Transformations, the Pythagorean Theorem, and Volume

Content Area: **Math**
Course(s):
Time Period: **MP1**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
Students understand the concept of translating figures and how the resulting images are related to the preimages. Students will also understand the concept of reflecting figures and recognize how the image isa flip of a preimage and that the two figures are an equal distance away from the line of reflection. Students will also develop their understanding of dilation to find and graph images. In this unit students will also recognize that congruent figures have the same shape and size. They know that congruent images are created by a sequence of reflections, rotations, and translations. In this unit, students recognize that parallel lines and a transversal create sets of angles. They will learn that there are angle-based relationships that can be used to categorize and determine the measures of unknown angles. Students will also build on their understanding of angle-based relationships and determine the missing measurements of interior and exterior angles of triangles. Students further their conceptual understanding of surface area by applying what they know to find the surface area of cylinders and cones. They identify the two-dimensional surfaces that make up three-dimensional figures and draw nets as a strategy for finding surface areas of cylinders and cones. In this unit students will also understand that volume is a measure of capacity. They relate volumes of cones, cylinders, and spheres to the volume of three-dimensional figures they know. They use these relationships to generalize volume formulas for cones, cylinders, and spheres.	Unit 4B allows students to develop procedure and fluency related to the topics related to geometry including, congruence, similarity, transformations and the Pythagorean Theorem. In this unit students also develop conceptual understanding related these topics. These are foundational skills for upper level geometry classes. This unit also helps students to see the connection between algebraic reasoning and geometry. The lessons in this unit will also help students to develop a geometric view of the world around them and how these skills are visible in the world around them.

NJSLS

8.G.A	Understand congruence and similarity using physical models, transparencies, or geometry software
8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations:
8.G.A.1.a	Lines are transformed to lines, and line segments to line segments of the same length.
8.G.A.1.b	Angles are transformed to angles of the same measure.
8.G.A.1.c	Parallel lines are transformed to parallel lines.
8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
8.G.B	Understand and apply the Pythagorean Theorem
8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.
8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
8.G.C	Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres
8.G.C.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Standards for Mathematical Practice

1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure
8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
● A translation (slide) is a transformation that moves every point of a figure the same distance and the same direction. ● A reflection (flip) creates images that have the same size and shape, but are different orientation. The preimage and image are the same distance from the line of reflection but on opposite sides. ● A rotation is created by moving each point of the preimage around a fixed point. The image and preimage have the same size and shape. ● When one transformation will not map a preimage into its image, a sequence of transformations is needed. ● A sequence of translations, reflections, and rotations can map one figure to another without changing its shape or size. ● A dilation is a transformation that changes the size of a figure. In a dilation, the preimage and image have the same shape, angle measures, and proportions. ● Two-dimensional figures are similar if there is a sequence of translations, reflections, rotations, and dilations that map one figure onto the other. ● If parallel lines are intersected by a transversal, then corresponding and alternate interior angles are congruent, and same-side interior angles are supplementary. ● The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles. ● If two angels in one triangle are congruent to two angles in another ● Formulas for finding the areas of polygons, such as rectangles, squares, triangles, and circles, can be used to find the surface areas of cylinders, cones, and spheres. ● Finding the volume of a cylinder is an extension of finding the volume of a rectangular prism. The volume of a rectangular prism is the product of the area of its base and its height. Similarly, the volume of a cylinder is equal to the product of the area of its circular base and its height. ● The volume of a cone is the volume of a cylinder given that the bases have the same radius and the heights are the same. The formula for the volume of a cone is where is the area of its circular base and is the height of the cone. ● The volumes of a sphere and cone are	● How does a translation affect the properties of a two-dimensional figure? ● How does a reflection affect the properties of a two-dimensional figure? ● How does a roation affect the properties of a two-dimensional figure? ● How can you use a sequence of transformations to map a preimage to its image? ● How does a sequence of translations, reflections, and rotations result in congruent figures? ● What is the relationship between a preimage and an image after a dilation? ● How are similar figures related by a series of transformations? ● What are the relationships among angles that are created when a line intersects two parallel lines? ● How are the interior and exterior angles of a triangle related? ● How can you use angle measures to determine if two triangles are similar? ● How can you show that two figures are either congruent or similar to one another? How are the areas of polygons used to find the surface area formulas for three-dimensional figures? ● How is the volume of a cylinder related to the volume of a rectangular prism? ● How is the volume of a cone related to the volume of a cylinder? ● How is the volume of a sphere related to the volume of a cone? ● How are the formulas for the volume of a cylinder, cone, and sphere related to one another? • How does a translation affect the properties of a two-dimensional figure? • How does a reflection affect the properties of a two-dimensional figure? • How does a rotation affect the properties of a two-dimensional figure? • How can you use a sequence of transformations to map a preimage to its image? • How does a sequence of translations, reflections, and rotations result in congruent figures? • What is the relationship between a preimage and its image after a dilation? • How are similar figures related by a sequence of transformations? • What are the relationships among angles that are created when a

proportionally related. The volume of a sphere is twice the volume of a cone that has the same circular base and height. The formula for the volume of a sphere is , where is the radius of the sphere. A translation (slide) is a transformation that moves every point of a figure the same distance and the same direction. • A reflection (flip) creates images that have the same size and shape, but different orientation. The preimage and image are the same distance from the line of reflection but on opposite sides. • A rotation is created by moving each point of the preimage around a fixed point. The image and preimage have the same size, shape, and orientation. • When one transformation will not map a preimage into its image, a sequence of transformations is needed. • A sequence of translations, reflections, and rotations can map one figure to another without changing its shape or size. • A dilation is a transformation that changes the size of a figure. In a dilation, the preimage and image have the same shape, angle measures, and proportions. • Two-dimensional figures are similar if there is a sequence of translations, reflections, and dilations that map one figure onto the other. • If parallel lines are intersected by a transversal, then corresponding and alternate interior angles are congruent, and same-side interior angles are supplementary. • The Pythagorean Theorem can be used to determine if a triangle is a right triangle and to find the missing side length of a triangle. • If a triangle has side length such that a2 + b2 = c2, the triangle is a right triangle. • The Pythagorean Theorem and its converse can be used to solve real-world problems that involve right triangles. Both can be used to determine the unknown leg lengths of a right triangle, or to identify or verify whether a triangle is a right triangle. • The Pythagorean Theorem can be used to find the distance between any two points on a coordinate plane by drawing a line to connect the points and using it as the hypotenuse of a right triangle where the legs are the horizontal and vertical distances.	line intersects two parallel lines? • How are the interior and exterior angles of a triangle related? • How can you use angle measures to determine whether two triangles are similar? • How does the Pythagorean Theorem relate the side lengths of a right triangle? • How can you determine whether a triangle is a right triangle? • What types of problems can be solved using the Pythagorean Theorem. • How can you use the Pythagorean Theorem to find the distance between two points?

Instructional Focus

Learning Targets

● Understand translations
● Translate a figure on a coordinate plane
● Describe a translation
● Understand and describe a reflection
● Reflect two-dimensional figures

● Identify and perform a rotation
● Determine how a rotation affects a two-dimensional figure
● Understand a sequence of transformations
● Describe and perform a sequence of transformations
● Understand congruence of figures using a series of transformations
● Identify congruent figures
● Understand dilations
● Dilate to enlarge or reduce a figure in a coordinate plane
● Understand similarity
● Complete a similarity transformation
● Identify similar figures
● Understand the relationships of angles formed by parallel lines and a transversal
● Find unknown angle measures
● Understand the relationship of the interior angles of a triangle.
● Find unknown angle measures.
● Determine whether triangles are similar
● Solve problems involving similar triangles

●Understand the Pythagorean Theorem. Given two side lengths of a right triangle, use the Pythagorean Theorem to find the length of the third side.

●Understand why the Converse of the Pythagorean Theorem is true.

● Apply the Converse of the Pythagorean Theorem to identify right triangles. Use the Converse of the Pythagorean to analyze two-dimensional shapes.

●Apply the Pythagorean Theorem and its converse to solve real-world problems. Apply the Pythagorean Theorem to solve problems that involve three dimensions.

●Apply the Pythagorean Theorem to find the distance between two points on a map or coordinate plane. Find the perimeter of a figure on a coordinate plane. Identify the coordinates of the third vertex of a triangle on the coordinate plane.

● Find the surface areas of cylinders, cones, and spheres
● Recognize the relationship between the volume of a rectangular prism and the volume of a cylinder
● Solve real-world problems involving the volume of a cylinder
● Use the formula for volume of a cylinder to find an unknown measure
● Recognize the relationship between the volume of a cylinder and the volume of a cone
● Find the volume of a cone. Given the circumference of the base, find the volume of a cone
● Recognize the relationship between the volume of a cone and the volume of a sphere
● Find the volume of a sphere. Given the surface area, find the volume of a sphere
● Find the volume of a composite figure

Prerequisite Skills

Draw geometric figures Solve problems involving area, circumference, surface area, and volume Understand percent and proportional relationships Represent polygons on the coordinate plane Draw, construct and describe geometrical figures and their relationships between them Extend knowledge of rational and irrational numbers Apply knowledge of surface area and volume of prisms.
Common Misconceptions
Students often confuse the vocabulary base, length, height, “B” (base area), when moving between 2- and 3-dimensional figures. whatever face the prism is sitting on is the base of the figure. Confuse the terms circumference and area with what they represent in a circle. Students may only express the positive square root in the solution. Students often think cube roots also have 2 solutions. Cube and square root problems are division by 2 or 3. Students mislabel the legs and hypotenuse of a right triangle or incorrectly substitute values to use the Pythagorean Theorem. When using PT in a coordinate plane, students often find the slope of the line instead of the length of the line. Students attempt to memorize Transformation rules instead of learning how to use them. Students may forget to move each point using the same rule

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Transformations • Pythagorean Theorem • Volume	• Geometry (Grade 6 and 7) • Triangles (grade 7) • Solve Problems Using Equations (Grade 7) • Number System (Grade 8)	Math Diagnostic and Intervention System Activities

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks	• Topic Tests (Common Assessments) • Unit 4 Benchmark (Link-It)

• MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	

Resources

Key Resources	Supplemental Resources
Savvas EnVision Accelerated Math 7 Pacing Guide	• IXL • Delta Math • Desmos • Khan Academy

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.

Interdisciplinary Connections

L.KL.7.2.A	Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases.
SL.PE.7.1.A	Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion.
SL.PE.7.1.C	Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed.
SL.PE.7.1.D	Acknowledge new information expressed by others and, when warranted, modify their own views.
6-8.MS-ETS1-3.4	Analyzing and Interpreting Data