Data and Modeling

Content Area: **Math**
Course(s):
Time Period: **MP1**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
Unit 4 focuses on extending knowledge of functions to include the exponential function. Students learn to identify, write, graph, and transform exponential functions. Students use exponential functions to model real-world situations to make predictions. In this unit students explore sequences and how they can be used to model real-world situations. This unit will also extends students' knowledge of functions to include radical functions. Students identify the key features of the graphs of radical functions. They also learn to transform functions, combine functions, and find inverse functions. Unit 4 will also extend students' knowledge of dot plots, and histograms. Students identify that standard deviation is used to compare a specific value to other values. Students understand how to find joint, marginal, and relative frequencies. Students learn methods to interpret data displays and create inferences based on the data.	In this unit students continue to develop their ability to analyze and model using data. This is a skill that allows students to become informed members of society and consumers. The ability to compare data is quickly becoming an asset in today's world.

NJSLS

MATH.9-12.S.ID.A.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).
MATH.9-12.F.BF.A.1	Write a function that describes a relationship between two quantities.
MATH.9-12.S.ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
MATH.9-12.F.BF.A.1.b	Combine standard function types using arithmetic operations.
MATH.9-12.S.ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

MATH.9-12.S.ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
MATH.9-12.S.ID.B.6	Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
MATH.9-12.S.ID.B.6.a	Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.N.Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MATH.9-12.S.ID.B.6.b	Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.
MATH.9-12.S.ID.B.6.c	Fit a linear function for a scatter plot that suggests a linear association.
MATH.9-12.S.ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
MATH.9-12.S.ID.C.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.
MATH.9-12.S.ID.C.9	Distinguish between correlation and causation.
MATH.9-12.A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MATH.9-12.F.IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.A.SSE.A.1.b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MATH.9-12.F.LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.
MATH.9-12.F.LE.A.1.c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MATH.9-12.F.LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MATH.9-12.F.LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MATH.9-12.A.SSE.B.3.c	Use the properties of exponents to transform expressions for exponential functions.
MATH.9-12.F.LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• An exponential function models that relationship between two quantities that differ by a constant ratio. Exponential functions are modeled using f(x) = abx where a is the initial amount and b is the constant ratio. • An exponential growth function increases by a fixed percent over each interval. An exponential decay function decreases by a fixed percent over each interval. Exponential growth and decay functions can be used to model many real-world situations. • Geometric sequences are number sequences in which each term is related to the next by a common ratio. They can be represented by recursive and explicit formulas. Exponential functions can represent geometric sequences. • Recursive and explicit formulas are used to describe arithmetic sequences. Arithmetic sequences, similar to linear functions, relate quantities that increase at a constant rate. • When data presented in a scatter plot suggests a linear function, a line can be fitted to the data and a linear function can be written to represent the relationship. • When a line is fitted to a set of data, the closer the data points are to the line, the stronger the correlation. A plot pf residuals can be used to determine the line of best fit, which is the trend line that most closely models the data.	• What are the characteristics of exponential functions? • What kinds of situations can be modeled with exponential growth or exponential decay functions? • How are geometric sequences related to exponential functions? • How are arithmetic sequences related to linear functions? • How can you use a scatter plot to describe the relationship between two data sets? • How can you evaluate the goodness of fit of a line of best fit for a paired data set? • How can you determine whether a linear, exponential, or quadratic function best models data? • Do horizontal and vertical translations work in the same way for all types of functions? • What change to a function will result in a vertical or horizontal stretch or compression of its graph? • How can you extend addition, subtraction, and multiplication from numbers to functions? • How can you use inverse functions to help solve problems? • What information about data sets can you get from different data displays? • How can you use the measures of center and spread to compare data sets? • How does the shape of a data set help you understand the data? • Why does the way in which data are spread out matter?

• Many real-world situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly. • Linear, quadratic, and exponential functions are differentiated by their average rates of change over different intervals. A linear function models a relationship between x and y in which the differences between successive y-values are constant. A quadratic function models a relationship in which the second differences, or the difference between the first differences, are constant. An exponential function models a relationship where the ratios of consecutive y-values are constant. values, axis of symmetry, and end behavior - are used to identify and compare functions. • Changes to the input and output of a function in the form f(x - h) + k result in a translation of the graph of the function. Adding a constant k to the output of the function shifts the graph vertically. Subtracting a constant h from the input of the function shifts the graph horizontally. • The graphs of functions are transformed when the input and output are multiplied by varying factors of k. Multiplying the output by a factor of k stretches or compresses the graph vertically. Multiplying the input by a factor of k stretches or compresses the graph horizontally. • Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly. • Functions can be combined in the same way as numbers, expressions, and polynomials. Addition, subtraction, and multiplication can be used to find (f + g)(x), (f - g)(x), and (f ·g)(x) • Dot plots, histograms, and box plots provide different information about the data sets they represent. Dot plots show the frequency of data and clearly show clusters, gaps, and outliers. Histograms show the distribution of values within a data set and shape of the data. Box plots show the center and spread of a distribution. • Measures of center and variability are used to compare data sets displayed in dot plots, box plots, and histograms. Dot plots show how	• How can you use two-way frequency tables to analyze data?

much the data vary. Box plots show the minimum, maximum, and center of the data. Histograms show ranges of data. • When the shape of the data display is symmetric, the mean is equal (or approximately equal) to the median. When the shape of the data display is skewed right or skewed left, the mean and median are not equal. • Standard deviation is a measure of spread, or variability. It indicates by how much the values in a data set deviate from the mean. • Two-way frequency tables show relationships between two sets of categorical data. Entries in the table can be frequency counts or relative frequency. Two-way frequency tables are used to analyze data and make inferences about population. • Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.	

Instructional Focus

Learning Targets
• Sketch graphs showing key features of exponential functions. • Write exponential functions using tables and graphs. • Compare linear and exponential functions. • Construct exponential growth and decay functions given a description of a relationship. • Recognize if a situation can be modeled with exponential growth or exponential decay, and interpret the parameters of the model in context. • Find explicit and recursive formulas for geometric sequences. • Translate between recursive and explicit formulas for geometric sequences. • Construct exponential functions to represent geometric sequences. • Write arithmetic and geometric sequences both recursively and with an explicit formula. • Use explicit formulas and recursive formulas to model real-world situations. • Fit a function to linear data shown in a scatter plot and use fitted functions to solve problems in the context of the data. • Interpret the slope of a trend line within the context of data. • Compute and interpret the correlation coefficient for linear data. • Plot and analyze residuals to assess the fit of a function.

- Distinguish between correlation and causation.
- Compare a model to a data set by analyzing and evaluating residuals.
- Use mathematical modeling to represent a problem situation and to propose a solution.
- Test and verify the appropriateness of their math models.
- Explain why the results from their mathematical models might not align exactly with the problem situation.
- Determine which model - linear, exponential, or quadratic - best fits a set of data.
- Use fitted function to solve problems in the context of data.
- Graph translations of absolute value, exponential, quadratic, and radical functions.
- Determine how combining translations affects the key features of the graph of a function.
- Identify the effect on the graph of a function of multiplying the output by -1.
- Identify the effect on the graph of a function of replacing f(x) by kf(x) or f(kx) for specific values of k.
- Use mathematical modeling to represent a problem situation and to propose a solution.
- Test and verify the appropriateness of their math models.
- Represent data using dot plots, box plots, and histograms.
- Interpret the data displayed in dot plots, box plots, and histograms within the context of the data that is represents.
- Use measures of center to interpret and compare data sets displayed in dot plots, box plots, and histograms.
- Explain and account for the effect of outliers on measures of center and variability.
- Use measures of variability, such as the MAD and IQR, to interpret and compare data sets.
- Interpret and compare difference in the shape, center, and spread of data of different data sets.
- Determine the relationship between the mean and median of a data set when the shape of the data is evenly spread, skewed right, or skewed left.
- Interpret the differences in the variability or spread in the context of a data set.
- Calculate the standard deviation of a data set and use it to compare and interpret data sets.
- Organize and summarize categorical data by creating two-way frequency tables.
- Calculate and interpret joint and marginal frequencies, joint and marginal relative frequencies, and conditional relative frequencies, and use them to make inferences about a population.
- Use mathematical modeling to represent a problem situation and to propose a solution.
- Test and verify the appropriateness of the math model.
- Explain why the results from the mathematical model might not align exactly with the problem situation.

Prerequisite Skills

- properties of quadratics functions and graphs
- zeros of an equation, polynomial function
- finding zeros, graphing, factoring
- rational and irrational numbers
- graphing linear functions, coordinate plane
- graphing a quadratic in standard form, domain and range.

- graphing a quadratic in standard form, maximum (vertex)
- graphing a linear function, slope, intercepts
- Interpreting functions in any given form.
- graphing quadratics in vertex form
- rate of change, slope
- rate of change, slope, quadratics and linear
- rate of change
- vertex form of quadratics, completing the square
- factoring quadratics
- quadratics, FOIL, order of operations
- Linear models
- exponential functions
- Area, creating functions, ratios
- Transformations using function notation.
- Comparing tables and functions, plugging in information
- Analyzing graphs and how a function increases or decreases over an interval.
- exponential functions, growth rate
- creating a histogram, mean, data analysis
- box plots and data analysis
- dot plots and data analysis
- surveys, data analysis
- distribution and data analysis
- Scatter plots, graphing lines, using linear functions
- scatter plots and determining the best model
- scatter plots, line of best fit
- linear regression

• data analysis • correlation
Common Misconceptions
• Creating scatter plots and recognizing linear trends in data. • Regression techniques to describe approximately linear functional relationships between quantities. • Graphical representation and knowledge of the context to make judgments about the appropriateness of linear models.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Understanding Exponential Functions • Model with Exponential Functions • Transformations of Functions • Analyzing Key Features in Functions • Compare Data • Analyze Data	• Solve Linear Equations (grade 8) • Graphing Linear Equations (Grade 8/ Algebra () • Translations of Functions (Algebra I) • Average Rate of Change (Grade 8/ Algebra I) • Dot Plots, Box Plots, and Histograms (Grade 8) • Calculate Measures of Center (Grade 8)	• IXL • Math Diagnostic and Intervention System Activities

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections	• Topic Tests • Unit Benchmark (Link-It)

• Performance Tasks	

Resources

Key Resources	Supplemental Resources
• Savvas EnVison Algebra I • Pacing Guide	• IXL • Delta Math • Desmos • Khan Academy

Career Readiness, Life Literacies, and Key Skills

PFL.9.1.12.PB.2	Prioritize financial decisions by considering alternatives and possible consequences.
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.
TECH.9.4.12.IML.1	Compare search browsers and recognize features that allow for filtering of information.
TECH.9.4.12.IML.7	Develop an argument to support a claim regarding a current workplace or societal/ethical issue such as climate change (e.g., NJSLSA.W1, 7.1.AL.PRSNT.4).

Interdisciplinary Connections

ELA.W.AW.9–10.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient textual and non-textual evidence.
ELA.W.AW.9–10.1.B	Develop claim(s) and counterclaims using sound reasoning, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate manner that anticipates the audience's knowledge level and concerns.
9-12.HS-PS1-7	Use mathematical representations to support the claim that atoms, and therefore mass, are conserved during a chemical reaction.
9-12.HS-PS2-2	Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.
9-12.HS-PS2-4	Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.

9-12.HS-PS2-1.4.1	Analyze data using tools, technologies, and/or models (e.g., computational, mathematical) in order to make valid and reliable scientific claims or determine an optimal design solution.
9-12.HS-PS2-4.5	Mathematical and computational thinking at the 9–12 level builds on K–8 and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.
9-12.HS-PS1-7.5	Mathematical and computational thinking at the 9–12 level builds on K–8 and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.