

Integers and Rational Numbers

Content Area: **Math**
Course(s):
Time Period: **MP1**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
<p>Students will reinforce their understanding of integers and their opposites. They will use this understanding to further extend their knowledge of integer operations by utilizing number lines and algorithms. Students will also write rational numbers as fractions and decimals within this unit. Students will learn about terminating and repeating decimals as well as be able to recognize a decimal as its specific type.</p> <p>Students build on their understanding of real numbers in order to classify numbers as rational or irrational. Students understand that repeating and terminating decimals can be represented as an equivalent rational number in fraction form. Students will also understand the relationship between squares, square roots, cubes, and cube roots. Students recognize that irrational numbers do not have an exact decimal representation and can be approximated. Students will learn that irrational square roots can be approximated by using perfect squares and decimal approximations. Learning how to estimate very small and very large quantities using powers of 10 is a critical step in learning to compare these quantities. Recognizing how to write very small and very large numbers in scientific notation makes it possible to add, subtract, multiply and divide these numbers using less effort. Students will also understand that any nonzero number raised to the power of zero is equal to one. Students will use patterns to predict and understand integer exponent relationships. Lastly, students will learn the properties of exponents and how to use these properties to add, subtract, multiply, and divide exponential expressions.</p>	<p>Unit 1 builds students procedural skill and fluency related to integers and rational numbers. In this unit students also develop conceptual understanding related to these topics. These are key foundational skills that have real world application. It will be important for students to work fluently with positive and negative real numbers through out their academic careers and in their non-academic lives. Unit 1 also builds students procedural skill and fluency related to real numbers and solving equations. In this unit students also develop conceptual understanding related to these topics. Being able to represent situations using real numbers is a transferable life skill. The skills and understandings developed in this unit will allow students to effectively analyze the world around them. Understanding real numbers and their applications are also foundational skills for upper level mathematics as well as real world situations.</p>

MATH.8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.
MATH.8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
MATH.8.NS.A.3	Understand that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MATH.8.EE.A	Work with radicals and integer exponents
ELA.L.KL.7.2.A	Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases.
MATH.8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions.
MATH.8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number.
MATH.8.EE.A.2.a	Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
MATH.8.EE.A.2.b	Simplify numerical radicals, limiting to square roots (i.e., nonperfect squares).
MATH.8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
MATH.7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
MATH.7.NS.A.1.a	Describe situations in which opposite quantities combine to make 0.
MATH.8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
MATH.7.NS.A.1.b	Understand $p + q$ as the number located a distance $ q $ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
MATH.7.NS.A.1.c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
MATH.8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
MATH.7.NS.A.1.d	Apply properties of operations as strategies to add and subtract rational numbers.
MATH.7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
MATH.8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

MATH.7.NS.A.2.a	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
MATH.7.NS.A.2.b	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
MATH.7.NS.A.2.c	Apply properties of operations as strategies to multiply and divide rational numbers.
MATH.8.EE.C.7.a	Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
MATH.7.NS.A.2.d	Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
MATH.8.EE.C.7.b	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
MATH.7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
ELA.SL.PE.7.1.A	Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion.
ELA.SL.PE.7.1.C	Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
<ul style="list-style-type: none"> • An integer and its opposite are the same distance from 0 on a number line and have a sum of 0. • Rational numbers expressed as fractions can be written in decimal form. • Adding integers requires adding or subtracting their absolute values and understanding the sign of 	<p>Unit Essential Questions</p> <ul style="list-style-type: none"> • How are integers and their opposites related? • How are rational numbers written as decimals? • How do you use what you know about absolute value to add integers? • How is subtracting integers related to adding

the sum.

- Subtracting a number is the same as adding the numbers additive inverse.
- Adding and subtracting integers is related to adding and subtracting other rational numbers.
- The sign of a product is determined by the sign of the factors in a multiplication expression.
- The same properties used to multiply integers also apply when multiplying rational numbers.
- The relationship between multiplication and division can be useful when dividing positive and negative integers.
- Dividing rational numbers is similar to dividing integers. The sign of the quotient depends on the signs of the dividend and divisor.
- Problems involving rational numbers can be solved by making sense of the quantities and their relationships to each other.
- Many real-world problems can be represented with a mathematical model, but that model may not represent a real-world situation exactly.

- Repeating decimals can be represented as an equivalent rational number.
- Every real number is either a rational number or an irrational number.
- Rational and irrational numbers can be compared and ordered using decimal approximations.
- To find the square root of a number, find the factor whose square is equal to that number. To find a cube root, find the factor whose cube is equal to that number.
- Solve equations with squares by taking the square root of each side of the equation. Solve equations with cubes by taking the cube root of each side of the equation.
- The properties of exponents are used to simplify expressions by adding, subtracting, multiplying, or dividing either the base or the exponents.
- Any nonzero number raised to the power of zero is equal to 1. Any non-zero number raised to a negative power is equal to its multiplicative reciprocal.
- An estimate of a very small or very large quantity can be written as a single digit times a power of ten.
- Scientific notation is an efficient way to write very small or very large numbers.
- Many real-world problem situations can be represented with a mathematical model, but that model may not represent a real-world situation.

integers?

- How are adding and subtracting integers related to adding and subtracting other rational numbers?
- How do the signs of factors affect their product?
- How is multiplying rational numbers like multiplying integers?
- How does dividing integers relate to multiplying integers?
- How is dividing rational numbers like dividing integers?
- How do you decide which rational number operations to use to solve problems?
- How do operations with integers relate to the same operations with rational numbers?
- How can you determine the correct operation to use to solve problems?
- How can you write repeating decimals as fractions?
- How is an irrational number different from a rational number?
- How can you compare and order rational and irrational numbers?
- How do you evaluate square roots and cube roots?
- How can you solve equations with squares and cubes?
- How do properties of integer exponents help you to write equivalent expressions?
- What do the Zero Exponent and Negative Exponent Properties mean?
- When would you use a power of 10 to estimate a quantity?
- What is scientific notation and why is it used?
- How does using scientific notation help when computing with very large or very small numbers?
- What are real numbers?
- How are real numbers used to solve problems?

exactly.

- Operating with numbers in scientific notation is an efficient way to add, subtract, multiply, and divide very large or very small numbers.

Instructional Focus

Learning Targets

Students will ...

- Understand how integers and their opposites are related
- Identify rational numbers and write them in decimal form
- Add positive and negative integers
- Model integer addition in real-life applications
- Understand subtraction of integers as adding the additive inverse,
- Use properties of operations to add and subtract rational numbers
- Multiply positive and negative integers
- Apply integer multiplication to real-life applications
- Find the product of rational numbers
- Understand how to divide integers by applying the rules of multiplying integers
- Determine equivalencies among integer quotients
- Understand how the signs of integers in a multiplication sentence relate to the sign in a related division statement
- Decide which operations to use to solve problems
- Use precision when solving problems with rational numbers
- Use mathematical modeling to represent a problem situation and to propose a solution
- Test and verify the appropriateness of their math models
- Locate repeating decimals on a number line
- Write repeating decimals as fractions
- Classify a number as rational or irrational
- Understand the concepts of square roots and perfect squares
- Approximate square roots using perfect squares
- Compare and order rational and irrational numbers
- Evaluate square roots and cube roots to solve problems
- Evaluate perfect squares and perfect cubes
- Solve equations involving perfect squares and perfect cubes
- Solve equations involving perfect squares or cubes
- Solve equations involving imperfect squares or cubes
- Multiply and divide expressions with integer exponents
- Find the power of a power
- Simplify exponential expressions using the Zero Exponent Property and the Negative Exponent Property
- Estimate and compare very large and very small quantities using powers of 10
- Write very large and very small numbers in scientific notation

- Convert scientific notation to standard form
- Use mathematical modeling to represent a problem situation to propose a solution
- Add, subtract, multiply and divide in scientific notation

Prerequisite Skills

- Fluently divide, whole numbers & fractions
- Fluently add, subtract, multiply, and divide decimals
- Identify and represent integers
- Order and compare integers
- Identify and describe absolute values of integers
- Describe quantities with positive and negative numbers

Common Misconceptions

Knowing only a limited number of models for interpreting fractions; for example, not recognizing fractions as locations on a number line or as division calculations. Understanding the symbol “-“ as subtraction and as a negative. Believe that all rational numbers are positive. Trouble using models to illustrate operations or to connect operations on objects to algorithms. Using rules for integers incorrectly. Working with negative fractions and decimals.

Students commonly misinterpret exponents as multiplication instead of repeated multiplication. It may be difficult for students to remember how to apply properties of exponents and that the square root of a number has 2 possible solutions. For Real Number Subgroups, students often think integers are irrational. Also, that the most restrictive subgroup is the only classification of a number.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
<ul style="list-style-type: none"> • Integers • Rational Numbers 	<ul style="list-style-type: none"> • Number Opposites (Grade 6) • Rational Number Operations (Grade 6) 	Math Diagnostic and Intervention System Activities

Assessment

Formative Assessment	Summative Assessment
<ul style="list-style-type: none"> • Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks 	<ul style="list-style-type: none"> • Topic Tests (Common Assessments) • Unit 1 Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
<ul style="list-style-type: none"> • Savvas EnVision Accelerated Math 7 • <u>Pacing Guide</u> 	<ul style="list-style-type: none"> • IXL • Delta Math • Desmos • Khan Academy

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.

Interdisciplinary Connections

ELA.L.KL.7.2.A	Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases.
ELA.SL.PE.7.1.A	Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion.
ELA.SL.PE.7.1.C	Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed.
ELA.SL.PE.7.1.D	Acknowledge new information expressed by others and, when warranted, modify their own views.

There are systematic processes for evaluating solutions with respect to how well they meet the criteria and constraints of a problem.

Analyzing and Interpreting Data

6-8.MS-ETS1-3.4	Analyzing and Interpreting Data
6-8.MS-ETS1-3.4.1	Analyze and interpret data to determine similarities and differences in findings.
6-8.MS-ETS1-3.ETS1.B.1	There are systematic processes for evaluating solutions with respect to how well they meet the criteria and constraints of a problem.
6-8.MS-ETS1-3.ETS1.B.2	Sometimes parts of different solutions can be combined to create a solution that is better than any of its predecessors.