calculating rates and understanding how quantities change in relation to one another will help students grasp the concept of slope as a rate of change in linear functions.	

world problem • Write linear equations in two variables using slope intercept form • Interpret the slope and the intercept of a linear model • Write and graph linear equations using point-slope form • Analyze different forms of a line to interpret the slope and -intercept of a linear model in the context of data • Write and graph linear equations in standard form • Use linear equations in standard form to interpret the x- and y-intercepts in the context of given data • Write Equations from Tables, Graphs, Equations • Graph Linear Equations in all Three Forms • Multiple Representations of Relations • Determine if the Relation is a Function (Include Mapping Diagrams and VLT) • Determine the Domain and Range of a Relation and Continuous Graphs • Determine a reasonable domain and identify constraints on the domain based on the context of a real-world problem		

• Evaluate Functions • Write Relations in Function Notation • Average Rate of Change • Identify Linear Functions from Tables, Graphs, and Equations • Write and evaluate linear functions using function notation • Graph a linear function and relate the domain of a function to its graph • Interpret functions represented by graphs, tables, verbal descriptions, and function notation in terms of a context		

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests • Unit 3 Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
EnVision 7th Grade	iXL

EnVision 8th Grade EnVision Algebra 1 Pacing Guide	Delta Math ThatQuiz.org Desmos Khan Academy Teacher Made Worksheets

Career Readiness, Life Literacies, and Key Skills

9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
9.4.12.CT.4	Participate in online strategy and planning sessions for course-based, school-based, or other project and determine the strategies that contribute to effective outcomes.
9.4.12.IML.1	Compare search browsers and recognize features that allow for filtering of information.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

Interdisciplinary Connections

PFL.9.1.12.B.6	Design and utilize a simulated budget to monitor progress of financial plans.
PFL.9.1.12.D.1	Calculate short- and long-term returns on various investments (e.g., stocks, bonds, mutual funds, IRAs, deferred pension plans, and so on).
9-12.HS-LS2-1.3.1	students understand the significance of a phenomenon is dependent on the scale, proportion, and quantity at which it occurs. They recognize patterns observable at one scale may not be observable or exist at other scales, and some systems can only be studied indirectly as they are too small, too large, too fast, or too slow to observe directly. Students use orders of magnitude to understand how a model at one scale relates to a model at another scale. They use algebraic thinking to examine scientific data and predict the effect of a change in one variable on another (e.g., linear growth vs. exponential growth).
9-12.HS-LS3-3.3.1	Algebraic thinking is used to examine scientific data and predict the effect of a change in one variable on another (e.g., linear growth vs. exponential growth).
9-12.HS-PS2-2	Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.