Polynomials and Quadratic Functions

Content Area: **Math**
Course(s):
Time Period: **MP1**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
Unit 3 focuses on extending polynomials. Students identify the parts and factors of polynomials. Students understand how to factor trinomials using the greatest common factor, binomial factors, and special patterns. Students learn methods to add, subtract, and multiply polynomials. This unit also focuses on extending students' previous understanding of functions to include quadratic functions: graphing them, using them to model real-world situations, and comparing them to liniear and exponential functions. Students also expand their knowledge of quadratic functions. Students learn to solve quadratic equations using tables, graphs, and factoring. Students also solve quadratic equations using square roots, completing the square, and the quadratic formula. Students learn different methods, such as graphing, elimination, and substitution, for solving linear-quadratic systems.	In unit 3 students utilize and apply concepts and skills from previous units. Students will demonstrate their reasoning abilities and their growth as independent problems solvers. This unit works to develop students into efficient and effective problem solvers.

NJSLS

MATH.9-12.F.BF.A.1	Write a function that describes a relationship between two quantities.
MATH.9-12.A.APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MATH.9-12.A.APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MATH.9-12.S.ID.B.6.a	Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.S.ID.B.6.b	Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.
MATH.9-12.A.CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
MATH.9-12.A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.A.REI.B.4.a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
MATH.9-12.A.REI.B.4.b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
MATH.9-12.F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MATH.9-12.F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MATH.9-12.F.IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MATH.9-12.F.IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MATH.9-12.A.REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
MATH.9-12.F.IF.C.8.a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MATH.9-12.F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MATH.9-12.A.SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MATH.9-12.A.SSE.A.1.a	Interpret parts of an expression, such as terms, factors, and coefficients.
MATH.9-12.A.SSE.A.1.b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.
MATH.9-12.A.SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MATH.9-12.A.SSE.B.3.a	Factor a quadratic expression to reveal the zeros of the function it defines.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions
• A polynomial is a monomial or the sum or differences of two or more monomials (terms). Polynomials can be added or subtracted by combining like terms. Polynomials are closed under addition or subtraction, similar to integers. • Polynomials can be multiplied by applying the Distributive Property or by using a table. They form a system similar to integers. Therefore, polynomials are closed under multiplication. • The product of the square of a binomial in the form $(a + b)^2$ is always the square of the first term, plus twice the product of the first and last terms, plus the square of the last term. The product of a sum and a difference of two binomials in the form $(a + b)(a - b)$ is always the difference of two squares, • The greatest common factor of the terms of a polynomial is the greatest common factor of the coefficients and the variables or variables, using the number of instances of the variable that are common to each term. • When a trinomial is in the form $x^2 + bx + c$, the factors are found by identifying a pair of integers factors of c that have a sum of b and then using the factors to write binomials that have	• How does adding or subtracting polynomials compare to adding or subtracting integers? • How does multiplying polynomials compare to multiplying integers? • What patterns are there in the product of the square of a binomial and the product of a sum and a difference? • How is factoring a polynomial similar to factoring integers? • How does recognizing patterns in the signs of the terms help you factor polynomials? • How is factoring a quadratic trinomial when $a \neq 1$ similar to factoring a quadratic trinomial when $a=1$? • What special patterns are helpful when factoring a perfect-square trinomial and the difference of two squares? • What is the quadratic parent function and how can you recognize the key features of its graph? • How can the vertex form of a quadratic function help you sketch the graph of a function? • How is the standard form of a quadratic function different from the vertex form? • What kinds of real-world situations can be modeled by quadratic functions? • How can you determine whether a linear, exponential, or quadratic function best models data?

a product equal to the trinomial. • Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly. • A quadratic trinomial in the form $ax^2 + bx + c$ when $a \neq 1$ can either be factored by grouping or by substitution. • When a trinomial has the pattern $a^2 + 2ab + b^2$, then it can be factored as $(a + b)^2$ or $(a - b)^2$ respectively. If a binomial has the pattern $a^2 - b^2$, then it can be factored as $(a + b)(a - b)$. • A quadratic function is a polynomial function in one or more variables in which the highest degree term is of the second degree. The graph of a quadratic function f(x) = ax2 is a parabola. The value of the leading coefficient a determines both the width of the parabola and the direction the parabola opens (upward or downward). • The structure of a quadratic function in vertex form reveals the vertex and axis of symmetry of the graph it represents. • The standard form of a quadratic function is f(x) = ax2 + bx + c, where c is the y-coordinate of the y-intercept and the axis of symmetry is the line x = -b/(2a). • A quadratic function can be used to model area and vertical motion problems. These models can be written in the same form as the quadratic function f(x) = ax2 + bx + c using key features to interpret and understand the situation. • Many real-world situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly. • Linear, quadratic, and exponential functions are differentiated by their average rates of change over different intervals. A linear function models a relationship between x and y in which the	• How can graphs and tables help you solve quadratic equations? • How does factoring help you solve quadratic equations? • How does rewriting radicals in different forms help you communicate your answers? • How can square roots be used to solve quadratic equations? • How is the technique of completing the square helpful for analyzing quadratic functions? • When should you use the quadratic formula to solve equations? • How is solving linear-quadratic systems of equations similar to and different from solving systems of linear equations?

differences
between successive y-values are constant. A quadratic function models a relationship in which the
second differences, or the difference between the first differences, are constant. An exponential
function models a relationship where the ratios of consecutive y-values are constant.

Critical Knowledge and Skills

- A quadratic equation is an equation of the second degree. It can have 0, 1, or 2 solutions. The x-intercepts of a graph and the zeros in a table can be used to identify the real solutions.
- In the standard form of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, the factors of the equation
determine the solutions. The Zero-Product Property states that for all real numbers a and b, if ab = 0,
then either a= 0 or b = 0.
- Properties of exponents are used to rewrite radical expressions in different forms. A radical expression is written in the simplest form when there is no perfect square factors other than 1 in the radicand.
- When a quadratic equation is in the form $ax^2 + bx + c$, it can be solved by isolating the ax^2 term,
simplifying to remove the coefficients, and then taking the square root of each side of the equation.
- To complete the square, add the square of half of the coefficient of x to each side of the equation.
Completing the square is useful when solving quadratic equations that are not factorable.
- The quadratic formula can be used to solve every quadratic equation and is particularly useful for those that cannot be easily factored. The discriminant of the quadratic formula indicates the number of solutions of the equation.
- Many real-world problem situations can be represented with a mathematical model, but that model
might not represent the real-world situation exactly.
- For any system of two equations in two

variables, the solution consists of the ordered pairs that satisfy both equations.	

Instructional Focus

Learning Targets

- Identify the parts of a polynomial.
- Classify polynomials by number of terms and by degree.
- Write a polynomial in standard form.
- Add or subtract two polynomials.
- Use the Distributive Property with polynomials, recognizing that polynomials are closed under multiplication.
- Multiply polynomials using a table and an area model.
- Determine the square of a binomial.
- Find the product of a sum and difference of two squares.
- Solve real-world problems involving the square of a binomial.
- Find the greatest common factor of the terms of a polynomial.
- Use the structure of a polynomial to rewrite it in factored form.
- Factor polynomials that represent real-world problems.
- Factor a trinomial in the form x2+bx+x by finding two binomial factors whose product is equal to the trinomial.
- Identify and use patterns in the signs of the coefficients of the terms of a trinomial expression.
- Use mathematical modeling to represent a problem situation and to propose a solution.
- Test and verify the appropriateness of their math models.
- Explain why the results from their mathematical models might not align exactly with the problem situation.
- Identify the common factor of the coefficients in the terms of a trinomial expression when a does not equal 1.
- Write a quadratic trinomial as a product of two binomial factors.
- Identify and factor a trinomial that is a perfect square or a binomial that is a difference of two squares.
- Factor special cases of polynomials within the context of real-world problems.
- Identify key features of the graph of a quadratic function using graphs, tables, and equations.
- Explain the effect of the value of a on the quadratic parent function.
- Identify key features of the graph of quadratic functions written in vertex form.
- Graph quadratic functions in vertex form.
- Graph quadratic functions in standard form and show intercepts, maxima, and minima.
- Determine how the values of a, b, and c affect the graph of $f(x)=ax^2+bx+c$.
- Identify key features of parabolas.
- Compare properties of quadratic functions presented in different forms (algebraically, in a table, graphically).
- Use quadratic functions fitted to data to model real-world situations.

- Use the vertical motion model to write an equation.
- Compare a model to a data set by analyzing and evaluating residuals.
- Use mathematical modeling to represent a problem situation and to propose a solution.
- Test and verify the appropriateness of their math models.
- Explain why the results from their mathematical models might not align exactly with the problem situation.
- Determine which model - linear, exponential, or quadratic - best fits a set of data.
- Use fitted function to solve problems in the context of data.
- Use a graph to identify the x-intercepts as solutions of a quadratic equation.
- Use a graphing calculator to make a table of values to approximate or solve a quadratic equation.
- Use the Zero-Product Property and factoring to find the solutions of a quadratic equation.
- Apply factoring to solve real-world problems.
- Use the zeros of a quadratic equation to sketch a graph.
- Write the factored form of a quadratic function from a graph.
- Use properties of exponents to rewrite radical expressions.
- Multiply radical expressions.
- Write a radical expression to model or represent a real-world problem.
- Solve quadratic equations by finding square roots.
- Determine reasonable solutions for real-world problems.
- Solve a quadratic trinomial by completing the square to transform a quadratic equation into a perfect square trinomial.
- Use completing the square to write a quadratic equation in vertex form.
- Derive the quadratic formula by completing the square.
- Solve quadratic equations in one variable by using the quadratic formula.
- Use the discriminant to determine the number and type of solutions to a quadratic equation.
- Use mathematical modeling to represent a problem situation.□ Test and verify the appropriateness of their math models.
- Explain why the results might not exactly match the problem situation.
- Describe a linear-quadratic system of equations.
- Solve a linear-quadratic system of equations by graphing, elimination, or substitution.

Prerequisite Skills

- Graphing on a coordinate plane, converting meters to kilometers, graphing, converting seconds to hours
- Volume
- converting
- absolute value, parts of an expression re: negative and positive signs
- percentage growth
- surface area (circle and cylinder)
- solving equations and justifying steps
- solving for another variable with no numbers or coefficients and justifying your answer

- solving equations and inequalities
- solving inequalities
- writing and solving an equation
- writing, solving, comparing through mathematical explanation
- equations – solving for another variable
- graphing equations
- graphing inequalities
- writing inequalities

Common Misconceptions

- In solving systems graphically, students may mistake a system with infinite solutions for a system with no solutions.
- When solving by substitution, students may substitute into the same equation they used to isolate the variable.
- Students may attempt to solve a system of linear equations using a less efficient method.
- When students are graphing a system of inequalities, they might shade the wrong side of the line.
- When students are graphing a system of inequalities, they might confuse whether the boundary line should be solid or dashed.
- When students are graphing a system of inequalities, they might not know what to do if a point falls ON one of the lines in the system.
- When students are graphing a system of inequalities whose graph forms a set of parallel lines, they might assume that the system has no solutions.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• Addition, Subtraction, and Multiplication • Products • Factors of Polynomials	• Factoring (Grade 8) • Operations and Properties (Grade 8) • Polynomials (Grade 8)	• IXL • Math Diagnostic and Intervention System

• Quadratic Functions • Modeling with Quadratic Functions • Solving Quadratic Equations • Solving Linear-Quadratic Systems of Equations	• Multiplying Exponents (Grade 8) • Analyzing Graphs of Functions (Grade 8) • Using Functions to Model Relationships (Grade 8/Algebra I) • Solving Linear Equations (Grade 8/Algebra I) • Graphing Quadratic Functions (Algebra I)	Activities

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests • Unit Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
• Savvas EnVision Algebra I • Pacing Guide	• IXL • Delta Math • Desmos • Khan Academy

Career Readiness, Life Literacies, and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.

CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Interdisciplinary Connections

ELA.K-12.L.KL	Knowledge of Language: By the end of grade 12, apply knowledge of language and command of vocabulary to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.
ELA.RI.MF.9–10.6	Analyze, integrate, and evaluate multiple interpretations (e.g., charts, graphs, diagrams, videos) of a single text or text/s presented in different formats (visually, quantitatively) as well as in words in order to address a question or solve a problem.
ELA.W.IW.9–10.2.D	Use precise language and domain-specific vocabulary to manage the complexity of the topic.
ELA.SL.PE.9–10.1	Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 9–10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
ELA.SL.PE.9–10.1.B	Collaborate with peers to set rules for discussions (e.g., informal consensus, taking votes on key issues, presentation of alternate views); develop clear goals and assessment criteria (e.g., student developed rubric) and assign individual roles as needed.
ELA.SL.PE.9–10.1.C	Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions.
ELA.SL.PE.9–10.1.D	Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.
ELA.SL.ES.9–10.3	Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, identifying any false reasoning or distorted evidence.
ELA.SL.PI.9–10.4	Present information, findings, and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
9-12.HS-PS1-5.1.1	students observe patterns in systems at different scales and cite patterns as empirical evidence for causality in supporting their explanations of phenomena. They recognize classifications or explanations used at one scale may not be useful or need revision using a different scale; thus requiring improved investigations and experiments. They use mathematical representations to identify certain patterns and analyze patterns of performance in order to reengineer and improve a designed system.
9-12.HS-PS3-1.PS3.B.3	Mathematical expressions, which quantify how the stored energy in a system depends on its configuration (e.g. relative positions of charged particles, compression of a spring) and how kinetic energy depends on mass and speed, allow the concept of conservation of energy to be used to predict and describe system behavior.